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Dynamical Pattern Recognition of Univariate Time Series from the Structural Stability Via Deterministic Learning

Hao Li[1,a,*]

[1]*School of Electrical Engineering and Automation, Henan Polytechnic University, Jiaozuo, 454003, China*
[a]*lh1739246516@outlook.com*
**Corresponding author*

Abstract: *In order to achieve dynamical pattern recognition of univariate time series from the perspective of structural stability, this paper proposes a method based on Extended State Observer (ESO) and deterministic learning for addressing the issue of recognizing the structural stability of the topological structure of dynamical patterns in univariate time series. The ESO is capable of reconstructing the system states and their unknown dynamics during the observation phase, while the deterministic learning effectively resolves the challenges associated with obtaining and modeling the derivative information of the dynamics. During the learning phase, the ESO is employed to recover state trajectories from sampled output signals. These trajectories utilize the regressed trajectories as inputs, satisfying the partially persistent excitation (PE) condition, thereby accurately approximating the unknown dynamics through the estimated regressed trajectories. Subsequently, we establish a novel recognition error system that approximates the first-order derivatives of the system dynamics using a finite difference method, thus avoiding the need for re-modeling the test patterns and enabling the identification of dynamic behaviors from the topological structural perspective. Finally, we theoretically demonstrate that the residuals of the recognition error system reflect the structural stability differences in system dynamics between the training and test patterns, and simulation further substantiate the effectiveness and accuracy of the proposed method.*

Keywords: *Extended State Observer; Structural Stability; Univariate Time Series; Dynamical Pattern Recognition; Deterministic Learning*

1. Introduction

Dynamical pattern recognition, as a complex and challenging field, has garnered widespread attention in the fields of biomedicine, control, and human activity recognition[1]. Compared with static patterns represented by vectors, dynamical patterns contain a large amount of information distributed over time and can usually be represented as time series. Due to various limitations in practical applications, dynamical patterns are often obtained only for univariate time series[2]. Therefore, it is necessary to carry out dynamical pattern recognition tasks from the perspective of univariate time series classification/recognition. Recently, for the learning problem of dynamical systems, a deterministic learning theory was proposed to ensure partially PE conditions during the recognition process and to enable locally accurate RBFN modeling of nonlinear dynamics[3]. The theory is further applied to dynamical pattern recognition by proposing a unified recognition framework based on dynamic differences.

The contributions of this paper are: 1) The problem of observing unknown dynamical patterns is solved by estimating the system state and unknown dynamics using ESO and storing the training patterns as time-invariant information using deterministic learning, which clearly demonstrates a locally accurate NN approximation along the periodic pattern trajectory. 2) A recognition error system based on structural stability is constructed, where the dynamics partial derivatives are approximated using first-order differences during the recognition phase, avoiding the need for learning modeling of the test patterns. 3) It is shown that these estimator state errors can be used to describe the structural stability differences between dynamical systems related to the test and training time series, allowing for rapid generation of recognition results based on the average L_1 norm of the output errors.

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2. Preliminary knowledge and problem formulation

2.1 Preliminary knowledge

An RBFN with an optimal weight vector $W^{\star} \in R^{N}$ and a sufficiently large number of neural nodes can be used to approximate an arbitrary continuous function $f(x): R^{n} \rightarrow R$ within any accuracy range $\varepsilon^{\star} > 0$. That is, $f(x) = W^{*T}S(x) + \varepsilon(x), \forall x \in \Omega_{x}$. $|\varepsilon(x)| < \varepsilon^{\star}$ denotes the ideal approximation error and $S(x) = [s_{1}(x), \dots, s_{N}(x)]^{T}$, where $s_{i}(x): R^{n} \rightarrow R, i = 1, \dots, N$ is the radial basis function. The common basis function can be chosen as the Gaussian function $s_{i}(x) = \exp[-((x-\xi_{i})^{T}(x-\xi_{i}))/\eta^{2}]$, where $\xi_{i} \in R^{n}$ and η are the center and the width of the receptive field, respectively. Inspired by the localization property of the RBFN, $f(x)$ can be modeled by a local RBFN that consists of a finite number of neural nodes placed in a local region along an arbitrary trajectory $x(t)$, i.e., $f(x) = W_{\zeta}^{*T}S_{\zeta}(x) + \varepsilon_{\zeta}(x) \forall x \in \Omega_{x}$, where $\varepsilon_{\zeta} = O(\varepsilon) = O(\varepsilon^{\star})$.

Lemma 1[4]. For almost any recurrent trajectory $x(k)$ that stays in Ω_{x}, and a local RBFN (whose nodes are regularly placed to cover Ω_{x}), $S_{\zeta}(x)$ satisfies the partial PE condition, i.e., for $c_{1} > 0$, $c_{2} > 0$, and δ_{k},

$$c_{1}I \leq \sum_{\tau=k}^{k+\delta_{k}-1} S_{\zeta}\big(X(\tau)\big)S_{\zeta}^{T}\big(X(\tau)\big) \leq c_{2}I, \forall k \geq 0 \tag{1}$$

Where $I \in R^{N_{\zeta} \times N_{\zeta}}$ Is the unit matrix.

Lemma 2 Consider the nonlinear dynamical systems f and f', if the state trajectory of any one system can be obtained through continuous transformation or homeomorphism mapping of the trajectory of the other system, then the two systems are topologically equivalent.

Lemma 3 Andronov's structural stability[5]: A system defined in a region $\Omega \subset R^{n}$ Is said to be structurally stable in a region $\Omega_{0} \subset \Omega$ if for any sufficiently C^{1}-close in Ω of another system there exists a region $U, V \subset \Omega, \Omega_{0} \subset U$, such that the system is topologically equivalent in U to the other system in V,

$$\| f - f' \|_{C^{1}} = \sup_{x \in \Omega} \left\{ \| f - f' \|_{C^{0}} + \| \frac{\partial f}{\partial x} - \frac{\partial f'}{\partial x} \|_{C^{0}} \right\}, \tag{2}$$

Where $\|\cdot\|_{C^{0}}$ denotes a vector norm in R^{n}.

2.2 Problem formulation

For an nth order nonlinear system with uncertain disturbances, it can be expressed in the following form:

$$y^{(n)} = f\big(y, y^{(1)}, \dots, y^{(n-1)}, w, d\big) + bu \tag{3}$$

Where w is the unmodeled dynamics, d is an external perturbation, and u is a known input. The nonlinear function $f(\cdot)$ is bounded but not measurable and represents uncertainty, i.e., the "total perturbation".

Let $y = x_{1}, \dots, y^{(n-1)} = x_{n}, f(x) = x_{n+1}, \dot{f}(x) = h$. The expanded nonlinear system is written in the form of an equation of state:

$$\begin{cases} \dot{x} = Ax + Bu + Eh \\ y = Cx \end{cases} \tag{4}$$

Where $x = [x_{1}, \dots, x_{n+1}] \in R^{n+1}$ And $y \in R$ denote the state inputs and measurable outputs, respectively. $A = [\begin{smallmatrix} 0 & I_{n \times n} \\ 0 & 0 \end{smallmatrix}] \in R^{(n+1)\times(n+1)}, B = [0, \dots, b, 0]^{T} \in R^{n+1}, C = [1,0,\dots,0] \in \mathrm{R}^{n+1}, E = [1,0,\dots,0]^{T} \in \mathrm{R}^{n+1}$ is the unit matrix. The nonlinear function $f(\cdot): R^{n} \rightarrow R$ is the dynamics in the dynamical pattern φ, which is usually assumed to be bounded but not measurable. The mode φ represents the state trajectory from x_{0}.

Realistically accurate system matrices A, B and C are often difficult to obtain, and the ESO requires minimal known information about the system, allowing it to be regarded as a model-free method. The observer has great dilated state tracking performance for a range of uncertain systems and is generalized

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over a range[6]. Take a second-order system as an example. After adding disturbance observation states, it is expanded into a third-order system. Construct a Luenberger observer for such a system after coordinate transformation, have

$$\begin{cases} \dot{\hat{x}} = A\hat{x} + Bu + L(y - \hat{y}) \\ y = C\hat{x} \end{cases} \tag{5}$$

Where $\hat{x}$ represents the state estimate, y is the output, $\hat{y}$ is the estimated output, and $L(y - \hat{y})$ is the feedback term used to adjust the state estimate. The term $y - \hat{y}$ represents the observation error. Write it in component form,

$$\begin{cases} \dot{\hat{x}}_1 = \hat{x}_2 + \beta_1(y - \hat{y}) \\ \dot{\hat{x}}_2 = \hat{x}_3 + bu + \beta_2(y - \hat{y}) \\ \dot{\hat{x}}_3 = \beta_3(y - \hat{y}) \\ \hat{y} = C\hat{x} = x_1 \end{cases} \tag{6}$$

Where β denotes the observer parameters. Let $e = x - \hat{x}$. By subtracting the observer (5) from the original system (4), we have

$$\dot{e} = A_e e + Eh \tag{7}$$

$$A_e = A - LC = \begin{bmatrix} -\beta_1 & 1 & 0 \\ -\beta_2 & 0 & 1 \\ -\beta_3 & 0 & 0 \end{bmatrix} \tag{8}$$

Where the characteristic polynomial of the matrix A_e is $f(\lambda) = \lambda^3 + \beta_1\lambda^2 + \beta_2\lambda + \beta_3$.

If we want to ensure that the observer error converges, all the eigenvalues should be located in the left half-plane. Assume the ideal characteristic polynomial is $f^*(\lambda) = (\lambda + w_o)^3$, this satisfies the requirement for all eigenvalues to be negative, and there is only one adjustable parameter. The w_o here can be related to the physical concept of bandwidth, drawing on the idea from Loop Shaping of balancing performance and noise. When disturbances are small, w_o can be taken larger. By expanding the cubic polynomial, we can obtain the relationship between β and bandwidth w_o: $\beta_1 = 3w_o, \beta_2 = 3w_o^2, \beta_3 = w_o^3$.

In this paper, we assume that the output sampling data is available. The univariate time series data $y^m(k)$ (i.e., $y^m(kT)$) sampled from $y^m(t)$ can be characterized using Eulerian system if we specify the sampling period $T > 0$, $m \in \{1, \dots, M\}$ and $M \in N_+$ to denote the different dynamical patterns:

$$\begin{cases} x^m(k+1) = x^m(k) + TAx^m(k) + TBu^m + TEh^m \\ y^m(k) = Cx^m(k) \end{cases} \tag{9}$$

Where $x^m(k_0) \in \omega_X$ and ω_X is a tight set. The training set $\varphi = \{y^m \mid m = 1, \dots, M\}$ contains M univariate time series with different dynamical patterns φ^m, which are generated by the dynamical system (9) with corresponding dynamics f^m. Similarly, the test univariate time series f^r can be described by the following dynamical system:

$$\begin{cases} x^r(k+1) = x^r(k) + TAx^r(k) + TBu^r + TEh^r \\ y^r(k) = Cx^r(k) \end{cases} \tag{10}$$

Assumption 1: the system state x^m is assumed to be uniformly bounded, i.e., $x^m(k) \in \Omega_X \subset R^n, \forall k \geq k_0$. Furthermore, the system trajectory starting from $x^m(0)$ is a cyclic motion, or a cyclic system trajectory.

For this expanded system (9), the following state observer can be built:

$$\begin{cases} e_1 = \hat{x}_1^m - y^m \\ \dot{\hat{x}}_1^m = \hat{x}_2^m - \beta_1\hat{x}_1 \\ \dot{\hat{x}}_2^m = \hat{x}_3^m - \beta_2|e_1|^{\frac{1}{2}}sign(e_1) \\ \vdots \\ \dot{\hat{x}}_n^m = \hat{x}_{n+1}^m - \beta_n|e_1|^{\frac{1}{2^{n-1}}}sign(e_1) + u^m \\ \dot{\hat{x}}_{n+1}^m = -\beta_{n+1}|e_1|^{\frac{1}{2^n}}sign(e_1) \end{cases} \tag{11}$$

Written in Euler discretization form as,

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$$\begin{cases} e_1(k) = \hat{x}_1^m(k) - y^m(k) \\ \hat{x}_1^m(k+1) = \hat{x}_1^m(k) + T(\hat{x}_2^m(k) - \beta_1 \hat{x}_1(k)) \\ \hat{x}_2^m(k+1) = \hat{x}_2^m(k) + T(\hat{x}_3^m(k) - \beta_2 |e_1|^{\frac{1}{2}} sign(e_1(k))) \\ \vdots \\ \hat{x}_n^m(k+1) = \hat{x}_n^m(k) + T(\hat{x}_{n+1}^m - \beta_n |e_1|^{\frac{1}{2^{n-1}}} sign(e_1(k))) \\ \hat{x}_{n+1}^m(k+1) = \hat{x}_{n+1}^m(k) - T\beta_{n+1} |e_1|^{\frac{1}{2^n}} sign(e_1(k)) \end{cases} \tag{12}$$

Where T represents the sampling period, and $\beta = [\beta_1, \ldots, \beta_{n+1}]$ denotes the observer parameters. The term $e_1(k)$ indicates the tracking error. The function $\mathrm{sign}(x)$ is a sign function that determines the sign of the input value based on the tracking error $e_1(k)$. The test pattern employs the same observer. With a proper choice of the parameter β, this system is able to estimate well the state variable $x^m(k)$ of the system under test and the real−time action of the dilated state $x_{n+1}^m(k) = f^m(x^m(k))$. $x_i^m(k) \rightarrow \hat{x}_i^m(k)$, $f^m(x^m(k)) \rightarrow \hat{x}_{n+1}^m(k)$.

3. Dynamic Modeling and Similarity Measurement

3.1 Dynamical pattern recognition from state observation

Consider the dynamic model $\varphi^m (m = 1, \ldots, M)$ of a univariate time series. Currently, observer-based deterministic learning methods can only observe the state trajectories $x^m(k)$ and $x^r(k)$ of the system. The representation of the unknown nonlinear dynamics $f^m(x^m(k))$ and $f^r(x^r(k))$ becomes inevitable if we want to realize the identification of dynamic systems from the perspective of structural stability. Existing deterministic learning methods based on structural stability, where the test pattern $f^r(x^r(k))$ and the training pattern $f^m(x^m(k))$ take the same representation of $f^r(x^r(k))$ by ${\bar{W}^r}^T S(\cdot)$, require the learning of the test pattern, which increases the complexity of the algorithm. The deterministic learning method based on ESO and structural stability proposed in this paper can be realized in three stages for the recognition task of dynamical systems, as shown in Fig. 1. First, the observation phase realizes the observation of the state trajectories $x^m(k)$ and $x^r(k)$ with the unknown nonlinear dynamics $f^m(x^m(k))$ and $f^r(x^r(k))$ by ESO. Second, the learning phase inputs the observed trajectories $x^m(k)$ into the RBFN recognizer, which is trained to obtain the time-invariant weight vector $\bar{W}^m$ to construct the dynamical system library. Finally, the identification stage constructs the structural stability identification error system to obtain the residuals based on the state trajectories $x^r(k)$ of the test pattern, the unknown nonlinear dynamics $f^r(x^r(k))$, and the weight vector $\bar{W}^m$. The algorithm flowchart is depicted in Fig. 1.

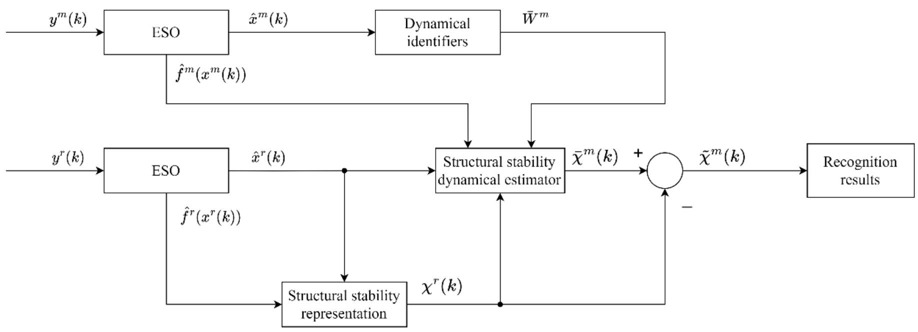

Figure 1 Algorithm flowchart

First, we reconstruct the underlying dynamics $f^m(x^m(k))$ of the univariate time series using the dilated state observer. Second, the following dynamical identifiers are constructed through deterministic learning:

$$\hat{\chi}^m(k+1) = \hat{x}_n^m(k) - T\alpha\big(\hat{\chi}^m(k) - \hat{x}_n^m(k)\big) + T{\hat{W}^m}^T(k) S\big(\hat{x}^m(k)\big) \tag{13}$$

Where $\hat{\chi}^m$ is the state of the dynamical rbf network. $\hat{x}^m = [\hat{x}_1^m, \ldots, \hat{x}_n^m]$ is the state estimate of the system (12) for (9). The gain $\alpha > 0$ is a design constant.

The adaptive learning law satisfies the following form:

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$$\widehat{W}^m(k+1) = \widehat{W}^m(k) - T\gamma S(\hat{x}^m(k))(\hat{\chi}^m(k+1) - \hat{x}_n^m(k+1)) \tag{14}$$

Where $\gamma = \gamma^t > 0$.

Consider the univariate time series $f^m(x^m(k))$ in y^m, the dilated state observer (12), the rbfn-based identifier (13), and the learning law (14), where $\widehat{W}^m(0) = 0$. under assumption 1, the locally accurate nn approximation of the unknown dynamics $f^m(\cdot)$ of the time series y^m as $x_n^m(k) \rightarrow \hat{x}_n^m(k)$ can be achieved by the constant rbfn $\bar{W}_n^{m^T} S_n(\cdot)$ to realize the

$$f^m(\cdot) = \bar{W}^{m^T} S(\cdot) + \varepsilon_1 \tag{15}$$

Where $\bar{W}^m = \frac{1}{k_b - k_a + 1} \sum_{k=k_a}^{k_b} \widehat{W}^m(k)$, $[k_a, k_b]$ denotes the time period following the transient process, and $\varepsilon_1 = 0(\varepsilon^\star)$ ($\varepsilon^\star$ is a small constant) is the approximation error along the estimated trajectory $\hat{\varphi}^m$.

3.2 Definition of similarity measure

Similarity measure is a key issue in pattern recognition. For dynamical patterns expressed by the trajectories of dynamical systems, existing methods measure the similarity between two dynamical patterns based on their dynamical differences, and we define the similarity between patterns from the perspective of structural stability. The local region $\Omega_{\varphi_\zeta^m}$ (or called the approximation region) can be expressed as follows:

$$\Omega_{\varphi_\zeta^m} := \left\{ X \mid dist(X, \varphi_\zeta^m) < d_m \Rightarrow \left| \bar{W}^{m^T} S(X) - f^m(X) \right| < \varepsilon_2 \right\} \tag{16}$$

Where $\mathrm{dist}(X, \varphi_\zeta^m)$ denotes the minimum euclidean distance, $d_m > 0$ is a constant denoting the size of the local region, and $\varepsilon_2 = O(\varepsilon_1)$ is the approximation error.

Considering structural stability, if the dynamical pattern φ^r is in a neighborhood of the dynamical pattern φ^m, the partial derivatives of the dynamics of the φ^r mode along the trajectories of the φ^m mode can be efficiently represented based on the results of the sampling-deterministic learning modeling: for all $X(k) \in \varphi^r$, there are

$$\begin{aligned} \frac{\partial f(X(k))}{\partial l(X(k))} &= \frac{\partial f(X(k))}{\partial x_j} Cos\langle l(X(k)), x_j \rangle \\ &= \bar{W}^T \frac{\partial S(X(k))}{\partial x_j} Cos\langle l(X(k)), x_j \rangle + \varepsilon_2' \end{aligned} \tag{17}$$

Where ε_2' denotes the approximation error of the kinetic bias. Based on the assumption of a sufficiently small sampling time, the direction of the bias at each moment is estimated by differencing the previous moment $l(X[k]) = \frac{X[k] - X[k-1]}{\|X[k] - X[k-1]\|}$. Based on the regression vector property of gaussian rbfn, the bias derivative of its regression vector is can be calculated based on the rbf neuron input and neuron center:

$$\frac{\partial S(X(k))}{\partial x_j} = \begin{bmatrix} -\frac{2s_1(X(k))(x_j - \xi_{1j})}{\eta^2} \\ \vdots \\ -\frac{2s_N(X(k))(x_j - \xi_{Nj})}{\eta^2} \end{bmatrix} \tag{18}$$

Where ξ_{1j} denotes the jth component of the ith neuron and $S(X)$ denotes the basis function.

Definition 1 Consider the test pattern φ^r And the training pattern φ^m, which are generated by the dynamical systems (10) and (9), respectively, and are sequences of sampled data that are periodic or regressive. If the state trajectory of the test pattern φ^r Is in the neighborhood of the state trajectory of the training pattern φ^m, and the differences between the corresponding system dynamics along the orbit of pattern φ^r Is small, i.e.

$$\max_{X[k] \in \phi^r} \left| f^r(X(k)) - f^m(X(k)) \right| + \lambda \left| \frac{\partial f^r(X(k))}{\partial l(X(k))} - \frac{\partial f^m(X(k))}{\partial l(X(k))} \right| < \varepsilon^* \tag{19}$$

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Where ε^* Denotes the similarity between dynamical patterns, λ is a constant value greater than 0, and, $\frac{\partial f(X(k))}{\partial l(X(k))}$ Denotes the directional derivative of the dynamics along the φ^r Trajectory in the direction $l(X(k))$ at each moment in time, and we claim that the test pattern φ^r is said to be similar to the training pattern φ^m, and when the test pattern φ^r is kept in the local region $\Omega_{\varphi_\zeta^m}$, we have the following definition.

Definition 2 Consider the test pattern φ^r and the training pattern φ^m, which are generated by the dynamical systems (10) and (9), respectively, and are sequences of sampled data that are periodic or regressive. If the state trajectory of the test pattern φ^r is in the neighborhood of the state trajectory of the training pattern φ^m, it satisfies:

$$\max_{X[k]\in\phi^r}\left|f^r\big(X(k)\big)-\bar{W}^{m^T}S\big(X(k)\big)\right|+\lambda\left|\frac{\partial f^r\big(X(k)\big)}{\partial l\big(X(k)\big)}-\frac{\partial\bar{W}^{m^T}S\big(X(k)\big)}{\partial l\big(X(k)\big)}\right|<\varepsilon^*+\lambda\xi^* \tag{20}$$

Where $\varepsilon^*+\lambda\xi^*$ denotes the similarity between dynamical patterns, $\frac{\partial f(X(k))}{\partial l(X(k))}$ denotes the directional derivative of the direction $l(X(k))$ at each moment of the dynamics along the trajectory of φ^r, and ξ^* is the structural stability approximation error. Then the test pattern φ^r is said to be similar to the training pattern φ^m.

4. Rapid dynamical pattern recognition

Based on the modeling of the test pattern with the dynamical estimators and the lemma 3, the recognition errors can be expressed by the following system of recognition errors:

$$\tilde{\chi}^m(k+1)=\gamma\tilde{\chi}^m(k)+T\left(\left|\bar{W}^{m^T}S\big(\hat{x}^r(k)\big)-f^r\big(\hat{x}^r(k)\big)\right|+\lambda\left|\frac{\partial\bar{W}^{m^T}S\big(\hat{x}^r(k)\big)}{\partial l\big(\hat{x}^r(k)\big)}-\frac{\partial f^r\big(\hat{x}^r(k)\big)}{\partial l\big(\hat{x}^r(k)\big)}\right|\right) \tag{21}$$

Where $\tilde{\chi}^m(k)=\parallel\bar{\chi}^m(k)-\chi^r(k)\parallel_{C^1}$ is the state tracking (or synchronization) error. $\bar{\chi}^m$ represents the state of the dynamical discrete estimator and $\hat{x}^r$ represents the input to the estimator.

Finally, we combine the average L_1 norm for recognition decisions:

$$\parallel\tilde{\chi}^m(k)\parallel_{A_1}=\frac{1}{K_p}\sum_{j=k-K_p+1}^{k}\mid\tilde{\chi}^m(j)\mid \tag{22}$$

Where K_p is a predetermined positive integer and denoted as the range of the average L_1 norm.

Theorem 1 consider the dynamical pattern φ^m of univariate time series of the training dataset $\varphi=\{y^m\mid m=1,\dots,M\}$, the test pattern φ^r of the periodic trajectory, and the period K_p of the average L_1 norm is a positive integer, and the dynamical estimator error $\tilde{\chi}^m(k)$ will converge to a neighborhood of zero, and the neighborhood size is approximately proportional to the dynamic difference in structural stability inherent in the test pattern trajectory φ^m and the structural stability dynamics difference inherent in the training pattern trajectory φ^m.

Proof. For ease of representation, we define the matching function:

$$H^{m,r}(k)\coloneqq\left|\bar{W}^{m^T}S\big(\hat{x}^r(k)\big)-f^r\big(\hat{x}^r(k)\big)\right|+\lambda\left|\frac{\partial\bar{W}^{m^T}S\big(\hat{x}^r(k)\big)}{\partial l\big(\hat{x}^r(k)\big)}-\frac{\partial f^r\big(\hat{x}^r(k)\big)}{\partial l\big(\hat{x}^r(k)\big)}\right| \tag{23}$$

Representation of $\tilde{\chi}^m(k)$ by iterative methods, it can be simplified as follows:

$$\tilde{\chi}^m(k)=\gamma^k\tilde{\chi}^m(0)+\sum_{j=0}^{k-1}T\gamma^{k-1-j}H^{m,r}(j) \tag{24}$$

Further, the average L_1 norm of the recognition error is satisfied:

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$$\begin{aligned}\|\tilde{\chi}^m(k)\|_{A1} &= \frac{1}{K_p}\sum_{j=k-K_p+1}^{k}|\tilde{\chi}^m(j)| \\ &= \frac{1}{K_p}\sum_{j=k-K_p+1}^{k}\left|\gamma^k\tilde{\chi}^m(0)+\sum_{j=0}^{k-1}T\gamma^{k-1-j}H^{m,r}(j)\right| \\ &= \frac{1}{K_p}\sum_{j=k-K_p+1}^{k}\left|\gamma^j\tilde{\chi}^m(0)\right|+\frac{1}{K_p}\sum_{j=k-K_p+1}^{k}\left|T\sum_{i=1}^{j}\gamma^{j-i}H^{m,r}(i)\right|\end{aligned} \tag{25}$$

For convenient representation, let $\tilde{\chi}^m(0)=0$. That is, $\|\tilde{\chi}^m(k)\|_{A1} = \frac{1}{K_p}\sum_{j=k-K_p+1}^{k}\left|T\sum_{i=1}^{j}\gamma^{j-i}H^{m,r}(i)\right|$. Since $|\gamma|<1$, V_1 decays exponentially to a neighborhood of 0 as time k increases. If K_p is chosen to be a cycle of the periodic trajectory, then we have

$$\begin{aligned}\|\tilde{\chi}^m(k)\|_{A1} &= \frac{1}{K_p}\sum_{j=k-K_p+1}^{k}\left|T\sum_{i=1}^{j}\gamma^{j-i}H^{m,r}(k)\right| \\ &< \frac{1}{K_p}\sum_{j=k-K_p+1}^{k}\frac{T|H^{m,r}(k)|}{1-\gamma} \\ &= \frac{T|H^{m,r}(k)|}{1-\gamma}\end{aligned} \tag{26}$$

With the above derivation, we obtain a representation of the average L_1 norm of the recognition error $\tilde{\chi}^m(k)$. As time k increases, $\|\tilde{\chi}^m(k)\|_{A1}$ will converge into a neighborhood of zero, and the size of the neighborhood is approximately proportional to the difference in the dynamics of structural stability inherent in the test pattern trajectory φ^m and the training pattern trajectory φ^m.

5. Simulation

To demonstrate the effectiveness of the proposed method, we consider the Duffing System:

$$\begin{cases}\dot{x}_1 = x_2 \\ \dot{x}_2 = -p_2x_1 - p_3x_1^3 - p_1x_2 + qcos(\omega t) \\ y = x_1,\end{cases} \tag{27}$$

Where $x=[x_1,x_2]^T$ is the state of the system and p_1, p_2, p_3, w and q are constant parameters. The system dynamics $f(x) = -p_2x_1 - p_3x_1^3 - p_1x_2$ is an unknown smooth nonlinear function and $qcos(wt)$ is a known periodic term. We choose different parameter vectors to generate different modes, $y^m(k), m=1,\dots 5$. We sample the output of the system at a sampling frequency of $1kHz$, $x(0)=[0.438; 0.07713]$. $y^r(k), r=6$, is the test pattern. The parameters corresponding to the above time series are shown in Table 1.

Table 1 Univariate time series parameters

parameter	training pattern					test pattern
	y^1	y^2	y^3	y^4	y^5	y^6
p_1	1.2	0.4	0.55	0.4	0.6	0.6
p_2	-1.5	-1.5	-1.1	-1.1	1.0	1.0
p_3	1.0	1.0	1.0	1.0	0.8	1.3
q	0.9	0.9	1,498	1.498	1.0	1.0
w	1.8	1.8	1.8	1.8	1.498	1.498

To simulate the noise present in actual data, Gaussian noise with a mean of 0 and a variance of 0.5×10^{-6} is added to the original signal x_1. This approach aims to enhance the robustness of the model, enabling it to better handle disturbances that may be encountered in real-world applications. For systems with disturbances, the ESO can achieve state trajectory reconstruction and has a filtering effect.

The state trajectories $x^m(k)$ of the dynamical system and the system dynamics $f^m(x^m(k))$ are first reconstructed using the dilated state observer (12). The observer gain is set to $\beta=[100,300,3000]$. Due

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to the high observer gain, only the phase diagram after 1s is shown in order to show the reconstruction effect more clearly. The state trajectories of the reconstructed training and test patterns are shown in Figure 2, and accurate state estimation can be achieved from sampled output measurements for both training and test patterns.

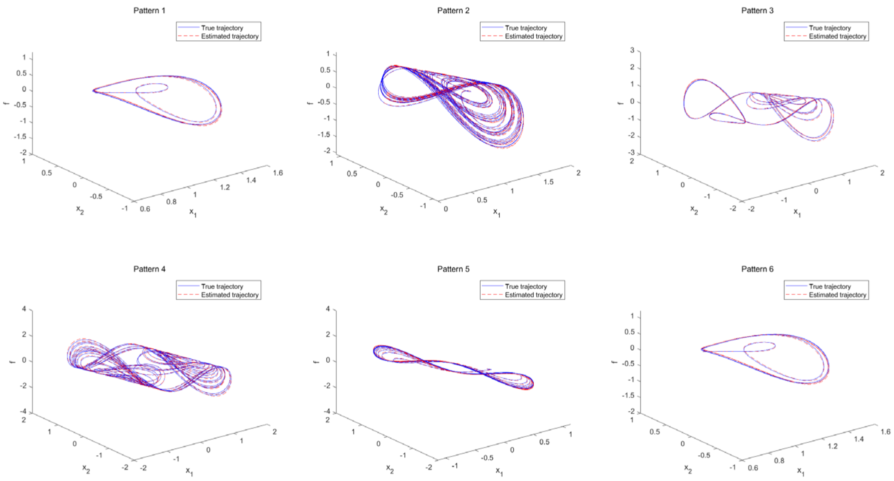

Figure 2 Reconstructing state trajectories and unknown dynamics

In order to achieve good approximation ability and relatively high excitation level of the RBFN, 441 neurons arranged in a regular lattice [-3, 3] × [-3, 3] with a width of $\eta = 0.3$ are used. The neuron weight vectors are adjusted according to the updating law (13), and the initial value of the weights is $\hat{W}^m(0) = 0$. The convergence of the weights is shown in Figure 3, demonstrating that the weight parameters achieve local convergence.

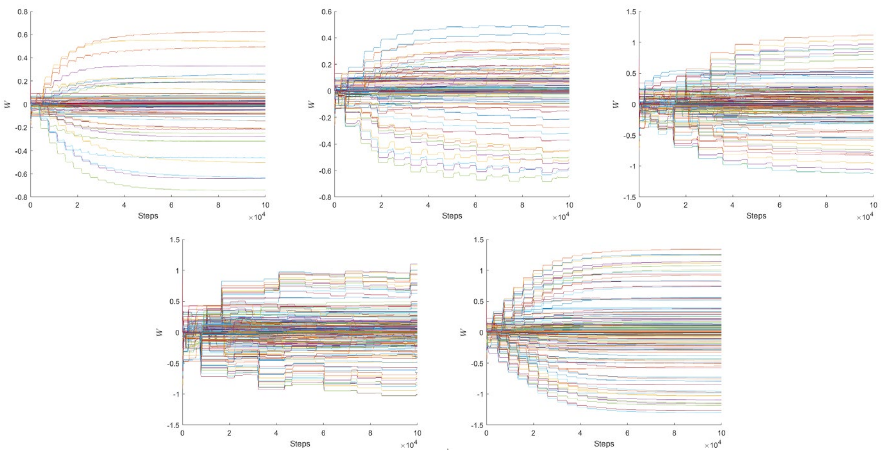

Figure 3 Parameter convergence

We adopt the RBFN based on deterministic learning to represent the dynamics along the trajectory and the partial derivatives of these dynamics. Figure 4 illustrates the effect of the phase space dynamic deviation approximation, where the dynamic partial derivative trajectories are consistent but exhibit some errors. Figure 5, based on the structural stability of the sampled data, presents the results of dynamical pattern recognition by assessing the average L_1 norm of the error.

Subsequently, a synthetic dataset was generated using dynamic systems $y^m(k), m = 1, \ldots 5$, with sampling data produced under different parameters. Keeping other parameters constant, this study employed various window ranges: $p_1 = [1,1.5]$, $p_1 = [0.38,0.48]$, $p_1 = [0.5,0.6]$, $p_1 = [0.38,0.42]$ and $p_1 = [0.5,1]$. Each window range was divided into 50 parameters, yielding a total of 250 data sets. From these, 100 data sets were randomly selected to calculate the accuracy of two methods, utilizing the Rand Index (RI) as the evaluation metric. The results based on the observer-based deterministic learning method yielded an RI of 0.976. In contrast, the results from the deterministic learning method based on ESO and structural stability achieved an RI of 1, indicating superior identification performance.

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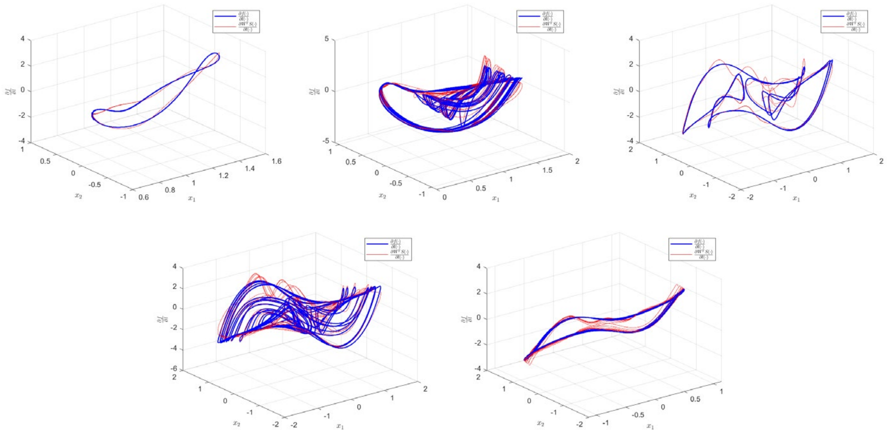

Figure 4 Approximation of partial derivatives of dynamics in phase space

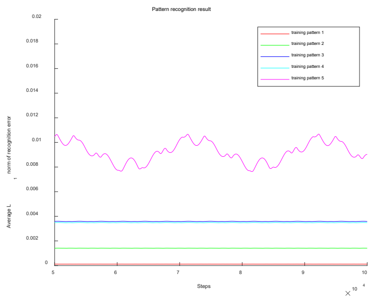

Figure 5 Recognition results

6. Conclusions

This paper proposes a deterministic learning method based on ESO and structural stability, aiming to achieve dynamical pattern recognition of univariate time series from the perspective of topological structural stability. These time series consist of output signals from dynamical systems. Specifically, ESO is used to estimate the state of the training dynamical patterns, and then RBFN is employed for local precise recognition of the inherent dynamics of univariate time series. In the recognition phase, the system dynamics partial derivatives are approximated using first-order differences. Structural stability state estimators and recognition error systems are constructed based on the state trajectories of the test patterns and the nonlinear system dynamics. The L_1 norm of the recognition error is used to identify the dynamics differences of the dynamical patterns, achieving dynamical pattern recognition from the perspective of structural stability.

References

[1] Ma Q, Liu Z, Zheng Z, et al. A survey on time-series pre-trained models[J]. IEEE Transactions on Knowledge and Data Engineering, 2024.
[2] Fu T. A review on time series data mining[J]. Engineering Applications of Artificial Intelligence, 2011, 24(1): 164-181.
[3] Wang C, Hill D J. Learning from neural control[J]. IEEE transactions on neural networks, 2006, 17(1): 130-146.
[4] Wang C, Hill D J. Deterministic learning theory for identification, recognition, and control[M]. CRC Press, 2018.
[5] Kuznetsov Y A, Kuznetsov I A, Kuznetsov Y. Elements of applied bifurcation theory[M]. New York: Springer, 1998.
[6] Han J. A class of extended state observers for uncertain systems[J]. Control and decision, 1995, 10(1): 85-88.