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Complete Convergence for Weighted Sums of Negatively Dependent Random Variables under Sub-Linear Expectations

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Abstract: *In this article, we study complete convergence theorems for weighted sums of negatively dependent random variables under the sub-linear expectations. Our results extend the corresponding results of complete convergence in the classical probability.*

Keywords: *Complete convergence, Negatively dependent, Random variables, Sub-linear expectations.*

1. Introduction

The complete convergence is a very important research field in probability limit theory.For classical probability space,it was first introduced by Hsu and Robbins in 1947 as follows:A sequence $\{X_n, n \geq 1\}$ of random variables converges completely to a constant C if $\sum_{n=1}^{\infty} P(|X_n - C| > \varepsilon) < \infty$ for all $\varepsilon > 0$. After that, many scholars have obtained the complete convergence on other random variable sequences, for example,wu qun-ying obtained the complete convergence for a sequence of independent and identically distributed random variables. Wu and Jiang etal gived the complete convergence and complete moment convergence for negatively dependent random variables sequence.

In the classical probability space, the additivity of the probabilities and the expectations are assumed. But in practice, such additivity assumption is not feasible in many areas of applications because the uncertainty phenomena cannot be modeled using additive probabilities or additive expectations. Therefore in recent years, some scholars began to extend the complete convergence of classical probability theory to sub-11linear space, for example, Ling obtained complete convergence of random variables sequence under sub-linear expectation space, Feng reseached complete convergence for weighted sums of negatively dependent random variables,Yu and Wu resarched the Marcinkiewicz type complete convergence for weighted sums under sub-linear expectations. Complete convergence for weighted sums are also important in sublinear expectation space, which can be applied to nonparametric regression models. The main purpose of this article is to establish complete convergence theorems for arrays of rowwise negatively dependent random variables under the sub-linear expectations.

2. Symbol Description

(1) $a_n << b_n$ denote that for sufficiently large n, there exist $c > 0$ such that $a_n \leq cb_n$;

(2) $I(\cdot)$ denote an indicator function.;

(3) $\log(x) = \max(1, \ln x)$.

3. The Basic Definition and Properties of Sub-Linear Expectation Space

We use the framework and notation of sub-linear expectation established by Peng.

Let (Ω, F) be a given measurable space, H be a linear space of real function defined on (Ω, F), such that if $X_1, X_2, \cdots, X_n \in \mathrm{H}$,then $\varphi(X_1, X_2, \cdots, X_n) \in \mathrm{H}$ for each

$\varphi \in C_{l,Lip}(R^n)$, where $C_{l,Lip}(R^n)$ denotes the linear space of local Lipschitz functions φ satisfying

$$|\varphi(x)-\varphi(y)| \le c(1+|x|^m+|y|^m)|x-y|, \forall x, y \in R^n$$

For some $C>0, m \in \mathrm{N}$ depending on φ. H is considered as a space of "random variables". If X is an element of set H, then we denote $X \in \mathrm{H}$.

Definition 1 A sub-linear expectation E is a function $\mathrm{E}: \mathrm{H} \to R$ satisfying the following properties: for all $X, Y \in \mathrm{H}$, we have

(a) Monotonicity: if $X \ge Y$ then $\mathrm{E}X \ge \mathrm{E}Y$;

(b) Constant preserving: $\mathrm{E}c = c$, for all $c \in R$;

(c) Sub-additivity: $\mathrm{E}(X+Y) \le \mathrm{E}X + \mathrm{E}Y$ whenever $\mathrm{E}X + \mathrm{E}Y$ is not of the form $+\infty-\infty$ or $-\infty+\infty$;

(d) Positive homogeneity: $\mathrm{E}(\lambda X) = \lambda \mathrm{E}X, \lambda > 0$.

E is called sub-linear expectation on H. Here $R=[-\infty,+\infty]$. The triple $(\Omega, \mathrm{H}, \mathrm{E})$ is called a sub-linear expectation space.Give a sub-linear expectation E, let us denote the conjugate expectation $\hat{\mathrm{E}}$ of E by $\hat{\mathrm{E}}X := -\mathrm{E}(-X), \forall X \in \mathrm{H}.$

From the definition of E, we can easily get that for $\forall X, Y \in \mathrm{H}$,

$$\hat{\mathrm{E}}X \le \mathrm{E}X,\ \mathrm{E}(X+c) = \mathrm{E}X + c,\ |\mathrm{E}(X-Y)| \le \mathrm{E}(|X-Y|), \mathrm{E}(X-Y) \ge \mathrm{E}X - \mathrm{E}Y.$$

For $\forall a \in R$, if $\mathrm{E}Y = \hat{\mathrm{E}}Y$, then $\mathrm{E}(X+aY) = \mathrm{E}X + a\mathrm{E}Y$.

Definition 2 Let $G \subset F$, a function $V: G \to [0,1]$ is called a capacity if :

(1) $V(\phi)=0,\ V(\Omega)=1$;

(2) $V(A) \le V(B)$ for $\forall A \subseteq B$, $A, B \in G$.

It is called to be sub-additive if $V(A \cup B) \le V(A) + V(B)$ for all $A, B \in G$ with $A \cup B \in G$. Let $(\Omega, \mathrm{H}, \mathrm{E})$ be a sub-linear expectation space ,and $\hat{\mathrm{E}}$ be the conjugate expectation of E. We denote a pair (V, v) of capacities by

$$V(A) = \inf\{\mathrm{E}\xi : I(A) \le \xi, \xi \in \mathrm{H}\}, v(A) = 1 - V(A^c), \forall A \in F.$$

where A^c is the complement set of A.

It is obvious that V is sub-additive and

$$v(A) \le V(A), \forall A \in F,\ V(A) = E[I(A)],\ v(A) = \hat{E}[I(A)], I(A) \in \mathrm{H}.$$

If $f \le I(A) \le g, g \in \mathrm{H}$, then

$$Ef \le V(A) \le Eg, \hat{E}f \le v(A) \le \hat{E}g$$

This implies Markov inequality: $\forall X \in \mathrm{H}, V(|X| \ge x) \le E(|X|^p) \big/ x^p, \forall x > 0, p > 0$ from $I(|X| \ge x) \le |X|^p / x^p \in \mathrm{H}$.

Next we introduce some important inequalities in sub-linear expectation spaces.

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(1) Jensen inequality: $\left(E|X|^r\right)^{\frac{1}{r}} \le \left(E|X|^s\right)^{\frac{1}{s}}, X \in \mathrm{H}, \forall 0 < r \le s.$

(2) Hölder inequality: $E|XY| \le E\left(|X|^p\right)^{\frac{1}{p}} E\left(|X|^q\right)^{\frac{1}{q}}, \forall X, Y \in \mathrm{H}, \frac{1}{p} + \frac{1}{q} = 1$

(3) C_r inequality: $E|X_1 + X_2 + \cdots + X_n|^r \le c_r\left(E|X_1|^r + E|X_2|^r + \cdots + E|X_n|^r\right),$

where $c_r = \begin{cases} 1, & 0 < r \le 1 \\ n^{r-1}, & r > 1 \end{cases}.$

Definition 3 Choquet integral is defined y : $C_V(X) = \int_0^{+\infty} V(X \ge t)dt + \int_{-\infty}^0 (V(X \ge t) - 1)dt$

Definition 4 (identical distribution)let X_1, X_2 be two n-dimensional random vector defined respectively in sub-linear expectation space $(\Omega_1, \mathrm{H}_1, E_1)$ and $(\Omega_2, \mathrm{H}_2, E_2)$ .They are called eidentically distributed,denoted by $X_1 \overset{d}{=} X_2$,if $E_1[\varphi(X_1)] = E_2[\varphi(X_2)], \forall \varphi \in C_{l,Lip}(R^n)$ .A sequence $\{X_n, n \ge 1\}$ of random variables is said to identically distributed if $X_i \overset{d}{=} X_1$,for each $i \ge 1$.

Definition 5 (independence) In a sub-linear expectation space (Ω, H, E) ,a random vector $Y = (Y_1, Y_2, \cdots, Y_n), Y_i \in \mathrm{H}$ is said to be independent to another random vector

$X = (X_1, X_2, \cdots, X_m), X_i \in \mathrm{H}$ under E if for each test function $\varphi \in C_{l,Lip}(R^m \times R^n)$,we have $E[\varphi(X, Y)] = E[E(\varphi(x,Y))|x = X]$,where $\overline{\varphi}(x) := E(|\varphi(x,Y)|) < \infty$ for all x and $E(|\overline{\varphi}(X)|) < \infty$.

Definition 6 (Negative dependence) In a sub-linear expectation space (Ω, H, E),a random vector $Y = (Y_1, Y_2, \cdots, Y_n)$, $Y_i \in \mathrm{H}$ is said to be negatively dependent to another random vector $X = (X_1, X_2, \cdots, X_m), X_i \in \mathrm{H}$ under E if for each pair of test function $\varphi_1 \in C_{l,Lip}(R^m)$and $\varphi_2 \in C_{l,Lip}(R^n)$,we have $E[\varphi_1(X)\varphi_2(Y)] \le E[\varphi_1(X)]\ E[\varphi_2(Y)]$,whenever either φ_1 and φ_2 are coordinate-wise non decreasing or φ_1 and φ_2 are coordinate-wise non increasing with

$$\varphi_1(X) \ge 0,\ E[\varphi_2(Y)] \ge 0,\ E[|\varphi_1(X)\varphi_2(Y)|] < \infty,\ E[|\varphi_1(X)|] < \infty,\ E[|\varphi_2(Y)|] < \infty.$$

Definition 7 (Negative dependence random variables)Let $\{X_n, n \ge 1\}$ be a sequence of random variables in the sub-linear expectation space (Ω, H, E). $X_1, X_2, \cdots$ are said to negatively dependent if X_{i+1} is negatively dependent to $(X_1, X_2, \cdots, X_i)$ for each $i \ge 1$.

It is obvious that,if $\{X_n, n \ge 1\}$ is a sequence of independent random variables and $f_1(x), f_2(x), \cdots \in C_{l,Lip}(R)$,then $\{f_n(X_n), n \ge 1\}$ is also a sequence of independent random variables ;if $\{X_n, n \ge 1\}$ is a sequence of negatively dependent random variables and $f_1(x)$, $f_2(x)$, $\cdots \in C_{l,Lip}(R)$ are non-decreasing(resp. non-decreasing)function,then

$\{f_n(X_n), n \ge 1\}$ is also a sequence of negatively dependent random variables .

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Definition8 An array of random variables $\{X_{ni}, 1 \le i \le k_n, n \ge 1\}$ in the sub-linear expectation space (Ω, H, E) is called rowwise negatively dependent random variables if for every given $n \ge 1$, $\{X_{ni}, 1 \le i \le k_n\}$ is a sequence of negatively dependent random variables in (Ω, H, E). Where $\{k_n, n \ge 1\}$ is a sequence of positive integers and $k_n \to \infty, n \to \infty$.

4. Main Result

In order to prove our result,we need the following lemma：

Lemma Let $\{X_{ni}, 1 \le i \le n, n \ge 1\}$ is an array of rowwise negatively dependent random variables , $EX_{ni} = 0$ .Assume that : $\max_{1 \le i \le n} |X_{ni}| = O((\log n)^{-1})$, $\sum_{i=1}^{n} EX_{ni}^2 = o((\log n)^{-1})$,then for any $\varepsilon > 0$, $\sum_{n=1}^{\infty} V\left(\sum_{i=1}^{n} X_{ni} > \varepsilon\right) < \infty$.

Theorem Let $\{X_{ni}, 1 \le i \le n, n \ge 1\}$ is an array of rowwise negatively dependent random variables in the sub-linear expectation space (Ω, H, E).Assume that there exist a random variable $X \in \mathrm{H}$ and a constant c satisfying

$$E(h(X_{ni})) \le cE(h(X)) \tag{1}$$

for all $n \ge 1, 1 \le i \le n, 0 \le h \in C_{l,Lip}(R)$.

$$E(|X|) \le C_V(|X|) < \infty \tag{2}$$

Let $\{a_{ni}, 1 \le i \le n, n \ge 1\}$ is an array of positive real numbers satisfying

$$\sum_{i=1}^{n} a_{ni} = O(n^{-\gamma}) \tag{3}$$

for some $\gamma > 0$,and

$$\sum_{i=1}^{n} a_{ni}^2 EX^2 g(a_{ni} X \log n) = o((\log n)^{-1}) \tag{4}$$

Then for any $\varepsilon > 0$,any positive integer number b , $b\gamma > 1$,

$$\sum_{n=1}^{\infty} V\left(\sum_{i=1}^{n} a_{ni}(X_{ni} - EX_{ni}) I\left(|a_{ni} X_{ni}| \le \frac{\varepsilon}{b}\right) > \varepsilon\right) < \infty \tag{5}$$

$$\sum_{n=1}^{\infty} V\left(\sum_{i=1}^{n} a_{ni}(X_{ni} - \hat{E}X_{ni}) I\left(|a_{ni} X_{ni}| \le \frac{\varepsilon}{b}\right) < -\varepsilon\right) < \infty \tag{6}$$

Proof We replace $\{X_{ni}, 1 \le i \le n, n \ge 1\}$ in formula (5) with $\{-X_{ni}, 1 \le i \le n, n \ge 1\}$ to get formula (6). If $\{X_{ni}, 1 \le i \le n, n \ge 1\}$ is an array of rowwise negatively dependent random variables, then $\{-X_{ni}, 1 \le i \le n, n \ge 1\}$ is also an array of rowwise negatively dependent random variables. Therefore we just need to prove formula (5). Without losing generality, we assume that $EX_{ni} = 0$. For $n \ge 1, and 1 \le i \le n$, we define

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$$X_{ni}(1)=X_{ni}I(|a_{ni}X_{ni}|\le(\log n)^{-1})+a_{ni}^{-1}(\log n)^{-1}I(a_{ni}X_{ni}>(\log n)^{-1}-a_{ni}^{-1}(\log n)^{-1}I(a_{ni}X_{ni}<-(\log n)^{-1}),$$

$$X_{ni}(2)=(X_{ni}-a_{ni}^{-1}(\log n)^{-1})I((\log n)^{-1}<a_{ni}X_{ni}\le\frac{\varepsilon}{b},$$

$$X_{ni}(3)=(X_{ni}+a_{ni}^{-1}(\log n)^{-1})I(-\frac{\varepsilon}{b}\le a_{ni}X_{ni}<-(\log n)^{-1},$$

$$X_{ni}(4)=-a_{ni}^{-1}(\log n)^{-1}I(a_{ni}X_{ni}>\frac{\varepsilon}{b})+a_{ni}^{-1}(\log n)^{-1}I(a_{ni}X_{ni}<-\frac{\varepsilon}{b}),$$

It is obvious that $a_{ni}X_{ni}I\left(|a_{ni}X_{ni}|\le\frac{\varepsilon}{b}\right)=a_{ni}X_{ni}(1)+a_{ni}X_{ni}(2)+a_{ni}X_{ni}(3)+a_{ni}X_{ni}(4)$.

Therefore we have

$$\sum_{n=1}^{\infty}V\left(\sum_{i=1}^{n}a_{ni}X_{ni}I\left(|a_{ni}X_{ni}|\le\frac{\varepsilon}{b}\right)>4\varepsilon\right)$$

$$\le\sum_{n=1}^{\infty}V\left(\sum_{i=1}^{n}a_{ni}X_{ni}(1)>\varepsilon\right)+\sum_{n=1}^{\infty}V\left(\sum_{i=1}^{n}a_{ni}X_{ni}(2)>\varepsilon\right)$$

$$+\sum_{n=1}^{\infty}V\left(\sum_{i=1}^{n}a_{ni}X_{ni}(3)>\varepsilon\right)+\sum_{n=1}^{\infty}V\left(\sum_{i=1}^{n}a_{ni}X_{ni}(4)>\varepsilon\right).$$

According to definition 8, we can easily know $\{a_{ni}X_{ni}(1)\}$ is an array of rowwise negatively dependent random variables.

We define a function $g(x)\in C_{l,Lip}(R)$ for $0<\mu<1$ as follows: $0\le g(x)\le 1$ for any x .When $|x|\le\mu$, $g(x)=1$; when $|x|>1$, $g(x)=0$. And then we have

$$I(|x|\le\mu)\le g(x)\le I(|x|\le 1),\ I(|x|>1)\le 1-g(x)\le I(|x|>\mu). \tag{7}$$

According to (7)and C_r inequality,we have

$$E(|X_{ni}(1)|^r)<<E(|X|^r g(\mu a_{ni}X\log n))+a_{ni}^{-r}(\log n)^{-r}E(1-g(a_{ni}X\log n))$$

$$\le E(|X|^r g(\mu a_{ni}X\log n))+a_{ni}^{-r}(\log n)^{-r}V(|a_{ni}X|>\mu(\log n)^{-1}) \tag{8}$$

Next we above prove $\sum_{n=1}^{\infty}V\left(\sum_{i=1}^{n}a_{ni}X_{ni}(1)>\varepsilon\right)<\infty$.

From Markov inequality and formula(1)(2)(3)(4)(8), we have

$$\sum_{i=1}^{n}E(a_{ni}(X_{ni}(1)-EX_{ni}(1)))^2\le\sum_{i=1}^{n}a_{ni}^2E(2X_{ni}^2(1)+2(EX_{ni}(1))^2)$$

$$\le cE(X_{ni}(1))^2\le\sum_{i=1}^{n}a_{ni}^2EX^2g(\mu a_{ni}X\log n)+(\log n)^{-2}\sum_{i=1}^{n}V(|a_{ni}X|>\mu(\log n)^{-1})$$

$$<<o(\log n)^{-1})+(\log n)^{-1}\sum_{i=1}^{n}a_{ni}E(|X|)$$

$$\le o((\log n)^{-1})+(\log n)^{p-2}O(n^{-\gamma})C_V(|X|)=o((\log n)^{-1}).$$

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Therefore from lemma we have $\sum_{n=1}^{\infty} V\left(\sum_{i=1}^{n} a_{ni}(X_{ni}(1) - EX_{ni}(1) > \frac{\varepsilon}{2}\right) < \infty$.

Because if $\left|\sum_{i=1}^{n} a_{ni} EX_{ni}(1)\right| \to 0, n \to \infty$ is proved,then $\sum_{n=1}^{\infty} V\left(\sum_{i=1}^{n} a_{ni} X_{ni}(1) > \varepsilon\right) < \infty$ is proved.

Next we prove $\left|\sum_{i=1}^{n} a_{ni} EX_{ni}(1)\right| \to 0, n \to \infty$.

$$\left|\sum_{i=1}^{n} a_{ni} EX_{ni}(1)\right| = \left|\sum_{i=1}^{n} a_{ni}(EX_{ni}(1) - EX_{ni})\right|$$

$$\leq \sum_{i=1}^{n} a_{ni} E|X_{ni} - X_{ni}(1)| << \sum_{i=1}^{n} a_{ni} E(|X|(1 - g(a_{ni} X \log n)))$$

$$\leq \sum_{i=1}^{n} a_{ni} E(|X|(1 - g(a_{ni} X \log n)))$$

$$<< \sum_{i=1}^{n} a_{ni} E|X| \leq cO(n^{-\gamma}) \to 0.$$

Thus $\sum_{n=1}^{\infty} V\left(\sum_{i=1}^{n} a_{ni} X_{ni}(1) > \varepsilon\right) < \infty$ is proved.

Next we prove $\sum_{n=1}^{\infty} V\left(\sum_{i=1}^{n} a_{ni} X_{ni}(2) > \varepsilon\right) < \infty$.

From $X_{ni}(2) = (X_{ni} - a_{ni}^{-1}(\log n)^{-1}) I((\log n)^{-1} < a_{ni} X_{ni} \leq \frac{\varepsilon}{b}$,

we can obtain $0 \leq a_{ni} X_{ni}(2) \leq\leq \frac{\varepsilon}{b}$, $\left|\sum_{i=1}^{n} a_{ni} X_{ni}(2)\right| = \sum_{i=1}^{n} a_{ni} X_{ni}(2) > \varepsilon$ .It means that there exists at least one positive integer so that $X_{ni}(2) \neq 0$.

Therefore, from Markov inequality and formula (1)-(4),we can obtain

$$V\left(\sum_{i=1}^{n} a_{ni} X_{ni}(2) > \varepsilon\right) \leq \sum_{1<k_1<\cdots<k_b \leq n} V\left(X_{nk_1}(2) \neq 0, \cdots, X_{nk_b}(2) \neq 0\right)$$

$$\leq \sum_{1<k_1<\cdots<k_b \leq n} V\left(\left|a_{nk_1} X_{nk_1}\right| \geq (\log n)^{-1}, \cdots, \left|a_{nk_b} X_{nk_b}\right| \geq (\log n)^{-1}\right)$$

$$<< \sum_{1<k_1<\cdots<k_b \leq n} E\left((1 - g(a_{nk_1} X \log n) \cdots (1 - g(a_{nk_b} X \log n)\right)$$

$$\leq \sum_{1<k_1<\cdots<k_b \leq n} E(1 - g(a_{nk_1} X \log n)) \cdots E(1 - g(a_{nk_b} X \log n))$$

$$\leq (\sum_{i=1}^{n} E(1 - g(a_{ni} X \log n)))^b$$

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$$\leq (\sum_{i=1}^{n} V(|a_{ni}X| > \mu(\log n)^{-1}))^{b}$$

$$<< \left((\log n)\sum_{i=1}^{n} a_{ni}E(|X|\ ) \right)^{b}$$

$$<< \left((\log n)(n^{-\gamma})E(|X|)\right)^{b}$$

$$\leq c\left((\log n)(n^{-\gamma})\right)^{b}.$$

From $\gamma \mathrm{b} > 1$, $\sum_{n=1}^{\infty} V\left(\sum_{i=1}^{n} a_{ni}X_{ni}(2) > \varepsilon\right) < \infty$ is proved.

By the same methods as the $\sum_{n=1}^{\infty} V\left(\sum_{i=1}^{n} a_{ni}X_{ni}(2) > \varepsilon\right) < \infty$,we can get

$$\sum_{n=1}^{\infty} V\left(\sum_{i=1}^{n} a_{ni}X_{ni}(3) > \varepsilon\right) < \infty .$$

Finally we prove $\sum_{n=1}^{\infty} V\left(\sum_{i=1}^{n} a_{ni}X_{ni}(4) > \varepsilon\right) < \infty$.

From

$$X_{ni}(4) = -a_{ni}^{-1}(\log n)^{-1} I(a_{ni}X_{ni} > \frac{\varepsilon}{b}) + a_{ni}^{-1}(\log n)^{-1} I(a_{ni}X_{ni} < -\frac{\varepsilon}{b}),$$

we obtain

$a_{ni}X_{ni}(4) \leq (\log n)^{-1} I(|a_{ni}X_{ni}| > \frac{\varepsilon}{b}) \leq \frac{\varepsilon}{b}$, thus there are at least b subscripts i so that

$|a_{ni}X_{ni}| > \frac{\varepsilon}{b}$,Therefore ,from Markov inequality ,formula (1)-(3)and(6),for $\gamma b > 1$,we can obtain

$$\sum_{n=1}^{\infty} V\left(\sum_{i=1}^{n} a_{ni}X_{ni}(4) > \varepsilon\right) << \sum_{n=1}^{\infty}\left(\sum_{i=1}^{n} E(1-g(\frac{a_{ni}X}{\varepsilon/b}))\right)^{b}$$

$$\leq \sum_{n=1}^{\infty}\left(\sum_{i=1}^{n} V(|a_{ni}X| > \varepsilon\mu/b\right)^{b}$$

$$\leq \sum_{n=1}^{\infty}\left(\sum_{i=1}^{n} a_{ni}E|X|(\varepsilon\mu/b)^{-1}\right)^{b}$$

$$<< \sum_{n=1}^{\infty}\left(E|X|O(n^{-\gamma})\right)^{b} < \infty .$$

Therefore $\sum_{n=1}^{\infty} V\left(\sum_{i=1}^{n} a_{ni}X_{ni}(4) > \varepsilon\right) < \infty$.

So far, the theorem is proved.

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