# **Best Pacing Strategy: Time Trial Optimization with Physiological and Power Simulation**

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Abstract: In this paper, we establish a dynamic programming model of the motion process and a dynamic simulation model of the riding process, and use the genetic algorithm to simulate and optimize the speed and force distribution of the rider during the riding process. Specifically, we established a dynamic power curve model determined by a physiological mechanism (lactate heart rate), and derived the dynamic power curves of different genders, different levels, and different types of riders. In addition, we established a timed trial simulation and pacing strategy optimization model based on the micro-element method and random velocity generation, and used genetic algorithm to solve it. Further, we build a tempo strategy optimization model for team games. The optimal solution is obtained by using the improved objective function of team competition in genetic algorithm. Finally, a sensitivity analysis was performed for the considered models.

Keywords: Self-feedback system, Micro-element, Genetic algorithm, Dynamic power curve

### 1. Introduction

Aesop's Fables tells the story of the tortoise and the rabbit, which warns people not to look down on their opponents, but also reveals a scientific truth: in addition to their own speed, choose an optimal work efficiency allocation is also particularly important.

The problem studied in this paper is related to the optimal work efficiency. In cycling road race, athletes need to choose and maintain the initial work rate in self-selected sports. The selection and distribution of working speed is an important factor that affects the final result of the competitors [1].

In this paper, we model a dynamic power curve determined by a physiological mechanism (lactate heart rate). The model incorporates a model of human circulatory movement dynamics and physiological constraints and the model's self-feedback mechanism. Accordingly, the dynamic power curves of different genders, different levels and different types of drivers are derived.

Secondly, we establish a timing test simulation and pacing strategy optimization model based on the micro-element method and random velocity generation, and use genetic algorithm to solve it. We discretize the time trial track and derive an environment matrix describing each track segment based on the data provided by the competitors. Then, the randomly generated speed-power matrix, penalty function and power-physical self-feedback system are used to determine the time cost of each random speed simulation, and use this as the objective function of the genetic algorithm to estimate the optimal solution of the time trial. We introduce the physiological parameters of various physical players for simulation. The simulation results are in good agreement with the actual competition results.

Finally, we extend the above-mentioned individual time trial optimization model to establish a rhythm strategy optimization model for team competition. In a team race, the difference in the order of the drivers can also have a huge impact on the power consumption of the drivers. At the same time, a permutation module is added on the basis of the original objective function. The optimal solution is obtained by using the improved objective function of team competition in genetic algorithm. In the sensitivity analysis section, the model shows that the model has high sensitivity to power deviation. The model is not very sensitive to low wind speeds, but increases with increasing wind speed.

## 2. Power-Physic Self-Feedback System

#### 2.1. Power-heartrate constraint model

Atkinson et al. 's research shows that it seems difficult for drivers to sustain drastic power changes, which can lead to various physiological indicators, including heart rate, being pushed to extremes and thus limiting human performance [2]. The study of [3] showed the variation and range of heart rate during exercise. Obviously, these short bursts require longer periods of low-power operation to rest and adjust. So, we decided on a special treatment. On the basis of discrete sections, some necessary intervals are placed between the sections where the boost takes place in the hope that factors such as the driver's heart rate will return to normal levels during the intervals. Blood lactate levels are also taken into account and must be brought back to normal before the next boost.

## 2.2. Power-Latic constraint Model

We analyzed the metabolism of lactic acid. Lactic acid in human body will be metabolized normally. When the rider's power is low, the metabolic rate of lactic acid in human body will be greater than the production of lactic acid, and at this time, lactic acid will slowly decrease [4]. Therefore, we can preliminarily obtain lactic acid changes under different cycling conditions, as shown in Fig. 1.

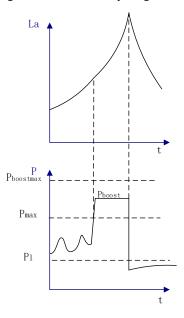


Figure 1: Changes of lactic acid in each state

Progressive exercise tests (GXTs) are commonly used to assess aerobic endurance performance. The rate of lactic acid production (Vlap) is exponentially related to exercise intensity (i.e., the rider's riding power (P)), according to the results obtained in incremental exercise tests at the institute of sports medicine at paderborn university, germany.

Therefore, the function between lactic acid formation rate  $(V_{lap})$  and riding power (P) can be given in the following form:

$$V_{lan} = k_{n1}(e^{k_{p2}*p} - 1) (1)$$

Where  $k_{p1}$ ,  $k_{p2}$  are parameters.

Therefore, we obtained the functional expression of the net production rate of lactic acid:

$$V_{la} = V_{lap} - V_{lae} = k_{p1}(e^{k_{p2}*p} - 1) - ke$$
(2)

The function of  $k_{p2}$  in this expression is to adjust the order of magnitude. Since the average riding power obtained before is about 400W~600W, we take  $k_{p2} = \frac{1}{100}$ , so that the left and right sides of the formula have the same order of magnitude.

In foster and Carr's study on the contribution value of aerobic and anaerobic energy system to the

total workload when athletes do full effort, the results are shown in the Fig.2 [5].

### **Proportional Energetic Contribution**

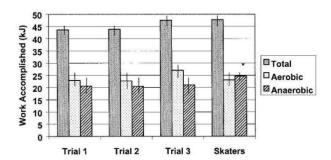


Figure 2: Proportional contribution to total work accomplished (kJ) by aerobic and anaerobic energy systems in the athletes

Before each test, the subjects were warmed up. Using the protocol described by Moritani et al. (1981), Pmax is calculated using four depletion exercises set to 90%, 95%, 100%, and 110% \_ VV O2max(Note: VV O2max refers to maximum oxygen uptake). When choosing these strengths, the time to exhaustion is between two and 15 minutes. The test was terminated, the time was recorded as the nearest second they had stopped pedaling, and the Pmax of each subject was determined from the graph of completed work and exhausted time. The power slope of the resulting linear relationship was defined as Pmax [6]. The experimental results are shown in Fig. 3.

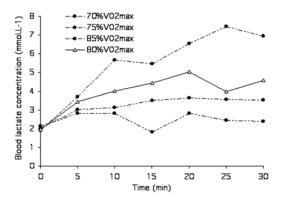


Figure 3: Determination of maximum power in a typical subject from the linear relationship between accomplished work (kJ) and exhaustion time (s).

According to the image results obtained in Fig. 3, the lactic acid concentration (La) and maximum power (Pmax) in human body were fitted in computer to find the most appropriate functional relationship. We can get:

$$P_{max} = k_{l1}[\arctan(La - \alpha)] + k_{l2}$$
(3)

At the same time, when La in human body is 0, that is, when lactic acid is not produced at the beginning of exercise, the rider's Pmax can be taken to the maximum (Pmax) Max, which is also determined by the ability of different competitors. So we get the following formula:

$$k_{l1}[\arctan(10 - \alpha)] + k_{l2} = 0$$
 (4)

$$La = VT2 (5)$$

$$k_{l1}[\arctan(0-\alpha)] + k_{l2} = (P_{max})_{max}$$
 (6)

# 2.3. P-E limitation model

According to the basic principles of kinematics and energy, the work done by the rider during the total cycle should not be greater than the energy available in his body. That is:

$$W_a = \int Pdt \le E_u \tag{7}$$

# 2.4. Results and the Application of the model

By referring to the results and data of cyclists of different genders and types, we draw the following data conclusions. The results are shown in table 1 and table 2.

Table 1: Data on male riders

type	Pl	Ke	VT2	(Pmax)max
Rouleur	500	1/3	6	1250
Specialist	600	1/3	6	1400
Climber	500	1/2	7	1200
Puncheur	600	1/3	6	1350
Sprinter	550	1/3	6	1250

Table 2: Data on female rider

type	Pl	Ke	VT2	(Pmax)max
Rouleur	475	1/3	6	1100
Specialist	550	1/3	6	1250
Climber	450	1/2	7	1000
Puncheur	550	1/3	6	1200
Sprinter	500	1/3	6	1150

The specific results are shown in table 3 and table 4.

Table 3: Male rider parameter results

Type	Kp1	Kp2	Ke	Kl1	K12	α
Rouleur	1/3	0.01	1/3	900	arctan-4	6
Specialist	1/2	0.01	1/3	100	arctan-4	6
Climber	1/2	0.01	1/2	900	arctan-3	7
Puncheur	1/3	0.01	1/3	1100	arctan-4	6
Sprinter	1/3	0.01	1/3	1200	arctan-4	6

Table 4: Female rider parameter results

Type	Kp1	Kp2	Ke	K11	K12	α
Rouleur	1/3	0.01	1/3	800	arctan-4	6
Specialist	1/2	0.01	1/3	900	arctan-4	6
Climber	1/2	0.01	1/2	800	arctan-3	7
Puncheur	1/3	0.01	1/3	950	arctan-4	6
Sprinter	1/3	0.01	1/3	1000	arctan-4	6

# 3. Time Trial Simulation and Pacing Strategy Optimization Model

# 3.1. Riding dynamical system

Force analysis is made on the ground resistance suffered by the rider in the process of riding. A diagram is shown in Fig.4.

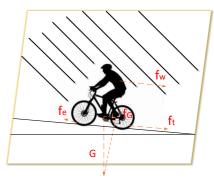


Figure 4: Force analysis of dynamic system

The power of gradient friction(Pt) can be obtained from the classical mechanics formula:

$$P_t = C_v * CCR * m * g * \cos(\arctan(Gr))$$
(8)

When the rider rides on a slope, the gravity component becomes resistance and the power of work done is:

$$P_G = C_v * m * g * Gr \tag{9}$$

Given wind speed  $V_w$  and wind direction Dw (relative to the speed direction), we can obtain tangential wind speed Va:

$$V_a = V_c - V_w * \cos(D_w) \tag{10}$$

Thus, the total wind resistance work power is:

$$P_{w} = \frac{1}{2} * V_{c} * \rho * V_{a} * |V_{a}| * area * 0.001$$
 (11)

To obtain the work power (Pe) done by the extra friction during turning:

$$P_e = C_v * CCR * m * g * \frac{1}{2} * (1 - \cos(Dr))$$
 (12)

Thus, we can give an expression for the power (Pf) of the work done to overcome the resistance:

$$P_f = P_t + P_G + P_w + P_e (13)$$

And then we have to think about the work that's done because of the change in velocity. Apply the law of conservation of energy:

$$P_{dv} = \frac{\Delta E}{t} = \frac{m * (v_2^2 - v_1^2)}{2t} \tag{14}$$

To sum up, we can get the total power P of the rider during riding:

$$P = P_f + P_{dv} \tag{15}$$

# 3.2. Genetic algorithm

After the description of the riding process is completed, we use genetic algorithm to optimize the pacing strategy. The classical genetic algorithm is shown in Fig.5.

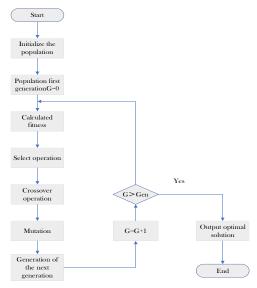


Figure 5: Flow chart of classical genetic algorithm

When the parameters related to the rider are entered into the program and the track parameters have been determined. We know that the velocity in parti is vi. We can get:

$$\begin{cases}
3.84 \le v_i \le 27.78 \\
P_i = g(v_i) \\
La_i = t(P_i)
\end{cases}$$

$$P_{\max(i+1)} = y(La_i)$$
(16)

Calculating the total ride time of the rider:

$$t_k = \sum_{i=1}^{R_{num}} \frac{R_{pd}}{v_i} \tag{17}$$

Putting the three track parameters into the model. The optimal efficiency planning was carried out for riders of different genders, types and levels.

Entering the track parameters and the different rider parameters for the Japan winter olympic cycling time trial. The results are shown in Fig.6.

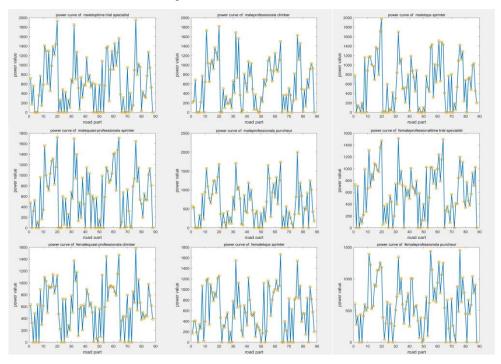


Figure 6: The best power distribution of different cyclists in Belgium world cup track Similarly, apply the same method to the self-designed track, and the result is shown in Fig. 7.

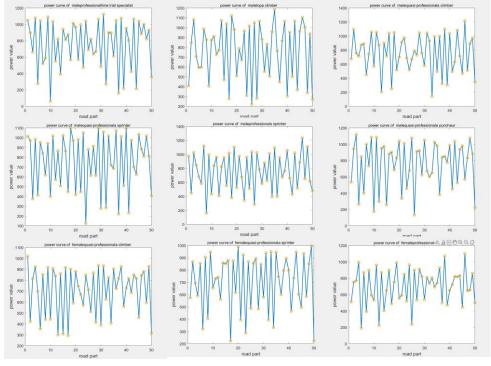


Figure 7: Optimal power distribution of different cyclists in self-made track

Through the above model, the optimal power curves corresponding to different racers on different circuits can be obtained. Here we randomly select 9 types of racers in the Belgium World Cup circuit and give their optimal power curves, which is shown in Fig.8.

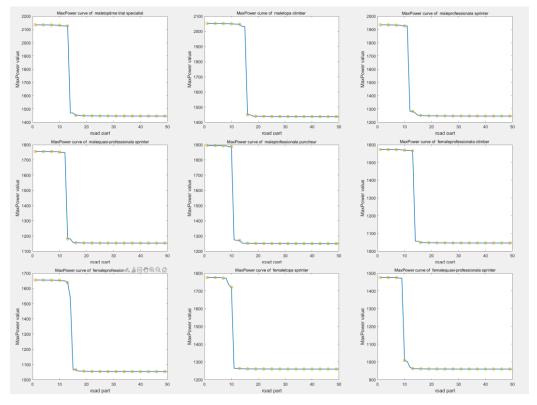


Figure 8: Best power curve of different players in Belgium world cup track

# 4. Team Time Trial Simulation Model

For the members who didn't fall behind, we sorted them according to certain rules. First, we numbered six members of a team from 1 to 6. Next, we define the amount of power remaining on the current journey:

$$P_{le(i,j)} = P_{\max(i,j)} - P_{(i,j)}$$
(17)

Similarly, the current remaining energy is defined:

$$E_{le} = E_u - E_c \tag{18}$$

We use the improved team competition objective function as the objective function of the genetic algorithm to optimize the solution. The results are constrained by the definition of drop-out member less than or equal to 2 and drop-out mechanism. The optimal speed and sorting strategy are obtained, which is shown in Fig.9.

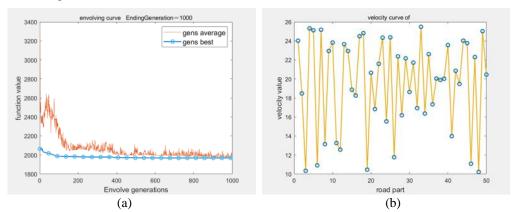


Figure 9: (a) Envoving curve, (b) Velocity curve

The results of the weather sensitivity analysis are shown in Fig. 10.

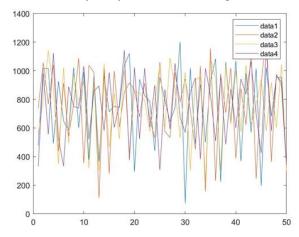


Figure 10: Weather sensitivity analysis results

The results of slope sensitivity analysis are shown in Fig. 11.

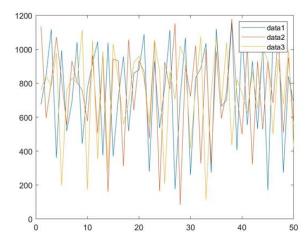


Figure 11: Slope sensitivity analysis results

The sensitivity analysis results of steering Angle are shown in Fig. 12.

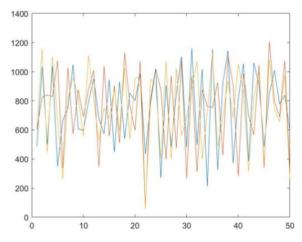


Figure 12: Steering angle sensitivity analysis results

# 5. Conclusion

In cycling road races, riders need to optimize their riding efficiency according to their own strengths. Therefore, in this paper, we need to establish a mathematical model for analysis. We built a dynamic

power curve model. The model incorporates a model of human circulatory movement dynamics and physiological constraints and the model's self-feedback mechanism. We established a timing test simulation and pacing strategy optimization model based on the micro-element method and random velocity generation, and solved it by genetic algorithm. We build a tempo strategy optimization model for team play. The optimal solution is obtained by using the improved objective function of team competition in genetic algorithm. Finally, a sensitivity analysis of the model was performed.

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