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Whale optimization algorithm based on Gaussian mutation and differential evolution

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Abstract: *Based on the defect that whale optimization algorithm is easy to fall into local optimization and slow convergence speed, this paper proposes an improved whale optimization algorithm based on Gauss mutation and differential evolution. The algorithm initializes the population through logistic chaos, improves the diversity and randomness of the population, and enhances the global search ability of the algorithm. Gaussian mutation and differential evolution are introduced to enhance the local search ability and improve the search accuracy. The performance of the improved optimization algorithm is tested by 12 benchmark functions. The results show that compared with the basic whale algorithm, particle swarm optimization algorithm and Multi-Verse Optimizer algorithm, IWOA algorithm can improve the convergence speed of the population and improve the search accuracy and stability of the algorithm to a certain extent.*

Keywords: *Chaos initialization, Gaussian mutation, Differential evolution, Benchmark functions*

1. Introduction

In recent years, many scholars have proposed many meta heuristic intelligent optimization algorithms. The first branch is based on the habits of animals in nature. For example, the bat algorithm that simulates bats sending and receiving ultrasonic waves for predation[1], the particle swarm optimization algorithm that simulates the foraging behavior of birds[2,3], the ant colony algorithm that simulates the phenomenon that ant colony finds the shortest path in the foraging process[4], the cuckoo algorithm that simulates cuckoo parasitic incubation habits[5,6], the gray wolf algorithm that simulates gray wolf hierarchy and predation process[7] Artificial bee colony (ABC) based on colony habits[8], etc. The second major branch of meta heuristics is physics based technology. This optimization algorithm usually imitates physical rules. The most popular are gravity search algorithm (GSA)[9], local search algorithm (LS)[10], ray optimization (RO)[11], small world optimization algorithm (SWOA)[12], Galaxy based search algorithm (GBSA)[13], curve space optimization (CSO)[14] and Multi-Verse Optimizer algorithm (MVO) based on multiverse theory in Physics[15]. Due to its simple principle, easy implementation and few adjustable parameters, meta heuristic algorithm has been widely used in the fields of bone structure size optimization, wireless sensor networks and power system.

Whale Optimization Algorithm (WOA) is a new swarm intelligence optimization algorithm proposed by Mirjalili of Griffith University in Australia in 2016 by observing the foraging behavior of whales[16]. The main idea of the algorithm is to solve the optimization problem by imitating the predation behavior of whales and fish. The main steps of whale predation are as follows: (1) Surrounding to the prey; (2) Spiral update location; (3) Random search. Although it has the advantages of simple operation, less parameter adjustment and strong ability to jump out of the local optimal solution, it has the disadvantages of easy falling into the local optimal solution, slow convergence speed and low convergence accuracy. Therefore, many scholars have improved the whale optimization algorithm in recent years.

In order to further improve the search ability of the algorithm, this paper proposes a whale optimization algorithm based on global search strategy. The algorithm uses Gaussian mutation to improve the local search ability, avoid falling into the local optimal solution, and improve the convergence speed of the algorithm. At the same time, inspired by the difference algorithm (DE)[17], a differential perturbation strategy is designed to improve the diversity of the population. The optimization ability of the IWOA algorithm is tested on multiple test functions. The results show that the algorithm has high optimization accuracy and fast convergence speed.

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The framework of this paper is as follows. In the first section, the development of the meta-heuristic algorithm is introduced, and an overview of the work of this paper is given. The rationale for the whale optimization algorithm is described in Section 2. Three strategies for improving the whale optimization algorithm are presented in Section 3. Experimental validation and result analysis are carried out in Section 4. In Section 5, the research of this paper is summarized and prospected.

2. Whale optimization algorithm (WOA)

Whales themselves are relatively large, and in the long process of evolution, they have formed a predation mode adapted to their own characteristics. Inspired by the way whales hunt, WOA algorithm includes three position updating methods: surrounded prey, rotating search and random search.

2.1. Surrounding to the prey

At this stage, humpbacks don't know where food is, and they work together in groups to obtain information about the location of the food. The whale that is closest to the food is used as a local optimal solution, sharing information about the prey that has been found so far, and other whales move to this position to form a circle around the food. The update formula of whale position in this link is:

$$\vec{D} = \left| \vec{C} . \vec{X}^*(t) - \vec{X}(t) \right| \tag{1}$$

$$\vec{X}(t+1) = \vec{X}^*(t) - \vec{A} \cdot \vec{D} \tag{2}$$

Where t indicates the current iteration, A and C as the coefficient vector, X^* to get the best solution for the current position vector, X is the position vector, It is worth mentioning here that X∗ should be updated with each iteration if a better solution is available. Vectors A and C can be calculated as follows:

$$\begin{cases} A = 2a \cdot r_1 - a \\ D = 2r_2 \\ a = 2 - 2t/t_{max} \end{cases} \tag{3}$$

Where r_1 and r_2 are uniformly distributed random numbers [0,1]; t is the number of iterations; t_{max} is the maximum number of iterations; A is the convergence factor, decreasing linearly from 2 to 0.

2.2. Spiral update location

In an upward spiral, the bubble attack acts as a gradual narrowing of the encirclement for food. Assuming that the probability to select the contraction enveloping mechanism and the spiral update position is 0.5, the spiral search expression is:

$$\vec{X}(t+1) = \vec{D}' \cdot e^{bl} \cdot \cos(2\pi l) + \vec{X}^*(t) \tag{4}$$

Where $\vec{D}' = \left| \vec{X}^*(t) - \vec{X}(t) \right|$ represents the distance from the best whale location ever obtained to the prey; b is the constant that defines the logarithmic spiral shape; l is the random number in [−1,1].

$$X(t+1) = \begin{cases} X^*(t) - A \cdot |X^*(t) - X(t)|, p < 0.5 \\ X^*(t) + D \cdot e^{bl} cos(2\pi l), p \geq 0.5 \end{cases} \tag{5}$$

2.3. Random Search

When the coefficient vector |A| > 1, it means that whales are swimming outside the constricted enclosure. At this time, random search is performed on individual whales to increase the search range and improve the global search ability. Its mathematical model is as follows:

$$X(\vec{t}+1) = \vec{X}_{rand} - \vec{A} \cdot \vec{D} \tag{6}$$

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In the formula, $\vec{D} = |\vec{C} \cdot \vec{X}_{rand} - \vec{X}|$, and X_{rand} is a random position vector selected from the current population (random whales).

3. Improved whale optimization algorithm(IWOA)

3.1. Chaos initialization

Logistic chaotic map is a typical representative of chaotic map, which can improve the diversity and randomness of population and enhance the global search ability of the algorithm. Because of its simple mathematical form, it is widely used. Its mathematical expression is:

$$X_{i+1} = \mu X_i * (1 - X_i) \tag{7}$$

Where $X_i \in [0,1]$; $\mu \in [0,1]$ is the logistic parameter. When μ is closer to 1, the value range of X_i will be evenly distributed to the whole [0,1] region.

3.2. Gaussian mutation

$$X(t+1) = (1+\mu) \cdot X(t) \tag{8}$$

Gaussian variation comes from Gaussian distribution, which specifically refers to replacing the original parameter value with a random number conforming to the normal distribution with mean μ and variance σ^2 [18]. According to the characteristics of normal distribution, the local search area of Gaussian variation can be better close to the original individual and has strong local search ability. For the optimization problem with a large number of local minima, the algorithm can find the global minima efficiently and accurately. At the same time, it also improves the robustness of the algorithm[19].The variation formula is as follows:

3.3. Differential variation strategy

Inspired by the mutation strategy of differential evolution algorithm, a random differential mutation strategy is proposed in this paper. We use the current whale individual, the current optimal individual and randomly selected whale individuals in the population to conduct random difference to generate new individuals,which generate individuals with good diversity to help the algorithm avoid falling into local optimum.Its specific expression is:

$$X(t+1) = r \times (X^* - X(t)) + r \times (X'(t) - X(t)) \tag{9}$$

Where X* is the position of the current optimal individual; r represents a random number generated between 0 and 1; X(t) is the initial position of the whale; X'(t) is the position of the whale after differential variation; X(t+1) is the updated value of differential mutation strategy.

The pseudo code of this article is as follows.

Set parameters and initialize whale population $X_i(i=1,2,...,n)$

Calculate the fitness value of each search agent

X^* = Optimal solution of current individual

while (t < maximum number of iterations)

for each search agent

Update a,A,C,l,and p

Update X^* by the Eq. (6)

if1 (p < 0.5)

if2 (|A| ≥ 1)

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Select a random search agent (X_{rand})

Update the position of the current search agent by the Eq. (5)

else if2 (|A| < 1)

Update the position of the current search agent by the Eq. (2)

end if2

else if1 (p ≥ 0.5 && p<0.9)

Update the position of the current search agent by the Eq. (4)

else if1 (p ≥ 0.9)

Update the position of the current search agent by the Eq. (7)

end if1

Update the position of the current search agent by the Eq. (8)

end for

Check if any search agent goes beyond the search space and amend it

Calculate the fitness of each search agent

Update X^* if there is a better solution

t = t+1

end while

return X^*

Table 1: Description of benchmark functions

Function	Dim	Search interval	f_{min}	Function	Dim	Search interval	f_{min}
$f_1(x)=\sum_{i=1}^{n} x_i^2$	30	[-100,100]	0	$f_7(x)=-20\exp(-0.2\sqrt{\frac{1}{n}\sum_{i=1}^{n} x_i^2})-\exp(\frac{1}{n}\sum_{i=1}^{n}\cos(2\pi x_i))+20+e$	30	[-32,32]	0
$f_2(x)=\sum_{i=1}^{n}\lvert x_i\rvert+\prod_{i=1}^{n}\lvert x_i\rvert$	30	[-10,10]	0	$f_8(x)=\frac{1}{4000}\sum_{i=1}^{n} x_i^2-\prod_{i=1}^{n}\cos(\frac{x_i}{\sqrt{i}})+1$	30	[-600,600]	0
$f_3(x)=\sum_{i=1}^{n-1}[100(x_{i+1}-x_i^2)^2+(x_i-1)^2]$	30	[-30,30]	0	$f_9(x)=(\frac{1}{500}+\sum_{j=1}^{25}\frac{1}{j+\sum_{i=1}^{2}(x_i-a_{ij})^6})^{-1}$	2	[-65,65]	1
$f_4(x)=\sum_{i=1}^{n}(x_i+0.5)^2$	30	[-100,100]	0	$f_{10}(x)=(\frac{1}{500}+\sum_{j=1}^{25}\frac{1}{j+\sum_{i=1}^{2}(x_i-a_{ij})^6})^{-1}$	2	[-5,5]	0.398
$f_5(x)=\sum_{i=1}^{n} ix_i^4+random[0,1)$	30	[-1.28,1.28]	0	$f_{11}(x)=-\sum_{i=1}^{4} c_i\exp(-\sum_{j=1}^{6} a_{ij}(x_j-p_{ij})^2)$	6	[0,1]	-3.32
$f_6(x)=\sum_{i=1}^{n}[x_i^2-10\cos(2\pi x_i)+10]$	30	[-5.12,5.12]	0	$f_{12}(x)=-\sum_{i=1}^{10}[(X-a_i)(X-a_i)^T+c_i]^{-1}$	4	[0,10]	-10.4028

4. Experiment and analysis

4.1. Experimental design and benchmark function

In order to verify the feasibility and superiority of IWOA algorithm, experiments are carried out on 12 different types of benchmark functions. The proposed algorithm is simulated in MATLAB R2020a. As shown in Table 1, there are 5 high-dimensional unimodal test function F1 ~ F5, 3 high-dimensional multimodal test functions F6 ~ F8 and 4 low-dimensional multimodal test functions F9 ~ F12. The optimization ability of IWOA can be fully tested through various types of benchmark functions.The optimization results of IWOA, WOA, PSO and MVO algorithms on the test function are compared.

In the experiment, the population size n = 30, the maximum number of iterations t = 500, the dimension of the objective function and the upper and lower bounds of the initial value are determined according to the benchmark functions in Table 1.

In order to avoid the contingency of optimization results and verify the stability of IWOA algorithm,

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the experimental results of 30 independent runs of each benchmark function are selected as experimental data. Each algorithm runs independently on each benchmark function for 30 times, and the average value and standard deviation of the searched optimal solution are used as the final evaluation index, as shown in Table 2.

4.2. Comparative analysis of algorithm performance

Table 2: Comparison of optimization results obtained for the benchmark functions

F	IWOA		WOA		PSO		MVO	
	ave	std	ave	std	ave	std	ave	std
F1	5.75E-85	2.79E-84	4.2E-82	2.26E-81	0.00014	0.00020	1.22247	0.33042
F2	2E-54	6.14E-54	6.85013E-54	3.31514E-53	0.04214	0.04542	1.15153	1.10483
F3	27.41275	0.38535	27.44	0.45542	96.71832	60.11559	461.92	690.66
F4	0.07010	0.04202	0.09209	0.05735	0.00010	8.28E-05	1.30400	0.30876
F5	0.00150	0.00191	0.00236	0.00212	0.12285	0.04496	0.03572	0.01699
F6	0	0	0	0	46.70423	11.62938	121.69	32.73
F7	4.32E-15	2.55E-15	4.79616E-15	2.696E-15	0.27602	0.50901	1.99600	0.53672
F8	0	0	0.000289	0.001582	0.00922	0.00772	0.83516	0.11371
F9	1.22955	0.62117	2.79625	3.27482	3.62717	2.56083	0.99800	0
F10	0.000797	0.0004	0.00083	0.00041	0.00058	0.00022	0.00543	0.00838
F11	-3.223	0.1001	-3.2204	0.13548	-3.26634	0.06052	-3.25304	0.06138
F12	-8.80298	2.4757	-8.51358	2.97446	-8.45653	3.08709	-8.65868	2.76309

The experimental results in Table 2 show that for one-dimensional peak functions F1 ~ F5, IWOA is better than the other three algorithms in terms of optimization stability and optimization accuracy (with smaller average value), and the stability of the algorithm is better (with smaller standard deviation); For functions F6 and F8, IWOA obtains the theoretical optimal value of the function, which is obviously better than PSO and MVO; For functions F9 ~ F12, the solution accuracy and stability of IWOA are better than WOA.

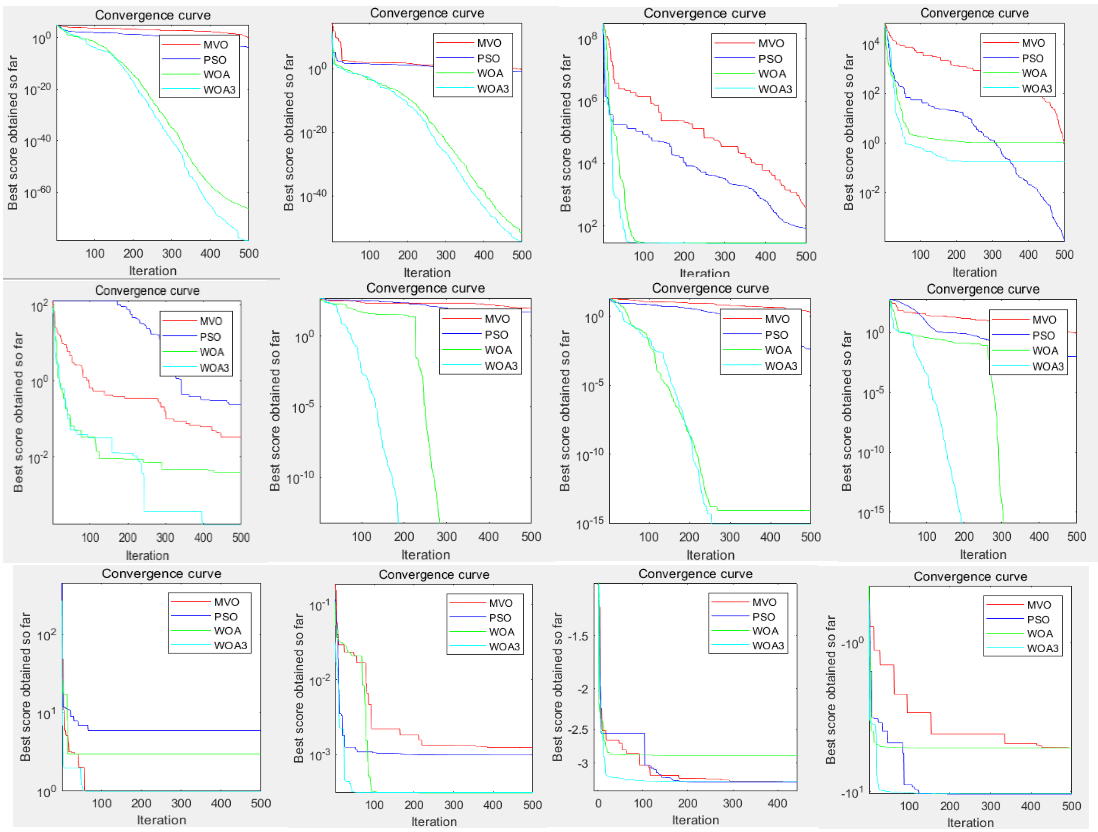

Figure 1: comparison of convergence curves of WOA and literature algorithms

In order to further compare the convergence performance of each algorithm, the fitness value (function value) convergence curve of F1 ~ F12 is drawn, as shown in Figure 1.

As can be seen from Figure 1, except for F4, the improved algorithm IWOA is superior to the other three algorithms in terms of convergence accuracy and convergence speed. The IWOA algorithm has obtained the global optimal solution after iterating about 100 times on the function F6, while WOA needs about 200 iterations to obtain the global optimal solution. Although the convergence accuracy of the IWOA algorithm is not significantly improved compared with the PSO and MVO algorithms on the function F4, the early convergence speed is significantly faster. In addition, PSO and MVO algorithms fall into local optimal solutions earlier on functions F1, F6, and F7, while IWOA can always find better solutions and avoid algorithm prematurity.

To sum up, the whale optimization algorithm updated by local Gaussian mutation and differential evolution can effectively balance the global search ability and local development ability of the algorithm, taking into account both the convergence accuracy and the convergence speed of the algorithm. Therefore, the improved whale optimization algorithm proposed in this paper is effective.

5. Conclusion

Whale optimization algorithm is an intelligent optimization algorithm with bionic optimization, but there are still some limitations in dealing with complex function optimization problems. Based on the optimization process of standard WOA, Gaussian mutation is introduced to jump out of local optimization to improve the convergence performance of the original whale algorithm and avoid falling into local optimization too early. What’s more, inspired by the mutation operator in the difference algorithm, the global solution is further updated and the global search ability is well improved. The results show that the IWOA algorithm has fast convergence speed and high convergence accuracy. It is suitable for structural engineering problems such as feature selection, water resource demand prediction, pressure vessel design and so on.

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