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Research on insurance decision problem based on break-even model

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Abstract: *With the frequent occurrence of extreme weather, it brings huge losses to the economy and increases the pressure on insurance companies to pay out. However, globally, there are still many shortcomings in natural disaster insurance that need to be addressed. Therefore, the problems related to insurance modelling and building protection modelling are explored. Firstly, a natural disaster risk assessment model based on AHP-entropy weighting method and TOPSISI was established, and natural disasters in the US and China were risk assessed into three risk levels. Secondly, an insurance breakeven model was established, breakeven validation was carried out, insurance schemes for two regions were designed, the relationship between the breakeven point and the four benchmark factors was clarified, and sensitivity analysis was carried out. Finally, an insurance decision model was developed to determine risk thresholds, and it was concluded that areas where extreme weather occurs should be included in insurance coverage.*

Keywords: *Extreme weather, AHP-Entropy Weight-TOPSIS, Insurance*

1. Introduction

In recent years, with the continuous change of climate, natural disasters caused by climate are also increasing, such as typhoons, tsunamis, wildfires, cold waves, floods, etc., especially tropical cyclones, which have caused nearly 2 trillion US dollars in losses[1].Due to the increasing number of natural disasters, both the insurance company and the policyholder have brought pressure, the cost of the insurance company has increased, and the insurance amount of the policyholder has increased.

Chinese and foreign scholars have conducted research on the impact of extreme weather on the decision-making of the insurance industry, explaining how weather changes affect the insurance industry from different aspects. Huang Yujie, this paper studies natural disaster risk modeling, insurance pricing optimization, reinsurance strategy, and generalized linear model parameter estimation, covering the theoretical and practical applications of continuous and discrete cases, but does not take into account the differential analysis in different places[1-2]. Zhou J, Bu Y, This paper studies the integration of EWM-TOPSIS and ARIMA models to assess natural disaster risk, support insurance decision-making and real estate development, and reveal the severity and potential benefits of disasters in Washington State. The above literature has studied the insurance of natural disasters from different aspects[3].

Firstly, it is necessary to establish a natural disaster risk assessment model based on AHP-entropy weight method and topography. Secondly, the insurance break-even model is established to verify the break-even point of different natural disaster risk levels. Further, insurance programmes for two regions are designed to clarify the relationship between the break-even point and the four benchmark indicators, and a sensitivity analysis of the insurance indicators is conducted.

2. Insurance Model

2.1 AHP-Entropy Weight-TOPSIS Natural Disaster Risk Assessment Model

Step1: Establish Hierarchical analysis architecture

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The quantitative analysis and evaluation of the situation affecting the risk of natural disasters in the United States is divided into two levels. The uppermost level is target level A,that is, to select the most appropriate effective indicators to evaluate the risk of natural disasters. The lowest layer is index layer B. In order to put the system structure of effective indicators B1, B2, B3, B4, B5, B6, B7 and B8 in Figure 1, the quantitative analysis and evaluation of the situation affecting the risk of natural disasters in China is divided into two levels, the toplayer is target layer Y, and the bottom layer is index layer Z, as shown in Figure 2.

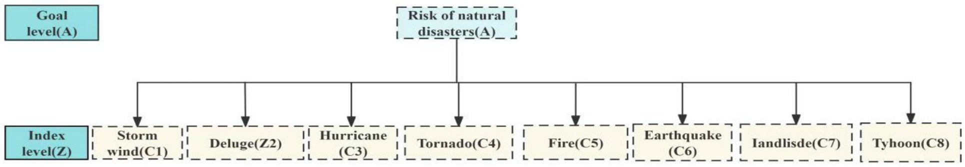

Figure 1: Natural disaster risk assessment layer in the United States

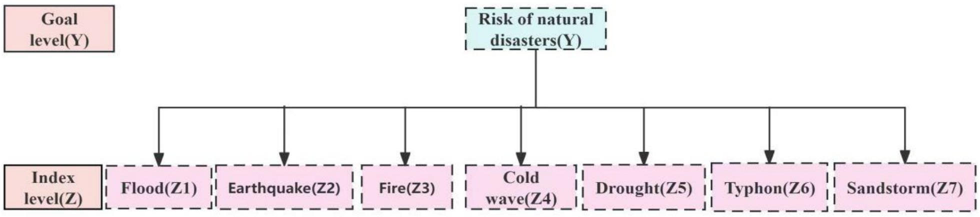

Figure 2: Natural disaster risk assessment layer in China

Next, will mainly calculate the risk assessment of natural disasters in the United States.

Step 2: Solve the AHP model

(1) Judging matrix A-B is obtained by means of expert scoring, as shown in the following Table 1.

Table 1: Judgment matrix A-B

Index	B1	B2	B3	B4	B5	B6	B7	B8
B1	1	2	3	4	4	5	5	5
B2	0.5	1	2	3	4	5	5	5
B3	0.333	0.5	1	3	4	5	5	5
B4	0.25	0.333	0.333	1	2	3	4	4
B5	0.25	0.25	0.25	0.5	1	2	2	2
B6	0.2	0.2	0.2	0.333	0.5	1	2	2
B7	0.2	0.2	0.2	0.25	0.5	0.5	1	2
B8	0.2	0.2	0.2	0.25	0.5	0.5	0.5	1

(2) The weights of A-B were solved by arithmetical average method, geometric average method and eigenvalue method respectively[4]. Finally, the weights obtained by the three methods were weighted to obtain the total weights determined by the final AHP, and the maximum eigenvalue $\lambda_{\max} = 8.435$ was obtained, and the values of CI and CR were calculated.

$$CI = \frac{\lambda_{\max} - n}{n-1} \tag{1}$$

$$CR = \frac{CI}{RI} \tag{2}$$

The result is $CR = 0.044 < 0.10$, which passes the consistency test.

Step 3: Use entropy weight method to calculate index weight

(1) The indicator matrix $A_{m\times n}$ is turned forward, $A_{m\times n}$ does not contain intermediate indicators, then small indicators need to be transformed into large indicators:

$$\max\{A_{m\times n}\} - a_{ij} \tag{3}$$

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(2) Indicator matrix $A_{m\times n}$ standardization:

$$\tilde{a}_{ij} = \frac{a_{ij} - \min\left\{a_{1j}, a_{2j}, \ldots a_{nj}\right\}}{\max\left\{a_{1j}, a_{2j}, \ldots a_{nj}\right\} - \min\left\{a_{1j}, a_{2j}, \ldots a_{nj}\right\}} \tag{4}$$

(3) The j index takes the proportion of the first i sample P_{ij} ,which is regarded as the probability used in entropy calculation:

$$p_{ij} = \frac{\tilde{a}_{ij}}{\sum_{i=1}^{n} \tilde{a}_{ij}} \tag{5}$$

(4) Calculate the information entropy of each indicator e_j, calculate the utility value, and normalize the entropy weight of each indicator. For the j indicator, the information entropy is as follows:

$$e_j = -\frac{1}{\ln n} \sum_{i=1}^{n} p_{ij} \ln\left(p_{ij}\right) \tag{6}$$

Step 4: Combine AHP-entropy weight method weights

(1) According to the principle of minimum relative information entropy, the entropy weight is used to modify the subjective weight, and the following model is solved:

$$\min y = \sum_{i=1}^{m} c_i \left(\ln\left(c_i\right)\right) - \left(\ln\left(w_{1i}\right)\right) - \left(\ln\left(w_{2i}\right)\right)$$
$$s.t. \sum_{i=1}^{m} = 1\left(c_i > 0\right) \tag{7}$$

(2) The Lagrange multiplier method is used to solve the problem, and the comprehensive weight c_i is obtained, in the following form:

$$c_i = \frac{\sqrt{w_{1i} w_{2i}}}{\sum_{i=1}^{m} \sqrt{w_{1i} w_{2i}}} \tag{8}$$

(3) Bring in w_{1i}, w_{2i}, which are the weights determined by AHP and entropy weight method respectively. The comprehensive weights are solved as shown in the Figure 3.

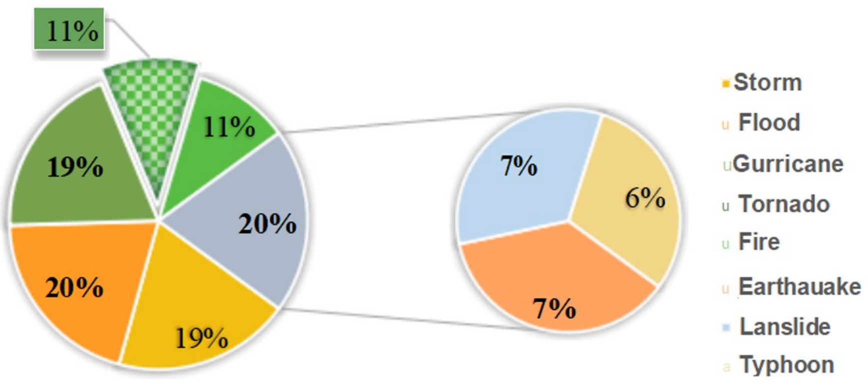

Figure 3: Indicator weight diagram

Step5: Use the good and bad solution distance method to calculate the natural disaster risk score.

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(1) According to the normalized index matrix $A_{m\times n}$, the optimal vector $A_j^+ = \max_{1\le i\le m}\left|A_{ij}\right|$ and the worst vector $A-j = \min_{1\le i\le m}\left|A_{ij}\right|$ are obtained

(2) Then the Euclidean distance between the index and the optimal vector is calculated.

$$D^+ = \sqrt{\sum_{i=1}^{n}\left(A_{ij} - A_J^+\right)^2} \tag{9}$$

Worst distance:

$$D^- = \sqrt{\sum_{i=1}^{n}\left(A_{ij} - A_j^-\right)^2} \tag{10}$$

(3) The relative proximity of the solution to the optimal value, that is, the score:

$$S_i = \frac{D_i^-}{D_I^+ + D_i^-} \tag{11}$$

After normalization, the natural disaster risk scores of the 50 states in the United States were obtained. The Table 2 shows the scores of the top four and the bottom four states.

Table 2: Normalized scores for eight states in the United States

Region	Score after normalization
Illinois	0.9994
Louisiana	0.9590
Oklahoma	0.9282
New York State	0.8970
Nebraska	0.3653
Oregon	0.3362
Wyoming	0.2906
Nevada	0.2545

Table 3: Normalized scores of six provincial administrative regions in China

Region	Score after normalization
Tibet Autonomous Region	0.9947
Qinghai Province	0.9863
Heilongjiang Province	0.9662
Hainan Province	0.4628
Jiangxi Province	0.4017
Fujian Province	0.3673

As can be seen from the above table, Nevada has the highest risk of natural disasters.

The natural disaster risk scores for China's 34 provincial administrations were then derived using the same calculation methodology as that used in the United States. The following table shows the scores for the top three and bottom three provincial administrations in China[5].

As can be seen from the above Table 3, the risk of natural disasters is highest in the Tibet Autonomous Region and lowest in Fujian Province.

2.2 Classification of Natural Disaster Risk LevelsResults

The level of risk of natural disasters is categorised into three levels, as shown in the Table 4 below.

Table 4: Risk levels of natural disasters

Level of risk	Scope
High risk	0.7~1
Medium risk	0.6~0.7
Low risk	0~0.6

The above figure shows the classification of the three types of risk levels. According to the risk levels, we determined the premium amounts of different levels in two regions and two countries. The following Table 5 and 6 shows the premium amounts of different levels in the United States and China respectively.

Table 5: Premium amounts of three classes in the United States

Level of risk	Premium amount (USD)
High risk	2000
Medium risk	4000
Low risk	6000

Table 6: Premium amounts of three classes in China

Level of risk	Premium amount (USD)
High risk	1000
Medium risk	2000
Low risk	3000

2.3 Insurance Breakeven Model

As to whether an insurance company should carryout insurance coverage in a place where extreme weather occurs frequently, considering that the profit of an insurance company must be greater than the cost, and the insurance company must bring benefits before it can carryout insurance coverage in this area, we need to establish an insurance breakeven model to achieve breakeven. The underwriting of insurance companies can be divided into two situations: information symmetry and information asymmetry[6]. We have rated natural disaster risk occurrence in each region of the United States and China above, so we only need to consider information symmetry here, and built the indicator weight diagram shown in Figure 4 below.

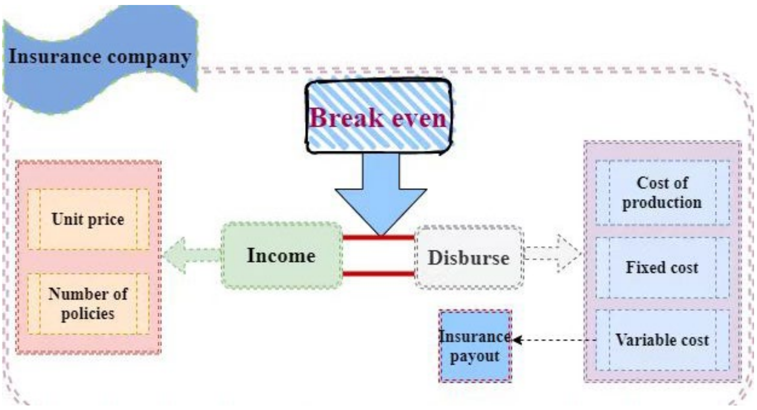

Figure 4: Indicator weight diagram

Step 1: It is necessary to take into account the total cost of the insurance company, and the amount of money spent by the insurance company in underwriting, through reading the references, you can know:

$$C_s = C_f + C_v + Q \tag{12}$$

Where C_s stands for total cost of production, C_f stands for fixed cost, C_v stands for unit variable cost and Q stands for production and sales volume.

Step 2: It is necessary to consider that insurance companies need to pay compensation for natural disasters caused by extreme climate after underwriting, so we need to calculate the claim rate, which is related to the number of natural disasters occurring in the region. Therefore, we can establish the equation of claim rate.

$$R_0 = r_z + r_s \tag{13}$$

Where R_0 represents the claim rate, r_z represents the disaster occurrence factor and r_s represents the disaster loss factor.

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Disaster occurrence coefficient refers to the frequency of a certain type of disaster over a period of time.

$$r_z = \frac{n}{T} \tag{14}$$

Where n represents the number of times a certain disaster occurs, and T represents a period of time.

The disaster factor refers to the extent to which a certain area is affected by a natural disaster.

$$r_s = \frac{p_0}{p_s} \tag{15}$$

Where P_0 is the number of people affected by the disaster, and P_s is the total population.

Step 3: After calculating the claim rate, it is necessary to calculate the amount of claims for natural disasters. Here, the present value coefficient of benefits needs to be considered, and the value of future cash at present needs to be known, so we can get the equation for calculating the total amount of claims:

$$C_r = R_0 \times Q \times C_a \times \frac{1}{(1+i)^n} \tag{16}$$

Where C_r represents the total amount claimed and C_a represents the average individual claim amount.

Step 4: Next, we need to consider the company's sales revenue and the income the company can earn from selling policies. We need to know the unit price of policies and the number of policies sold under different circumstances to calculate the income.

$$M = P \times Q \tag{17}$$

Where M represents sales revenue and P represents the average unit price sold. Q^* represents the break-even point reflected in policy volume.

Step 5: Consider breakeven, where profits and expenses need to be balanced.

$$M_1 = (P - T_b - T_v) \times Q \tag{18}$$

Where T_b and T_v refer to surcharges.

Step 6: After knowing that break-even income is required, it is necessary to calculate break- even sales income:

$$B^* = (P - T_b - T_v) \times Q^* = \frac{(P - T_b - T_v) \times C_f}{P - T_b - T_v - C_v} \tag{19}$$

Where B^* represents profit balancing sales revenue.

Step 7: Break-even capacity utilization:

$$E^* = \frac{Q^*}{Q_d} \times 100\% = \frac{C_f}{(P - T_b - T_v - C_v) \times Q_d} \times 100\% \tag{20}$$

Where Q_d represents the design and production capacity of the insurance scheme.

2.4 Model Solving

In this case, the regions we choose are the United States and China. In the above article, we also

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established the break-even insurance model for insurance companies that need to reach break-even for insurance coverage[7]. Next, we need to design the insurance schemes for the two regions to ensure the break-even of insurance companies and select the relevant indicators of insurance schemes to verify the break-even model.

(1) Insurance schemes in the United States

By searching for relevant data and information, we designed the insurance scheme in the United States, selected four indicators, namely, the expected claim rate, the average cost per policy, the average amount per claim, and the fixed cost, and determined the baseline value. As shown in Table 7 below.

Table 7: Baseline factors for United States insurance schemes

Index	Reference value
Expected claim rate	0.7
Average cost per policy	4000
Average amount per claim	100000
Fixed cost	50000

After determining the indicators and baseline values, we need to conduct break-even analysis through the insurance break-even model established above.

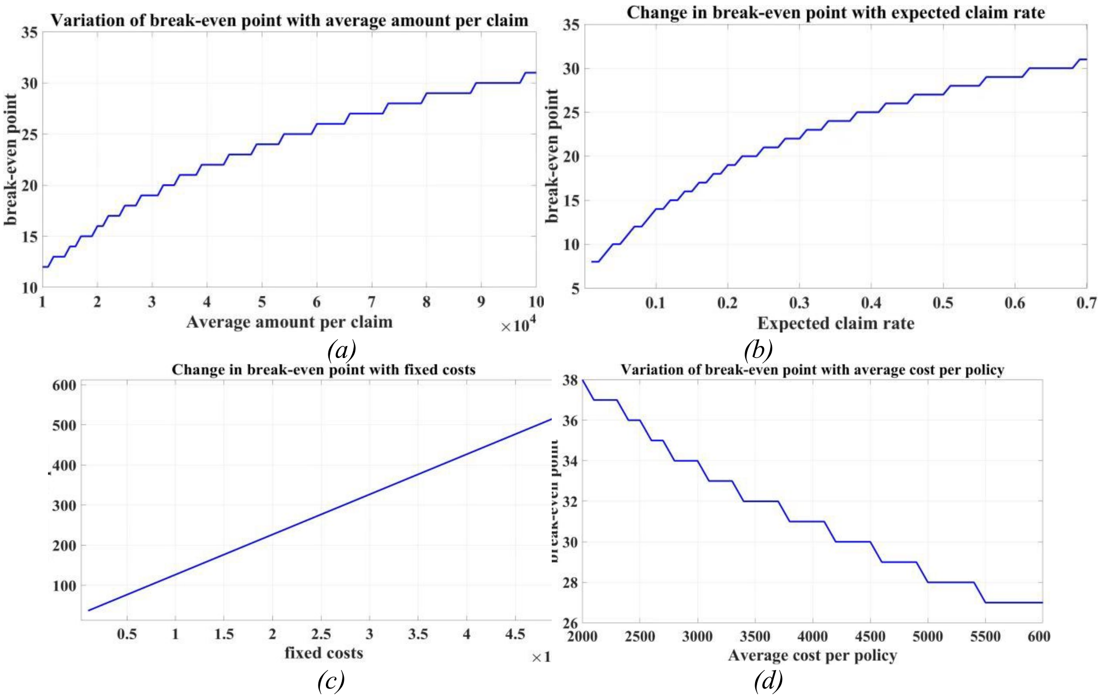

Figure 5: Relationship between the break-even point and the index in the United States

In the Figure 5 above, (a) represents the relationship between the break-even point and the expected claim rate,(b) represents the relationship between the average amount per claim and the break-even point,(c) represents the average cost per policy and the break-even point, and (d) represents the relationship between fixed costs and the break-even point[8]. As can be seen from the above four graphs, the higher the break-even, the greater the corresponding value. For American insurance companies, the lower the break-even point, the lower the number of policies sold or produced by the insurance company, the company can reach the break-even point, and the company is easy to make profits, if the break-even point is higher, the insurance company will be difficult to reach the break-even point[9]. A lower break-even point allows insurers to be more flexible and adapt to fluctuations in the market, reducing their own risk and attracting more investors.

(2) Insurance schemes in China

By searching relevant data and materials, we designed China's insurance program and selected four indicators, namely, expected claim rate, average cost per policy, average amount per claim, and fixed cost. The selection of these four indicators is the same as that of the insurance program in the United States, and the benchmark value of China's natural disaster insurance related indicators is also determined, as shown in Table 8 below.

Table 8: Baseline factors for insurance schemes in China

Index	Reference value
Expected claim rate	0.6
Average cost per policy	2000
Average amount per claim	100000
Fixed cost	30000

After determining the indicators and baseline values, we need to conduct a break-even analysis through the insurance break-even model established above.

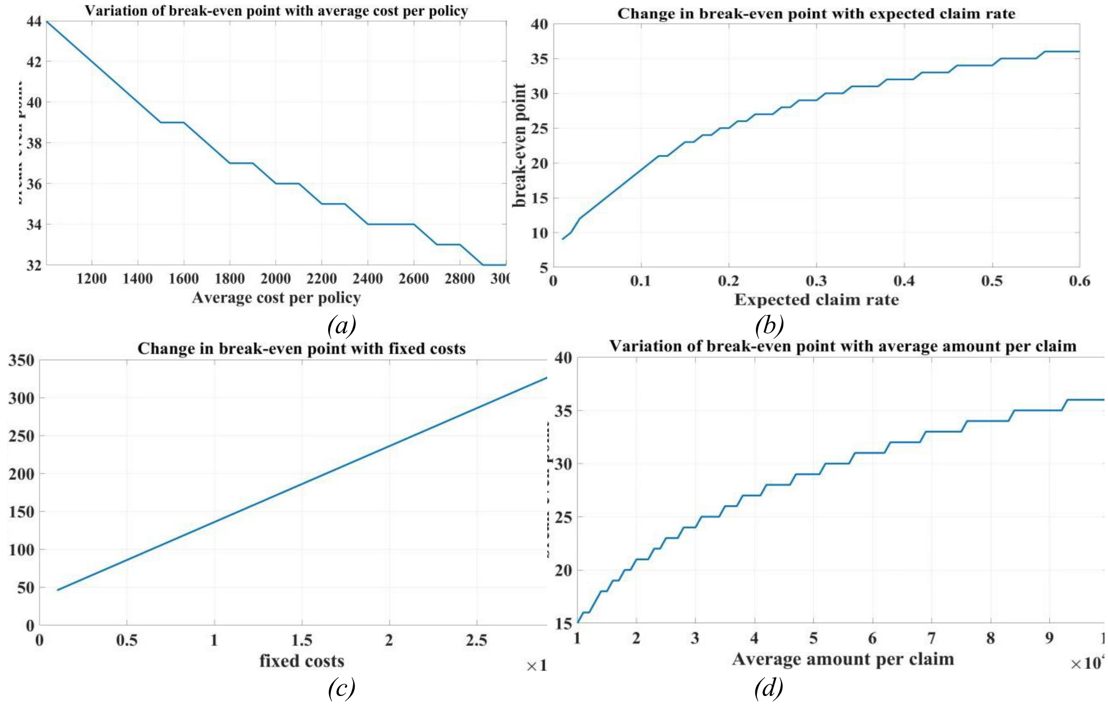

Figure 6: Relationship between break-even point and index in China

In the Figure 6 above, (a) represents the relationship between the break-even point and the expected claim rate, (b) represents the break-even point and the average cost per policy, (c) represents the average amount per claim and the break-even point, and (d) represents the fixed cost and the breakeven. The four charts all show that when the index rises, the break-even point will also increase. However, in the operation of enterprises, the lower the break-even point, the more beneficial the development of enterprises. In order to develop in areas with high risk of natural disasters, insurance companies should properly reduce the break-even point of the company, indicating that the insurance company can maintain a profitable state even with low premiums, and a high break-even point will make the insurance company face high cost and high-risk business.

The break-even point is the key indicator when an insurance company reaches a state of neither loss nor profit. For an insurance company, a lower breakeven point means that the company can break even with fewer policies sold, making it easier for the company to make a profit. A lower breakeven point also helps insurers to be more flexible in responding to market fluctuations, reducing operational risks and attracting investors. In particular, a lower breakeven point is important for insurers operating in areas with high risk of natural catastrophes, as it demonstrates that the company can remain profitable even when premiums are low. Too high a breakeven point, on the other hand, can expose insurers to the challenges of high-cost, high-risk operations.

3. Analysis of experimental results

3.1 Insurance schemes in the United States

By searching for relevant data and information, we designed the insurance scheme in the United States, selected four indicators, namely, the expected claim rate, the average cost per policy, the average amount per claim, and the fixed cost, and determined the baseline value, as shown in Table 9 below.

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Table 9: Baseline factors for United States insurance schemes

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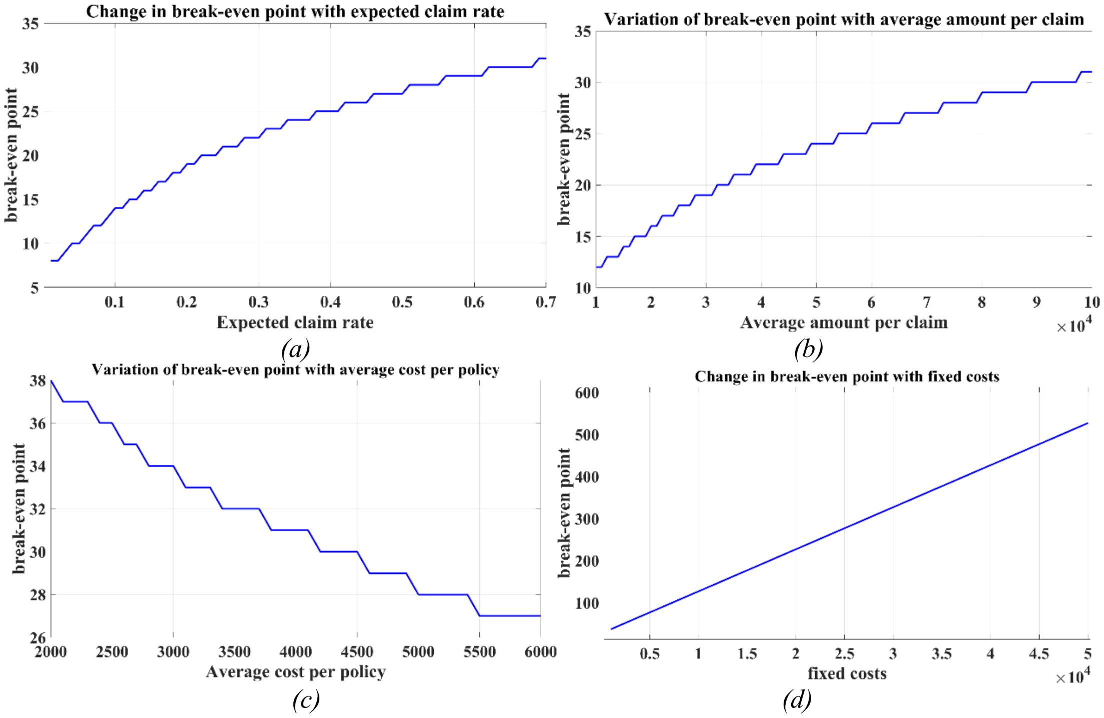

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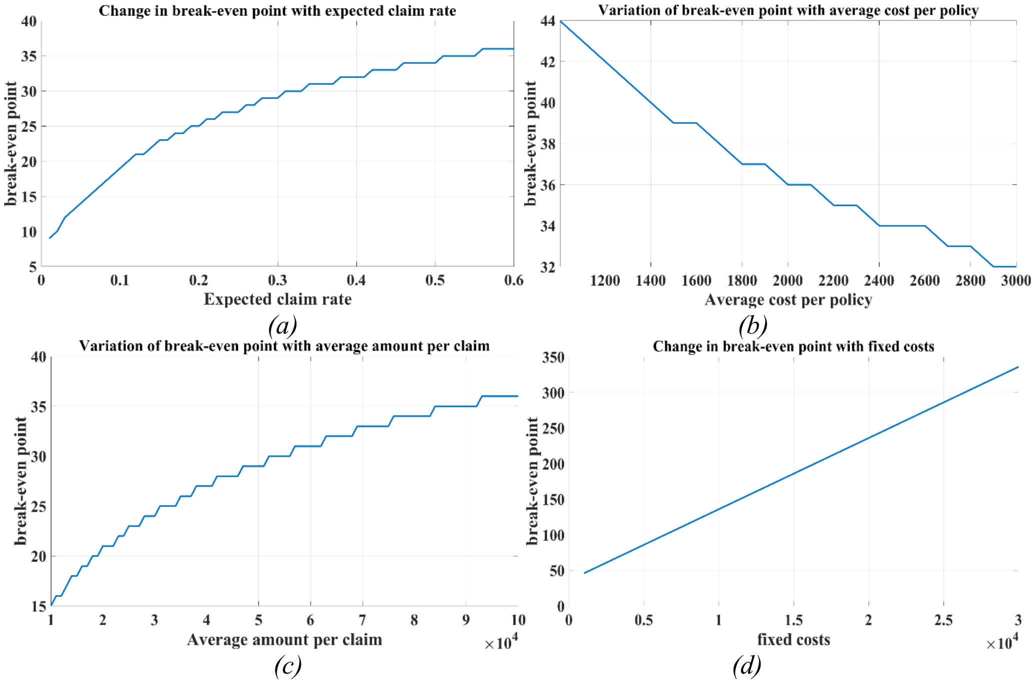

Figure 8: Relationship between break-even point and index in China

In the Figure 8 above, (a) represents the relationship between the break-even point and the expected claim rate, (b) represents the break-even point and the average cost per policy, (c) represents the average amount per claim and the break-even point, and (d) represents the fixed cost and the breakeven. The four charts all show that when the index rises, the break-even point will also increase.

3.3 Sensitivity Analysis of Indicators

In the above, we have analyzed the relationship between indicators and break-even point. Then we need to analyze the sensitivity of indicators to the model or break-even point to understand the impact of indicators on the insurance model.

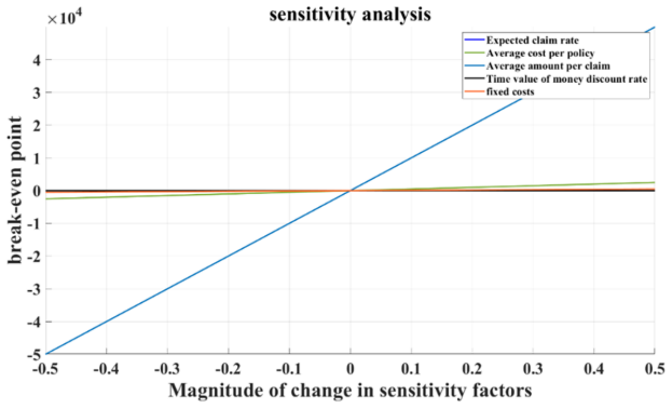

Figure 9: Sensitivity analysis of indicators

From the Figure 9 above, we can see that the average amount of each claim has the greatest influence on the model. As the amount of each claim increases, the break-even point also increases, while the other three indicators have no obvious influence on the break-even point.

In order to see the degree of influence of the other three indicators on the model, the sensitivity analysis diagram of these three indicators on the model is drawn separately.

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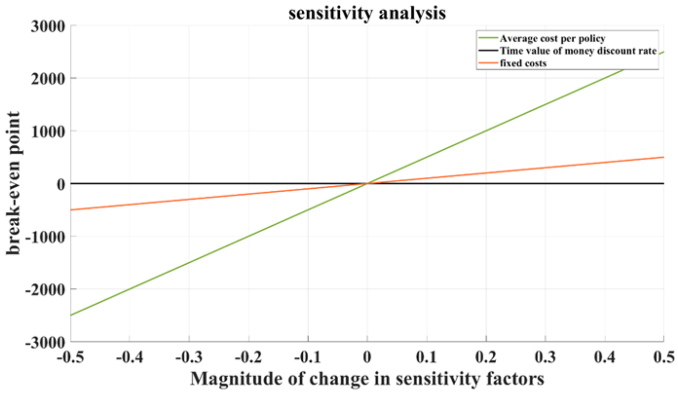

Figure 10: Sensitivity analysis

Figure 10 above shows the impact of expected claim rates, average cost per policy and fixed costs on the break-even point. As can be seen from the figure, the change in average cost per policy has a greater impact on the break-even point, while the other two indicators have a lesser impact.

In summary, it can be concluded that the index ranking of the degree of impact on the model is: average amount per claim, average cost per policy, fixed cost and expected claim rate.

3.4 Insurance Underwriting Decision Model

In the previous part, we evaluated the risk of natural disasters in the two places and planned the insurance plan of the insurance company[10]. Then we need to make a decision on whether the insurance company should carry out insurance coverage in the two places.

Step1: Determine the frequency of natural disasters, insurance rates and claim costs.

Step2: The financial risk threshold of the insurance company is 0.9. Step3: Calculate the risk value of the selected area:

$$R_k = W_n \times S_c \tag{21}$$

Where, R_k represents value-at-risk, W_n represents frequency of extreme weather natural disasters, and S_c represents average cost of claims.

Step4: Calculate the annual premium:

$$N = R_k \times F \tag{22}$$

Where, N represents the annual premium and F represents the premium rate.

Step 5: Calculate the total premium.

Step 6: Determine whether to cover:

$$PV < 0.9 \times TA \tag{23}$$

Where, PV stands for total premium and TA stands for total assets. If the total premium is less than the risk threshold multiplied by the total assets, coverage will be given in the region; otherwise, no coverage will be given.

In the end, it is concluded that insurance companies need to underwrite in areas with a lot of extreme weather events in the United States and China.

4. Conclusion

The frequent occurrence of extreme weather has brought great losses to the economy and increased the pressure of insurance companies to pay out. In this paper, the weights of eight major hazards in the United States and seven major hazards in China were firstly calculated by using the AHP-entropy

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weighting method, and then the natural disaster risk scores of each region of the two countries were determined by using the TOPSIS method, and three different risk levels were established. A breakeven model was then developed and the relationship between the breakeven point and the four benchmark factors in the insurance programme was found, and finally it was concluded that insurers need to underwrite in regions prone to extreme weather events in the US and China. Future research could further explore how technological innovations and data-driven approaches can improve insurers' ability to predict extreme weather events to meet the challenges posed by climate change.

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