Dynamic Analysis and Coupling Synchronization Control of A Class of Four-dimensional Chaotic Systems

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Abstract: *In this paper, for a class of four-dimensional complex chaotic systems, the stability of the equilibrium point of the system and the dissipation of the system are analyzed. Secondly, the complex chaotic attractors of the system are studied. At the same time, the chaotic behavior of the system is quantitatively analyzed by "0-1" test. Conclusion it is proved that the system has complex chaotic characteristics. Finally, through the establishment of the theory of coupling synchronization control for the four-dimensional chaotic system, the numerical analysis shows that the system can achieve synchronization control in a short time.*

Keywords: *"0-1" test, Chaotic attractor, Coupling synchronous control*

1. Introduction

Since the 1960s, nonlinear science has developed rapidly, which not only affects the existing scientific system, but also changes people's traditional view of the real world. Generally, nonlinear science includes chaotic, fractal and soliton. Chaotic is the most important part of nonlinear science. As a complex nonlinear motion, chaotic has been found in many fields, such as electronic circuits, bioengineering, energy and power, aerospace engineering and electrical machinery. Especially with the development and improvement of chaotic dynamics theory in the past three decades, Chaotic dynamic systems are widely used in communication security systems, so the construction of chaotic systems, especially the construction of complex chaotic systems, is still a research hotspot [1-2]. Since 1963, Lorenz discovered the first chaotic attractor in the three-dimensional autonomous chaotic system, and since then, people have continuously discovered new chaotic systems. Especially in the last decade, the construction and implementation of complex high-dimensional chaotic systems have become more and more important in practical engineering [3]. At present, these theoretical advances of chaotic systems have important applications in many fields, such as power electronic systems, robotics, lasers and secure communications [4-6].

In this paper, a kind of four-dimensional chaotic system is analyzed. The basic dynamic characteristics of the system are studied through the stability and equilibrium points of the system, the attractor diagram of the numerical simulation system and the "0-1" test [7-10]. At the same time, in order to better apply the system in practice, the coupling synchronous control of the system is designed and realized in theory, which further explains the objective existence of the system control design. The effectiveness and realizability of the method are verified by numerical simulation.

2. Chaotic System and Analysis

For a class of four-dimensional hyperchaotic systems, the dynamic equation is as follows:

$$\begin{cases} \dot{x} = -az \\ \dot{y} = (|y+1|-|y-1|) - y - w \\ \dot{z} = -bcx^2 + z + bw \\ \dot{w} = dy - dz \end{cases} \tag{1}$$

Where x, y, z, w are system variables, a, b, c and d are system parameters. When $a = 5, b = 9.25, c = 2.2, d = 3.5$ and initial value $x_0 = (0.1, 0.1, 0.1, 0.1)$, then the chaotic attractor of system (1) is shown in Figure 1. Through numerical calculation, it is concluded that the three Lyapunov exponents

of the system are $\lambda_1 = 0.1039, \lambda_2 = 0.0162, \lambda_3 = -0.0189, \lambda_4 = -34.5525$. Because there are two Lyapunov exponents greater than 0 and the dimension of the system is fractional, it can be determined that the system has hyperchaotic characteristics.
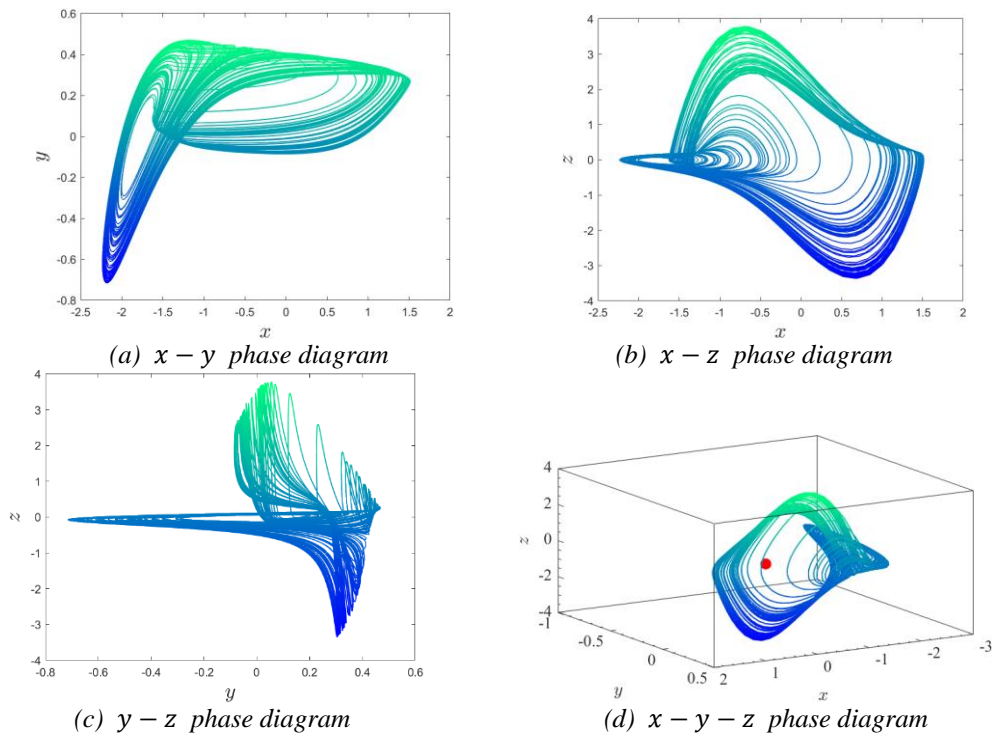

(a) $x - y$ phase diagram

(b) $x - z$ phase diagram

(c) $y - z$ phase diagram

(d) $x - y - z$ phase diagram

Figure 1: Chaotic phase diagram of system.

2.1. Dissipation

Because

$$\nabla V = \frac{\partial \dot{x}}{\partial x} + \frac{\partial \dot{y}}{\partial y} + \frac{\partial \dot{z}}{\partial z} + \frac{\partial \dot{w}}{\partial w} = -1 - 9.5(2.2x^2 - 1) \tag{2}$$

When $x > 1$ and $2.2x^2 - 1 > 0$, then the system (1) is dissipative, and converges exponentially $\frac{dV}{dt} = e^{-1-9.5(2.2x^2-1)}$, that is, volume element V_0 shrinks to volume element $Ve^{-1-9.5(2.2x^2-1)}$ at time t. This means that when $t \to \infty$, each volume element containing the trajectory of the system shrinks to zero at an exponential rate $-1 - 9.5(2.2x^2 - 1)$. Therefore, all system trajectories will eventually be limited to a set with zero volume, and its progressive motion is fixed on an attractor, which shows that the attractor exists.

2.2. Equilibrium Point and Stability

If the right side of each formula of system (1) is equal to zero, the system equilibrium point $E = (x_0, y_0, z_0, w_0)$ can be obtained, then there is a zero equilibrium point of the system, i.e. $E_0 = (0,0,0,0)$, which is represented by red dots in the chaotic phase diagram, as shown in Figure 1 (d).

The system is linearized at the equilibrium point $E_0 = (0,0,0,0)$, and the Jacobian matrix is obtained as:

$$J = \begin{pmatrix} 0 & 0 & -a & 0 \\ 0 & -1 & -1 & 0 \\ 0 & 0 & -b & b \\ 0 & d & -d & 0 \end{pmatrix} \tag{3}$$

Then the characteristic equation is

$$f(\lambda) = \lambda(\lambda^3 + (b+1)\lambda^2 + (bd+b)\lambda + 2bd) \tag{4}$$

The calculated characteristic value is: $\lambda_1 = 0, \lambda_2 = -6.1148, \lambda_3 = -2.0676 + 2.5128i, \lambda_4 =$

$-2.0676 - 2.5128i$, according to the Routh Hurwitz criterion, it can be determined that the equilibrium point is an unstable index type 2 equilibrium point.

2.3. System Poincare Map

Poincare mapping is a quantitative method to study the complex dynamic behavior of the system. Whether it is chaotic is judged by observing the distribution of the cut points on the Poincare section: when there are some dense points with fractal structure on the Poincare section, the motion is chaotic. In this overall model, take plane $z = 0.1$, obtain the corresponding Poincare section, and obtain the Poincare map of the system, as shown in Figure 2. It can be judged that system (1) is a chaotic system.
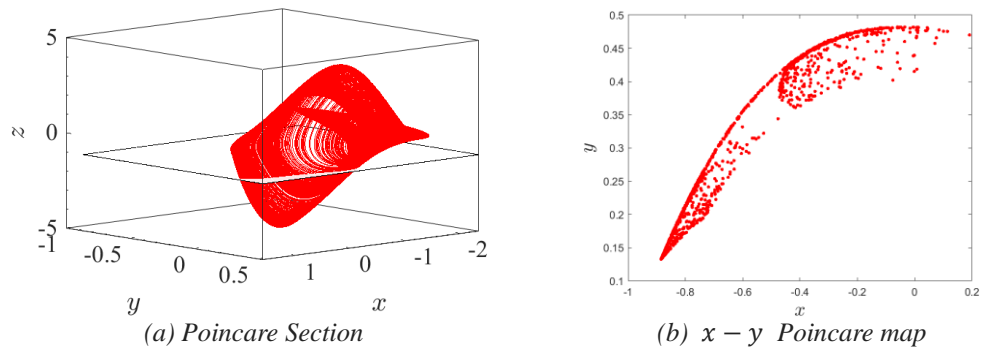

(a) Poincare Section   *(b) $x - y$ Poincare map*

Figure 2: Poincare map.

2.4. System "0-1" test

"0-1" test is a new method to judge whether the system is chaotic: it can directly judge whether the system is chaotic. It is tested by observing the trajectory of the dynamic system in the $p - s$ plane. If the trajectory rules in the $p - s$ plane are bounded, it indicates that the system is a non chaotic system. If the trajectory is unbounded and similar to Brownian motion, it is a chaotic state. As shown in Figure 3, the dynamic behavior of the system is analyzed by changing the system parameter d. when $d = 5.5$, the system is in a chaotic state; When $d = 8.5$, the system shows periodic behavior, and the analysis results are shown in Figure 3.
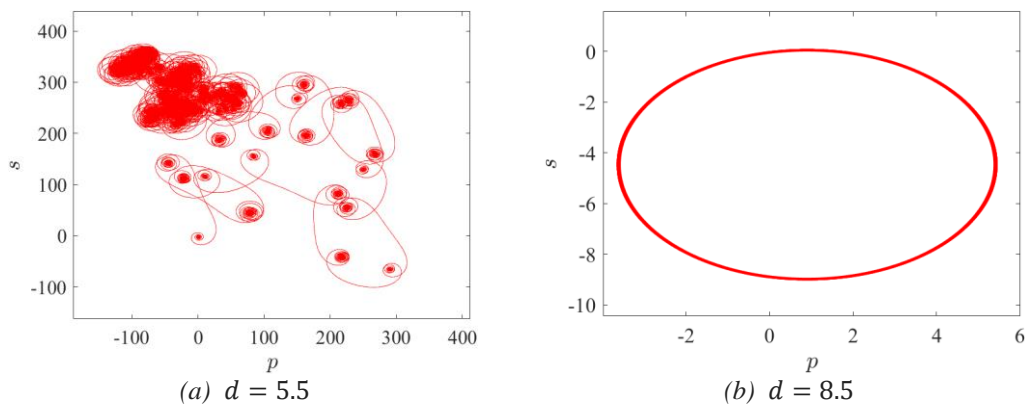

(a) $d = 5.5$ *(b) $d = 8.5$*

Figure 3: P-S plane trajectory of system.

3. System Coupling Synchronization

Using the system coupling contract method to realize the synchronous control of the system has achieved fruitful research results

$$\begin{cases} \dot{x} = -az \\ \dot{y} = (|y + 1| - |y + 1|) - y - w \\ \dot{z} = -bcx^2 + z + bw \\ \dot{w} = dy - dz \end{cases} \tag{5}$$

The response system is

$$\begin{cases} \dot{x}_m = -az_m + k_1(x_m - x) \\ \dot{y}_m = (|y_m + 1| - |y_m + 1|) - y_m - w_m + k_2(y_m - y) \\ \dot{z}_m = -bcx_m{}^2 + z_m + bw_m + k_3(z_m - z) \\ \dot{w}_m = dy_m - dz_m + k_4(w_m - w) \end{cases} \quad (6)$$

The error system is

$$\begin{cases} \dot{e}_1 = x_1 - x \\ \dot{e}_2 = y_1 - y \\ \dot{e}_3 = z_1 - z \\ \dot{e}_4 = w_1 - w \end{cases} \quad (7)$$

When the system control parameters are selected as $k_1 = 4, k_2 = 2, k_3 = 3, k_4 = 1$, and the parameter selection of the system (1) is consistent with the above, and the initial value of the drive system is $x_0 = (0.1, 0.1, 0.1, 0.1)$, the initial value of the drive system is $x_{m0} = (1,2,3,4)$. The synchronization sequence diagram and error diagram of systems (5) and (6) obtained by numerical simulation with MATLAB are shown in Figure 4 and Figure 5 respectively, which shows that the system has realized synchronization in a very short time, and finally realized the synchronization control of the system.
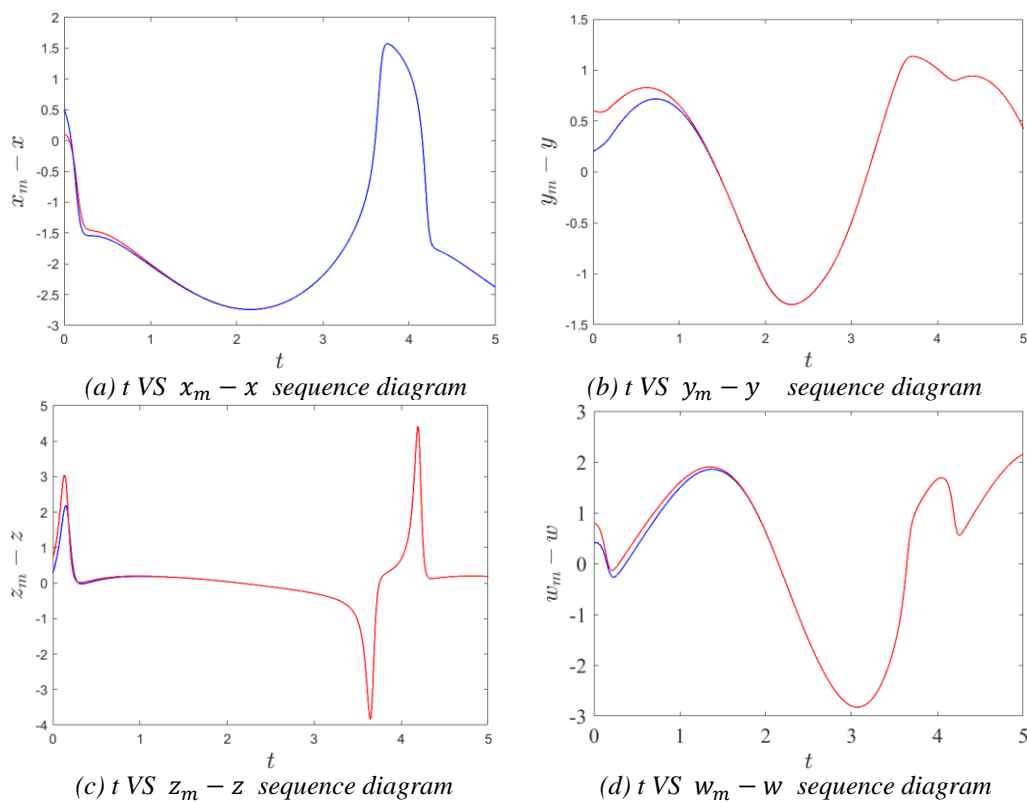

(a) t VS $x_m - x$ sequence diagram

(b) t VS $y_m - y$ sequence diagram

(c) t VS $z_m - z$ sequence diagram

(d) t VS $w_m - w$ sequence diagram
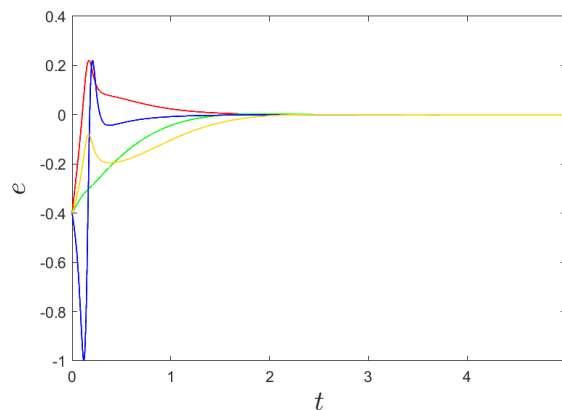
Figure 4: System state sequence diagram.

Figure 5: Synchronization error curve of system (1).

4. Conclusion

In this paper, the new four-dimensional chaotic system is numerically simulated and synchronized. By adjusting the parameters of the new system, the "0-1" test of the system is realized. The numerical simulation verifies the rich dynamic characteristics of the system. The coupling synchronization control of the system is theoretically analyzed, and the feasibility of the system synchronization control is verified by numerical simulation. The results provide a new idea for the design of communication encryption and chaos detection.

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