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Dynamic Analysis of an Improved Financial System with New Parameter

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Abstract: *To better understand and utilize the chaotic financial system, we introduce a quadratic term into the third equation of the original system, which represents the effect of external disturbances on the interest rate and the investment demand, etc., and thus establish a new four-dimensional chaotic system. The newly added parameters can effectively control the chaotic nature and stability of this system. By analyzing the Lyapunov exponent, chaotic attractor, phase diagram, and equilibrium point of the system, it is proved that the system has complex chaotic dynamics. In addition, because of the large influence of different initial values and the selection of different parameter values on the chaotic system, we used different initial values of the system and several typical new parameter values for simulation and observation, respectively, and from the results, it is consistent with our inference. All parts of the article are validated and illustrated by MATLAB numerical simulations.*

Keywords: *Financial System, Dynamics, Lyapunov exponent, Phase Diagrams, Numerical Simulation*

1. Introduction

Chaos is some deterministic nonlinear system that can give a non-periodic, seemingly chaotic output within a certain parameter range, that is, random motion from a definite system. The chaos phenomenon reveals that there is a bridge between certainty and randomness, which has far-reaching significance in scientific concepts. Since meteorologist Lorenz discovered the famous Lorenz system in 1963, the study of chaotic systems has attracted wide attention. Many new chaotic systems have been discovered, such as the Chen system and Liu system, etc. [1-4]. In recent years, with the development of nonlinear science, the application of chaos theory in economic and financial fields has been widely concerned. As we all know, the financial system is a very complex nonlinear system, which contains many complicated factors. In 1985, researchers first discovered the existence of chaotic behavior in the economic system, and then the dynamics analysis of a chaotic financial system and the improvement of the system have been widely studied and applied. The following aspects are mainly studied: Hopf bifurcation of complex financial systems, the relationship between parameters and Hopf bifurcation, chaotic motion [5-8], control and synchronization of three-dimensional chaotic financial systems [9-12].

Previously, based on the subprime crisis in 2007, we constructed a new hyperchaotic financial system by adding state variable, analyzed the new four-dimensional financial hyperchaotic system, and carried out dynamic analysis, bifurcation and equilibrium point studies, velocity feedback control, linear feedback control and synchronization studies on the new system [13-14]. This system has strong practical significance and application value. In fact, for a system, external disturbance is impossible to completely avoid, especially for economic systems, the impact of external conditions on the system is very obvious. Sometimes the disturbance of the economic system is not fixed, for example, the system is interfered with by an external pulse signal at one time, and is affected by other events at another time. Therefore, the study of the system under external interference has not only important theoretical significance, but also has great application value.

In this paper, an improved four-dimensional hyperchaotic financial system is constructed by adding a yz quadratic term and parameter m to the third equation of the four-dimensional financial system. The quadratic term and parameter represent the external disturbance of the world economy due to various extreme events in the recent past. By selecting appropriate parameters, the influence of external effects on the system can be controlled. For the improved system, the symmetry, Lyapunov index, phase diagram, and equilibrium point are analyzed and numerical simulation is given. The purpose of this paper is to provide some references for the regulation of this kind of economic system and provide new ideas for

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the stable operation of the economic system after the dynamic analysis of the financial chaotic system.

2. Construction of New Systems with External Disturbance Terms

The currently recognized four-dimensional hyperchaotic system is composed of four sub-blocks of production, money, inventory, and labor force, which are represented by four first-order differential equations. References [13] proposed a new financial chaotic system model. Where x, y, z, and w represent interest rate, investment demand， price index, and average profit rate respectively. Parameters a, b, c, d, and k are positive constants, representing savings, investment cost, elasticity of business demand, control parameters, etc. The chaotic financial system is given as follows:

$$\begin{cases} \dot{x} = -ax + z + w + xy \\ \dot{y} = -by - x^2 + 1 \\ \dot{z} = -x - cz \\ \dot{w} = -kw - dxy \end{cases} \tag{1}$$

Considering that changes in external conditions have a great impact on interest rate and investment demand, this paper improves the third equation of system (1), and by adding a quadratic term and new parameters, an improved financial system is obtained:

$$\begin{cases} \dot{x} = -ax + z + w + xy \\ \dot{y} = -by - x^2 + 1 \\ \dot{z} = -x - cz + myz \\ \dot{w} = -kw - dxy \end{cases} \tag{2}$$

Based on numerical and detailed theoretical analyses, we have chosen initial values of (0.01, 1.08, 0.06, 0.01) for the system (2), and when the parameters *a, b, c, d, m, k* are 0.9, 0.2, 1.55, 0.2, 0.1, 0.17. At this time, the new system (2) shows complex and rich behavior of hyperchaotic dynamics. The singular attractor of the system is shown in Figure 1, from which it is easy to see that the system also exhibits complex chaotic properties.

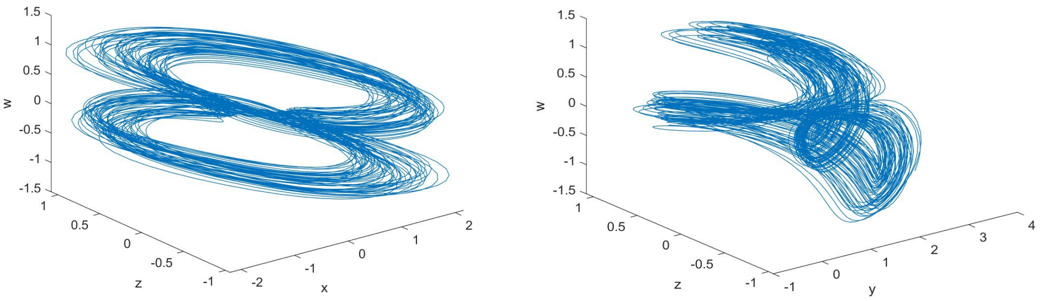

Figure 1: The chaotic attractor of the system (2).

By selecting the right parameters, the relevant authorities of the government can control the impact of external influences on the system. The effects of this quadratic term and the new parameters on the system are analyzed and simulated in the following section.

3. Dynamical Analyses and Simulations

3.1. Symmetry and Dissipativity

Under the following coordinate trans-formations $(x, y, z, w) \rightarrow (-x, y, -z, -w)$, the new hyperchaotic system (2) stays the same. So, the system is symmetric about the Y-axis, and under reflection in the Y-

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axis, all values of the system parameters are the symmetry persists.

The new system (2) has a divergence value of

$$\nabla V = \frac{\partial \dot{x}}{\partial x} + \frac{\partial \dot{y}}{\partial y} + \frac{\partial \dot{z}}{\partial z} + \frac{\partial \dot{w}}{\partial w} = -(a+b+c+k) \quad (3)$$

When $a + b + c + k > 0$, the new system is dissipative. This means that, in time t, the volume element V_0 is contracted to $V_0 e^{-(a+b+c+k)t}$. So as $t \to \infty$, the dynamical system (2) shrinks to zero at an exponential rate of $(a + b + c + k)$ such that each volume of the state variables shrinks to zero. Therefore, all state variables of the new system point to an attractor.

3.2. Lyapunov Exponents and Dimension

It is well known that the Lyapunov exponent plays a strong role in chaos theory, and its magnitude measures the divergence or convergence of trajectories in the neighborhood of the system's phase space. When more than one positive exponent exists in a four-dimensional system, the system exhibits hyperchaotic properties.

When the parameters of the system (2) *a, b, c, d, m, k* are 0.9, 0.2, 1.55, 0.2, 0.1, 0.17, the Lyapunov exponents obtained by numerical simulation software are 0.036125, 0.010689, 0.001687, -1.383413, and the Lyapunov dimension is calculated by the following formula as:

$$D_L = j + \frac{1}{\left|LE_{j+1}\right|} \sum_{i=1}^{j} LE_i = 3.050589 \quad (4)$$

In addition, we give the corresponding spectrogram of the Lyapunov exponent over time, as shown in Figure 2.

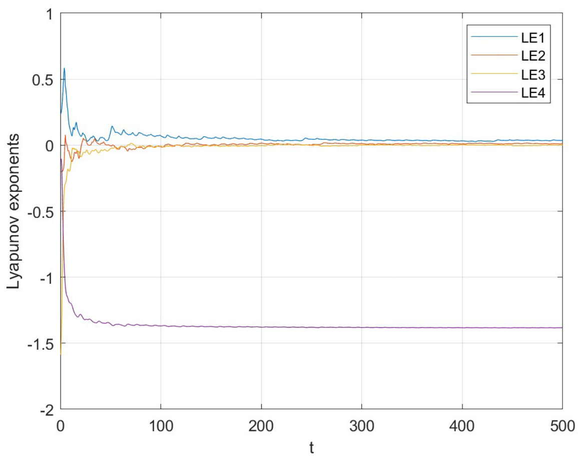

Figure 2: Lyapunov exponents spectrum of the system (2).

The above figure shows the nature of contraction in phase space in different directions. Therefore, from the theoretical analysis and numerical simulation, system (2) is a new hyperchaotic system.

3.3. Waveform and Phase Portraits

The time-domain waveforms of the four state variables in the four-dimensional chaotic system (2) are shown in Figure 3, and it is clear that the state variables are all with non-periodicity in continuous time, which also reflects the chaotic dynamics properties of the system.

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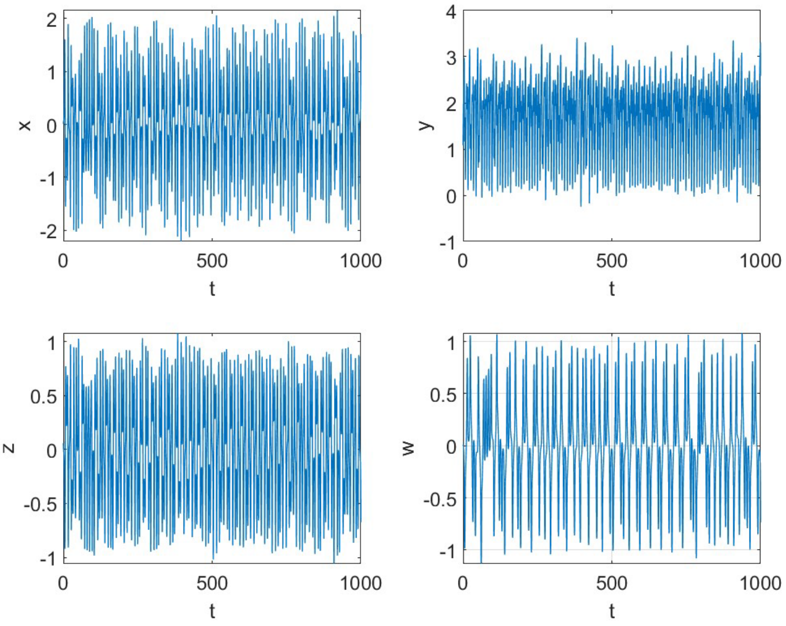

Figure 3: The waveforms of the system (2).

When the parameters *a, b, c, d, k, m* of system (2) are given as 0.9, 0.2, 1.55, 0.2, 0.17, 0.1, the different initial values *x, y, z and w* are chosen as (0.01, 1.08, 0.06, 0.01), the phase diagrams of system (2) in different planes are also given in Figure 4. From which the properties of the new system can be observed more clearly, with clearer chaotic properties and butterfly attractor shape compared to the original four-dimensional system.

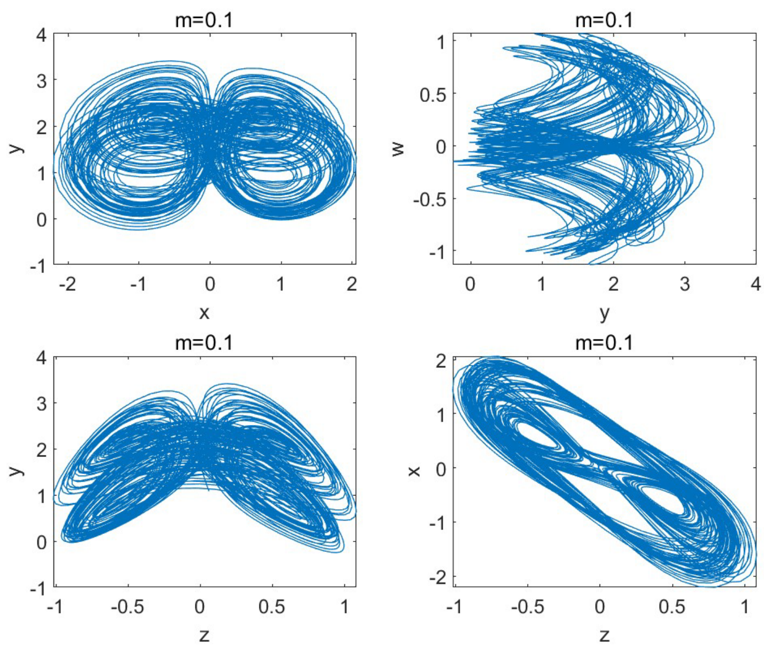

Figure 4: The phase diagrams of the system (2).

3.4. Demonstration of the System

By the determination of the above properties, our improved systems are also hyperchaotic attractors with inverse butterfly shapes. The chaotic dynamics behavior of the system may change when the parameter values or the initial values are changed. So, we will study the demonstration of the phase diagram of the system (2) for different initial values and different parameters value of *m*.

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When the parameters *a, b, c, d, k, m* of system (2) are given as 0.9, 0.2, 1.55, 0.2, 0.17, 0.1, the different initial values *x, y, z,* and *w* are chosen as 1.61, 1.68, 1.51, 1.8. To facilitate the comparison of the effect of different initial values on the system, we use numerical simulation to give a phase diagram in the same plane as Figure 4, as shown in Figure 5.

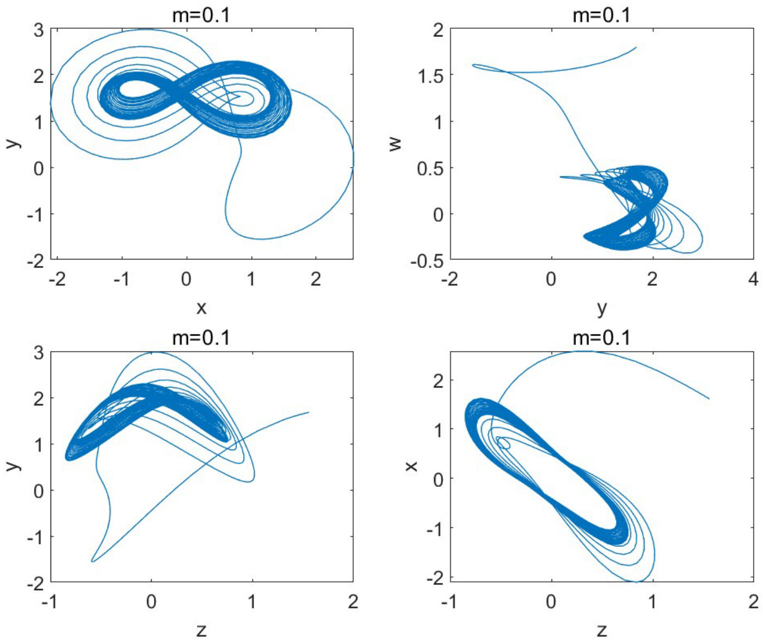

Figure 5: The phase diagrams with initial values (1.61, 1.68, 1.51, 1.8).

It is clear from the above figure that the trajectory of the system has changed considerably.

In addition, to better study the variation of the system, we will also discuss the effect on the system when different values of the parameter *m* are chosen. When the initial value is unchanged, *x, y, z,* and *w* are chosen as (0.01, 1.08, 0.06, 0.01), and the parameters *a, b, c, d, k* of the system (2) are given as 0.9, 0.2, 1.55, 0.2, 0.17, *m* is taken different values: -0.5, 0.5, 1.5. The phase diagrams in each plane corresponding to different parameters are given in Figures 6, 7, 8.

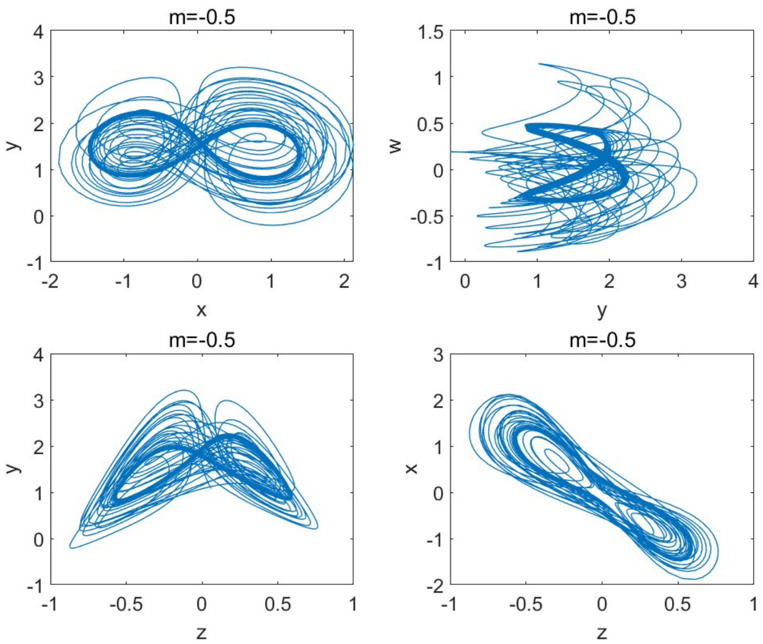

Figure 6: The phase diagrams with different m: -0.5.

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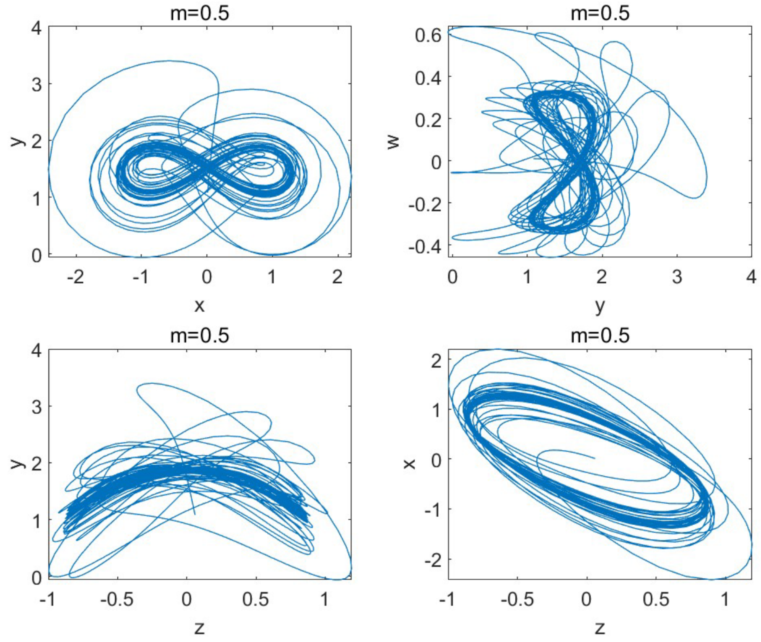

Figure 7: The phase diagrams with different m: 0.5.

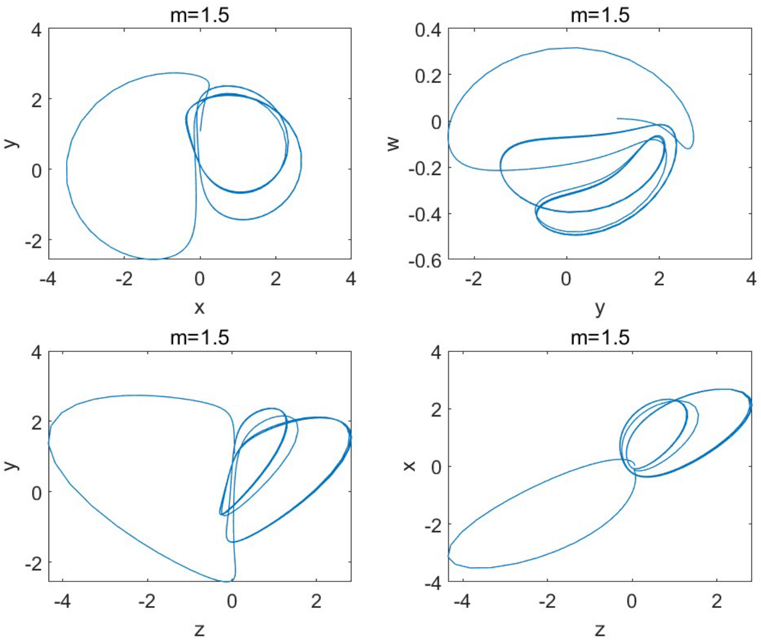

Figure 8: The phase diagrams with different m: 1.5.

As we can see from the figure, different m's still have a large effect on the system, and may make the system chaotic or periodic.

3.5. Equilibrium and Stability

The equilibrium of hyperchaotic system (2) satisfies the following equations:

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$$\begin{cases} -ax+z+w+xy=0 \\ -by-x^2+1=0 \\ -x-cz+myz=0 \\ -kw-dxy=0 \end{cases} \tag{5}$$

when the parameters are satisfied $(2md+abmk-bck+bcd)/m(d-k)\geq 0$, there are three equilibrium points:

$P_1=(0,1/b,0,0)$, $P_{2,3}=(\pm\vartheta,(1-\vartheta^2)/b,\pm(abk+(k-d)\vartheta^2)\vartheta/bk,\pm d\vartheta(\vartheta^2-1)/bk)$,

Where $\vartheta=\sqrt{(2md+abmk-bck+bcd)/m(d-k)}$.

The Jacobian matrix of the equilibrium points of the system (2) is

$$J=\begin{pmatrix} y-a & 0 & 1 & 1 \\ -2x & -b & 0 & 0 \\ -1 & mz & my-c & 0 \\ -dy & -dx & 0 & -k \end{pmatrix} \tag{6}$$

When we pick a parameter value of a=0.9, b=0.2, c=1.55, d=0.2, m=0.1 and k=0.17, the eigenvalues of the equilibrium point P_1 are 3.5943，0.1098, -0.1800，-1.4131. Hence, equilibrium P_1 is an unstable saddle point. At the equilibrium point P_2 and P_3, the corresponding characteristic equation roots are -8.8951, -1.7880, 0.1892+1.0252i, 0.1892-1.0252i. So, the two equilibrium points are unstable, too.

In this section, we use the central manifold canonical shape theory as well as the Routh-Hurwitz criterion to rule to analyze the stability of the equilibrium point of the system. Point P_1 is the equilibrium point of the system (2) for any parameter condition, and the other two are required to fulfill certain conditions. And the derived ones are unstable equilibrium points.

4. Conclusions

In this paper, an improved class of four-dimensional hyperchaotic systems is presented. For similar financial chaotic systems, any perturbation of external conditions can greatly affect the chaotic nature of the system, in order to be more in line with the current world economic situation, so this paper improves the original economic system by adding parameters and quadratic terms. Then, the article also studies the dynamic behavior of some chaotic systems from several aspects. From the analysis of various situations, it can be seen that the newly improved system has more complex chaotic characteristics compared with the ordinary chaotic system. It is believed that our system will be more valuable for theoretical research and practical application. Finally, numerical simulations give and verify the proposed conclusions.

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