Research on Hydropower Allocation Based on Multi-Objective Optimization and Bankruptcy Game Theory

Rui Zhang

School of Communications and Information Engineering, Nanjing University of Posts and Telecommunications, Nanjing, Jiangsu, 210023, China

Abstract: *In recent decades, droughts have become more frequent and severe due to global warming, and the amount of water in dams and reservoirs has decreased in many regions. To alleviate the water shortage problem, we modeled the water allocation in dams to achieve the optimal combination of water and electricity. A multi-objective optimization model for water allocation in dams was developed. A grid method was used to rasterize the dams and calculate the reservoir capacity of the dams. The multi-objective optimization model was used to improve water distribution schemes in five states based on the collection of water and electricity consumption data from five states in recent years. The consumption time for water transport and power generation, as well as the amount of water supplied, is minimized while meeting the water supply and power demand and providing surplus water for Mexico. In the next step, game theory was used to solve the conflict problem of allocating water among the four uses and to transform the problem into a utility non-transferable game model. Considering the differences in water use by state, the states were typed into three categories: hydro, industrial and non-industrial states. Different weights were assigned to the four types of water use in different types of states, taking into account the different importance attached to different water use in different regions. The allocation results for the different states were obtained.*

Keywords: *Grid Method; Game Theory; Multi-objective Optimization*

1. Introduction

As the global climate continues to heat up, the distribution of global water resources is changing. As a result, some rivers are running out of water, and dams in many areas are lack of water, leaving numerous areas unable to meet demand. At the same time, the decrease in electricity generation caused by low water flow is in conflict with people's increasing demand for electricity. Therefore, it is necessary to make the best use of water resources. In view of the above situation, it is necessary and urgent to establish a mathematical optimization model driven by water and electricity generation demand, so as to develop a reasonable and defensible water allocation plan for current and future water supply conditions [1].

Therefore, the main work of this paper is based on the data analysis of the water demand and electricity needs of the five states, taking into account rainfall as external water supply, a Multi-objective Optimization Model is established to determine the best water distribution method by regarding the time needed to transport water, the time needed to generate electricity and transmit electricity from hydroelectric power as the objective and classify states based on their water use in recent years. The conflict of interest between water and electricity production is solved by using Game Theory [2].

2. Assumptions and notations

2.1 Assumptions

We use the following assumptions.

Assumption 1: Additional water only comes from rainfall.

Assumption 2: General usage of water only include agriculture, industry and living.

Assumption 3: The weather of each state was only divided into two parts: dry season and rainy season.

ISSN 2616-5775 Vol. 5, Issue 10: 35-40, DOI: 10.25236/AJCIS.2022.051006

2.2 Notations

The primary notations used in this paper are listed as Table 1.

Table 1: Notations

Symbol	Significance
x_{ij}	The supply of water dam provided for different states
ω_i	The weight of indicators after using general contrast algorithm
n	The number of indicators in each subsystem
y_{ij}	The normalized value of indicators calculated through Genetic Algorithm
Y	The synthesis scores of sustainability evaluation
C	Efficiency matrix in 0-1 linear programming model

3. Model construction and solving

3.1 Multi-objective Optimization Model for Water Distribution in DAMS

3.1.1 Dam Storage Capacity Calculat

Considering that the water level of a Dam is inseparable from the water quantity, we established a Dam Storage Capacity Calculation Model to quantify the relationship between them. Considering the relatively flat terrain of the dam, choose Grid Method to solve this problem, in order to calculate accurately and quickly [3]. The grid method is to divide the water body into several cubes by using the established DEM Model, and then calculate the storage capacity of the whole reservoir by integrating the volume space of each cube. In order to more intuitively show the concrete effect after rasterization, simulated part of the structure of the dam as shown in the Figure 1.

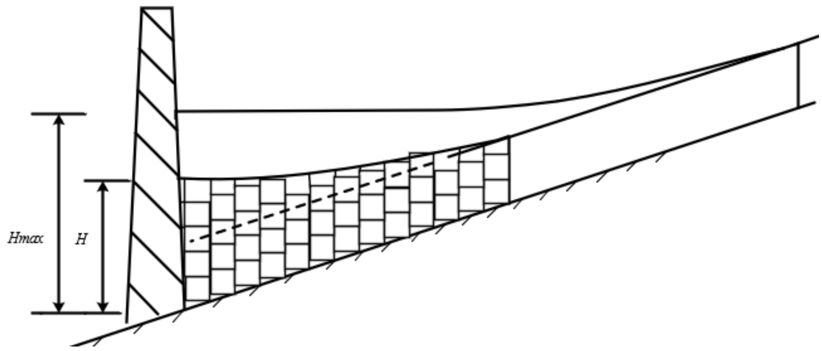

Figure 1: Part of the Structure of the Dam

3.1.2 Multi-objective Optimization Model

The Multi-objective Optimization Model is an model that uses mathematical programming methods to determine the optimal solution. In this problem, it is to determine how to allocate the water from the dam effectively under established goals (such as the minimum allocation time required to meet the water demand) and given constraints (such as the water level limit of the dam), solving the problem of the optimal combination of water supply and electricity generation.

Decision Variab: The variable i is used to represent two Dams: Hoover Dam when i =1 and Glen Canyon Dam when i=2.

Objective Function: The two targets can be attributed to the minimum amount of water needed to meet daily living and electricity needs, the minimum time for transporting water and generating electricity. The mission plan is to develop a rational and defensible water allocation plan to alleviate water shortages. An optimization objective is required to set the minimum water supply to meet the demand for water usage and electricity. Hence, the first objective function can be expressed as.

$$\min \sum_{i=1}^{2} \sum_{j=1}^{5} \left(x_{ij} + y_{ij} \right) \tag{1}$$

Another objective is to minimize the time required to transport water and hydroelectric power, so the second objective function can be expressed as.

$$\min t_1 + t_2 \tag{2}$$

Considering that the time consumed in transporting water is related to the distance from the dam to each state, the velocity of water and the amount of water supplied, t1 can be further written as follows.

$$t_1 = \sum_{i=1}^{2}\sum_{j=1}^{5}\frac{S_{ij}}{v_w}\cdot x_{ij} \tag{3}$$

The consumption of the time for hydroelectric power generation is related to the installed capacity of electric system, the energy conversion efficiency and other factors, so the value of t2 can be modified as.

$$t_2 = \sum_{j=1}^{5}\left(60?\frac{\mu_1\cdot y_{1j}}{Z_1} + 60?\frac{\mu_2\cdot y_{2j}}{Z_2}\right) \tag{4}$$

Constraints: Water distribution requires that the total amount of water supplied by the two dams to the five states plus the additional supply in each state should be greater than or equal to the state's water needs. Subject to the constraint by.

$$R_j < \sum_{i=1}^{2} x_{ij} + F_j \le C \tag{5}$$

In addition, the hydroelectric power generated by the dams would have to meet each state's electricity needs. The constraint reduces to.

$$E_j < \sum_{i=1}^{2} \mu_i\cdot y_{ij} \tag{6}$$

Since the supply of water cannot exceed the capacity of the dams, the following constraints are obtained.

$$\sum_{j=1}^{5}\left(x_{1j} + y_{1j}\right) \le C(M) \tag{7}$$

$$\sum_{j=1}^{5}\left(x_{2j} + y_{2j}\right) \le C(P) \tag{8}$$

Finally, there are certain constraints on the water level of the dams. Subject to the constraint by.

$$C(P)_{\min} \le C(P) - \sum_{j=1}^{5} x_{2j} - \sum_{j=1}^{5} y_{2j} - \xi \le C(P)_{\max} \tag{9}$$

$$175 \le P \le 216.4 \tag{10}$$

$$C(M)_{\min} \le C(M) - \sum_{j=1}^{5} x_{1j} - \sum_{j=1}^{5} y_{2j} + \xi \le C(M)_{\max} \tag{11}$$

$$180 \le M \le 221.4 \tag{12}$$

$$\sum_{j=1}^{5} x_{1j} + \sum_{j=1}^{5} y_{2j} \ge \varphi \tag{13}$$

3.1.3 Model Verification and Analysis

Based on collecting and organizing the data of water and electricity consumption in the five states over the years, we decided to use the established model to optimize the water distribution in the five states in September 2018 and propose a more reasonable distribution scheme. To give a better picture of

ISSN 2616-5775 Vol. 5, Issue 10: 35-40, DOI: 10.25236/AJCIS.2022.051006

how the scheme is distributed, we have drawn a bar chart of water supply for different uses in five states, as shown in Figure 2.

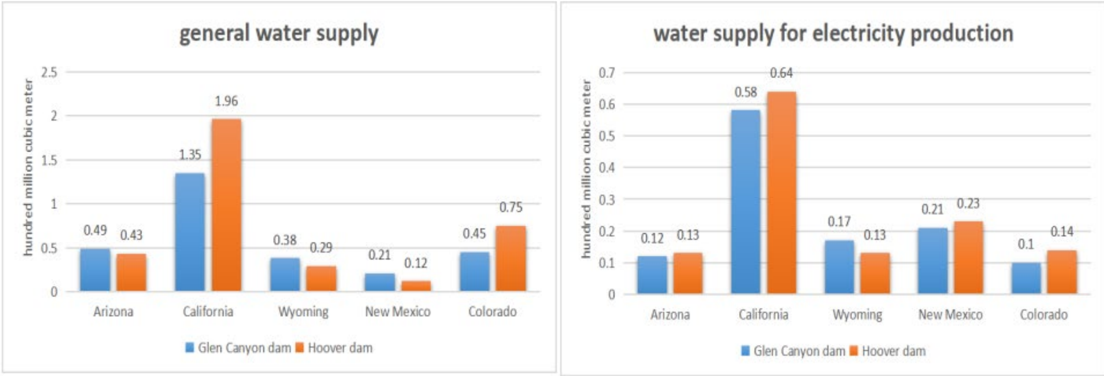

Figure 2: Water distribution plans for five states in September 2018

As can be seen from the graph, there are significant differences in the supply of water for general use and the supply of water used to produce electricity in different states. California has the largest supply of water for general production, and significantly more than the other four states, with a combined total of about 331 million cubic meters. Mineral-rich Wyoming uses significantly more water to produce electricity than any of the other four states, with a combined 19, 300 cubic meters.

At the same time, considering the geographical location differences of these five states, the dry rainy season has a great impact on water supply. Therefore, we applied our model for the dry rainy season again, and the results are shown in the Figure 3 below.

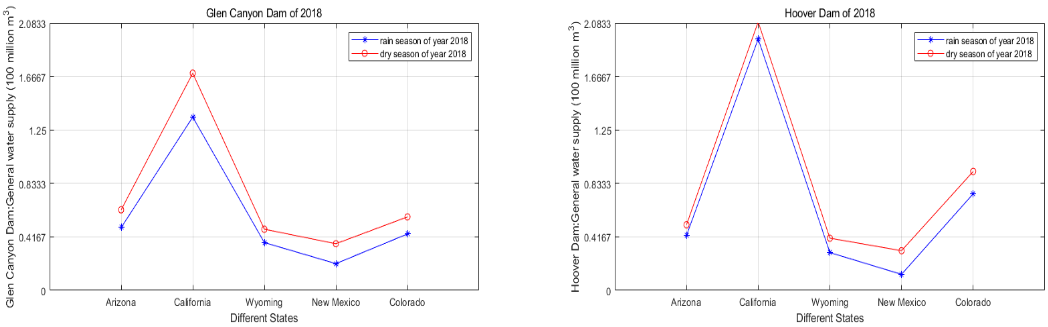

Figure 3: Water distribution plans for five states in dry rainy season

As can be seen from the figure above, water supply in each state is greater during the dry season than during the rainy season. For relatively dry Wyoming and New Mexico, the Hoover Dam significantly increased water supplies during the dry season to 488 million cubic meters and 371 million cubic meters respectively. The Glen Canyon Dam has a more pronounced change in water supply to Mediterranean California during the dry season, reaching 2.03 billion cubic meters. Our model tells us that Glen Canyon Dam should supply 51 million cubic meters of water to Hoover Dam in September 2018.

Time delay includes the time of water transportation and the time consumed by hydro power generation. So the total delay is.

$$\tau = \max\left(t_1, t_2\right) \tag{14}$$

Where t_1 and t_2 can be expressed respectively.

$$t_1 = \sum_{i=1}^{2}\sum_{j=1}^{5}\frac{S_{ij}}{v_w}\cdot x_{ij} \tag{15}$$

$$t_2 = \sum_{j=1}^{5}\left(\frac{\mu_1\cdot y_{1j}}{Z_1} + \frac{\mu_2\cdot y_{2j}}{Z_2}\right) \tag{16}$$

ISSN 2616-5775 Vol. 5, Issue 10: 35-40, DOI: 10.25236/AJCIS.2022.051006

3.2 Water Distribution based on Game Theory

3.2.1 Water Resources Allocation Model

The second question requires consideration of the competing interests between water used in general and water used for electricity production, and game theory can often provide a very effective solution to conflicts of interest. Therefore, we're trying to use game theory to get a better allocation of water use. The traditional game theory problem is that asset E is divided among creditors, and the creditors have more claims than asset E. Creditors are usually represented by set N, which in this case is water. Where $N=\{1,2,3,4\}$, respectively represents the water needed for agriculture, domestic use, residential use and power generation. Cooperation between subjects in water resource allocation conflicts can often produce greater economic, ecological and political utility. Therefore, we assume that subjects can resolve water resource allocation conflicts through consultation and negotiation. Nash bargaining game can not only describe most of the characteristics of water use allocation problem well, but also find the optimal solution of water resources allocation according to the linkage function of main utility function. Therefore, we combine the game theory and Nash bargaining game model to transform the problem of conflicting interests between different uses of water into a Utility Nontransferable Game Model.

The distribution scheme is represented by the vector X =(x1, x2, x3, x4), from which a Weighted Proportional Distribution Model can be obtained.

$$x(i)=\min\left(\lambda\omega_i r_i, r_i\right) \tag{17}$$

The distribution scheme is constrained by the following condition.

$$\begin{cases} \sum_{i\in N} x(i)=E \\ 0\le x_i\le r_i \end{cases} \tag{18}$$

In game theory, the utility function of the player is usually expressed as $\{n_i(X), i=1,2,3,4\}$. However, since water resources penetrate into all aspects of economic and social development, it is difficult to accurately determine its utility function. Therefore, we convert the utility function of water resources allocation into an interval linear function through satisfaction index[4].

The utility allocation set of water resource allocation problem can be expressed as: for any x∈ X, participants can obtain the corresponding utility $\{n_i(X), i=1,2,3,4\}$, then the only utility solution $\{u_i^N, i=1,2,3,4\}$ that meets the following maximization conditions is the solution under bargaining[5].

$$\begin{gathered} u_x^N(x)=\left\{ x\in X \middle| \max\prod_{i=1}^{4}\left[u_i(x)-u_i\left(d_i\right)\right]^{\omega_i}\right\} \\ \text{s.t. } : 0\le x_i; d_i\le x_i\le c_i; \sum_{i=1}^{4}x_i\le E; \sum_{i=1}^{4}\omega_i=1 \end{gathered} \tag{19}$$

3.2.2 Weight Allocation Model

As for the determination of weight ω_i, considering the differences between states, we first classified five different states. For the sake of the conflict of interest between water used for electricity production and water used in general, we first divided states into hydro and non-hydro states. After analyzing the data of water consumption in the United States, we find that most water resources in the United States are used for agricultural irrigation. Therefore, when distinguishing the states with hydro type and non-hydro type, we compare the total amount of residential water and industrial water with the amount of water used for power generation. If the water used to generate electricity is more than the combined amount of residential and industrial water, it is considered a hydro state; otherwise, it is a non-hydro state.

Among the non-hydro states, we further subdivide them into industrial states and non-industrial states. When industrial water consumption exceeds residential water consumption, it is an industrial state; otherwise, it is a non-industrial state. Since agriculture is the most important part, its weight ω_1 is fixed at 0.7 regardless of state type. For hydro states, we set the power generation weighting ω_4 at 0.2. Considering that more power generation is supplied to industry than to households, the weight of

ISSN 2616-5775 Vol. 5, Issue 10: 35-40, DOI: 10.25236/AJCIS.2022.051006

industrial water consumption is ω_2 =0.06, and that of residential water consumption is 0.04 ω_3 = . If the category is industrial state, then $\omega 2$ =0.15,$\omega 3$ =0.05,$\omega 4$ =0.1. If it is a non-industrial state, then ω_2 =0.08,ω_3 =0.12,ω_4 = 0.1.

3.2.3 Model Verification and Analysis

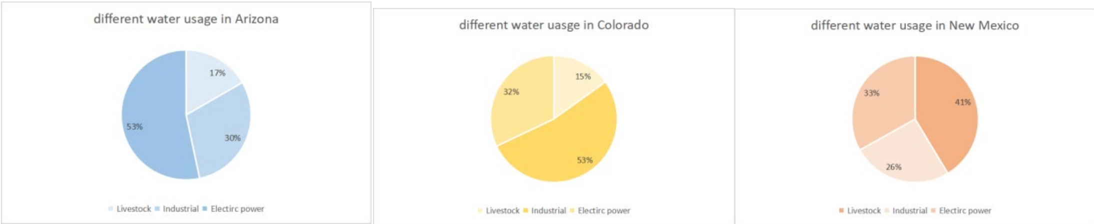

Figure 4: The Water Allocation for Different Uses

To test the rationality of the model, we selected the water consumption data of three states in May 2019 as their water demand, and calculated the water distribution amount of each state for each purpose. We found that water in all three states is allocated primarily to agriculture, so we compared water allocations for the other three uses. In order to display the results more intuitively, we draw the pie chart of water distribution as shown in Figure 4.

It can be seen that Arizona, as a hydro state, has more than half of the water supply for power generation, followed by 30% for industry and 17% domestic use. The industrial state of Colorado uses 53% of its water for industry, followed by 32 percent for power generation and 15% for domestic use. The distribution of water in non-industrial state New Mexico is evenly: 41% for domestic use, 33% for power generation, and 26% for industry. Based on the classification criteria we have developed, the amount of water supplied is distributed more reasonably, taking into account that different types of states require different uses of water.

4. Conclusion

By using the Multi-objective Optimization Model and Game Theory we effectively solve the problems of the water distribution scheme of five states. Under the precondition of the conflict problem of water resource allocation among different uses, we balance all the benefits perfectly and arrive the conclusion that compared with hydro state, industrial state will be allocated more water for industrial usage instead of electric power. When facing dry season, the water supply of two dams should be increased to some extents.

References

[1] Wang Yu Kai. The development and utilization of water resources in the West of the United States and the Project of Water Diversion from north to South. Henan Water conservancy and South-to-North Water Transfer, 2019, 07-0036-02.

[2] Barts, C. M. Regulation of sloping canal by automatic downstream control. Ph.D. thesis, Utah State Unversity. Logan, Utah. 1983.

[3] Rosalind H B, George F, Karl W F. The role of economics in transboundary restoration water management in the Colorado River Delta [J]. Water Resources and Economics, 2014, 8: 43-56.

[4] Shi G M, Wang J N, Zhang B et al. Pollution control costs of a transboundary river basin: Empirical tests of the fairness and stability of cost allocation mechanisms using game theory [J]. Journal of Environmental Management, 2016, 177(15): 145- 152.

[5] Qin J N, Fu X, Peng S M et al. Asymmetric benefit compensation model for resolving transboundary water management conflicts [J]. Water Resources Management, 2020, 34.