Higher Mathematical Modeling Thinking and Method Application

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ABSTRACT. Advanced mathematics is a public basic course in science and engineering of colleges and universities. It is an important course to train students' thinking training and master mathematical theory and methods. Introducing mathematical experiments in higher mathematics teaching, giving play to the role of basic experiments and exploration, application and comprehensive modeling experiments, and reforming university mathematics courses can cultivate innovative talents. The idea of mathematical modeling is to combine practical problems with mathematical theories, to abstract them into concrete, and to solve practical problems in life by mathematical methods. Integrating the idea of mathematical modeling into today's higher mathematics teaching is a new teaching method for higher mathematics teaching and a very meaningful quality education method. Combining examples, the thesis analyzes the process of solving advanced mathematics problems by using higher mathematics modeling in the teaching process, and strengthens students' theoretical cognition ability through mathematical modeling, guides students' self-learning ability training, and enhances students' application ability and practical ability. Inspire students' enthusiasm for learning.

KEYWORDS: Advanced mathematical modeling, case teaching, curriculum reform, mathematical thinking training

1. Research background

"Advanced Mathematics" is one of the important subject basic courses for science and engineering students in colleges and universities. Its teaching content involves unary function calculus, multivariate function calculus, spatial analytic geometry and preliminary vector algebra, series and differential equations [1]. Different subject majors have different requirements for higher mathematics content. The teachers need to combine or increase the corresponding content of the students' professional characteristics. For example, students of physics majors need to learn a combination of mathematics and physics after learning advanced mathematics. Courses - Mathematical physics equations, biology and medical students need to

increase knowledge of probability theory and statistics. Through the study of advanced mathematics courses, students' abstract thinking and logical thinking are well trained. It is the theoretical basis for the follow-up mathematics curriculum and professional courses. At the same time, it cultivates students' ability to use mathematics. Therefore, the study of advanced mathematics courses is the foundation of science and engineering majors [2]. However, most science and engineering graduates do not know how to use the mathematics knowledge they have learned to solve real-life problems in life. Even a large number of students think that mathematics is useless for professional learning and contradicts the cultivation of applied talents. Therefore, the key to reforming education and teaching in higher mathematics is to find a breakthrough and link with real life, that is, to establish a mathematical model to solve practical problems. Therefore, it is imperative to incorporate mathematical modeling ideas into the teaching of higher mathematics education in science and engineering. It must be done.

2. Mathematical modeling thinking and application

2.1 Overview of Modeling Thinking

The essence of modeling thought is the connection between practical problems and theoretical knowledge, clarifying the mathematical relationship between various variables in practical problems, and based on this, establish a scientific and reasonable mathematical model, concrete abstract problems, and use certain Mathematical methods, such as formula calculations, graph observations, and program operations, provide in-depth analysis and exploration of the established mathematical models to solve practical problems. The rational use of mathematical modeling ideas can combine practical problems with theoretical knowledge, thus training the modeler's thinking ability, logic ability and innovation ability, and has high requirements for model builders, not only must Solid professional knowledge, but also have a certain ability to apply mathematical knowledge and the ability to transform theoretical knowledge [3-4]. Therefore, the rational application of mathematical modeling ideas in higher mathematics teaching can effectively stimulate students' interest in learning, improve their initiative and enthusiasm, and thus improve teaching efficiency and learning efficiency. The basic concepts in higher mathematics are mathematical concepts abstracted to solve some practical problems, such as derivatives and definite integrals. Higher mathematics is a compulsory course for college students of science and engineering. It has many teaching contents, less teaching time, strong theoretical theory and high abstraction.

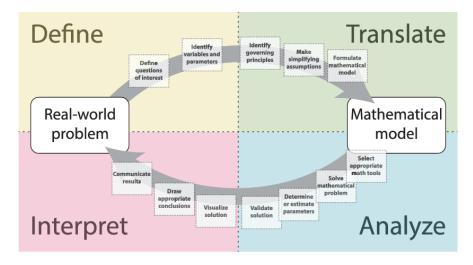


Figure. 1 Mathematical modeling thinking

2.2 Mathematical Modeling Application

Mathematical modeling plays a large role in cultivating students' ability to observe, imagine, and logically think and analyze and solve practical problems. For more than a decade, various mathematical modeling teaching and research activities with the theme of mathematical modeling competition have been spread all over the country. It is to cultivate students' courage in improving students' interest in learning, stimulating learning initiative and improving their ability to acquire knowledge. The tenacity, the solid work spirit and the good coordination ability to overcome difficulties have shown their important role in cultivating students' ability to apply knowledge, practical ability and innovative ability. However, limited to the size of the competition and the requirements for the level of participation, participation in the mathematical modeling competition is only a small number of students. Although many institutions offer mathematics modeling electives and mathematics modeling classes to students every year, the courses require students to have higher mathematics knowledge, so these courses are not suitable for popular education. To comprehensively improve the quality of college students and cultivate innovative application talents with complex spirits, the responsibility should fall on the traditional mathematics curriculum, and advanced mathematics is an ideal carrier. As a basic course, advanced mathematics is a compulsory course for science and engineering students. Introducing mathematical modeling in higher mathematics is undoubtedly the most influential.

Infiltrating the idea of mathematical modeling in higher mathematics teaching can restore the original knowledge of mathematics knowledge and cultivate students' awareness of applying mathematics knowledge to daily life and social practice. Mathematical modeling requires students to use mathematical language and tools to

simplify, abstract, translate, and summarize some of the real-world information, and to express quantitative relationships in mathematical forms, graphs, or tables, so that students can be trained and improved. Ability to express. After the mathematical modeling is actually solved, it is necessary to use the information of the real object to test to confirm the correctness of the result. The training of this step allows the students to learn to analyze the problem mathematically, objectively and dialectically. Get the best way to solve the problem. Therefore, mathematical modeling plays an important role in the teaching of advanced mathematics.

3. Create multiple ways to promote the cultivation of students' advanced mathematical modeling thinking

3.1 Integrating mathematical modeling ideas into mathematical concepts

Mathematical concepts are the most basic theoretical knowledge in mathematical science, and also the premise and basis for mathematical reasoning and argumentation. For example, basic experiments such as function limits, derivative differentials, and integrals, series operations, and check calculations, as well as function planes, stereoscopic plot experiments, function approximation experiments, and equation solving experiments. The understanding and mastery of mathematical concepts play a decisive role in mathematics learning. In the teaching of mathematical concepts, we must pay attention to the combination of its actual background. It not only allows students to see the predecessor of mathematical concepts, but also the corresponding real problems, and also experiencing the formation process of mathematical concepts, which is more helpful to understand the mathematical ideas contained in mathematical concepts.

For example, the summation series is part of the experiment. The purpose of the basic experiment is to grasp the commands for finding the sum of the finite sum and the number of series in Mathematical mathematics software; verify the convergence of the series.

Example 1. (Calculation Problem) Indian young legendary mathematician Ramanu Jin proposed a formula for describing the $\,\rho\,$ of a series:

$$\frac{1}{\rho} = \frac{2\sqrt{2}}{9801} \sum_{n=0}^{\infty} \frac{(4n)!(1103 + 26390n)}{n!^4 396^{4n}} \tag{1}$$

Try this formula to calculate the approximation of the ρ , and the result is accurate to 29 decimal places [5]. There are programming procedures as follows:

$$P1[n]: =2Sqrt[2]/9801*Sum[(4k)!*(26390k+1103)/396^(4k)/(k!)^4, \{k,0,n\}]$$

// Define the derivative at the right end of the above expression as a P1[n] function, where n is the number of terms to be summed.

s=N[1/P1[2], 29]; //N [] function retains 29 decimal places

Print ["s=", s]//output s value

s=3.1415926535897932384626490657

Print ["err=", N[p-s, 29]]//The error between the output exact p-value and the p-value calculated by this expression

Err=-5.6824*10-24

Using the mathematical software Mathematical or MATLAB to calculate the series sum function Sum [] and the square root function Sqrt [], the accuracy of the calculated value of this formula is very high, and the value of the parameter n is increased or decreased to give different error values. The value example is a typical basic experiment of applying mathematical software commands to perform complex calculations.

Example 2. When teaching the concept of definite integral, the following two models can be established: (1) the path of the linear motion of the shifting speed; (2) the variable force works along the straight line.

For question (1), if the speed is constant, you can use the formula: distance = $speed \times time$ to find, but the key to the problem is that the speed is changing. Therefore, it can be considered that "the whole is zero", and the time period is divided into a plurality of cells. When the time segment is divided enough, the speed of each segment can be considered to be close to a constant, and the velocity is multiplied by the time. Add the distances of all the short periods of time to get an approximation of the distance. In order to get a more accurate value, of course, the segmentation should be refined, the finer and more precise! To get the exact value, the segmentation must be infinitely fine, even if the length of each segment is zero, then the limit of the sum of the distances on all inter-cell segments is the desired route. Finally, you can get the expression of the distance:

$$s = \lim_{\lambda \to 0} \sum_{i=1}^{n} v(f_i) \Delta t_i \tag{2}$$

Where v(t) is the speed function, f_i is a time taken at the time interval $\left[t_{i-1},\,t_i\right]$ after the division, Δt_i is the time length of each small segment, and λ is the largest of the small time intervals. Similarly, the expression of question (2) can be obtained.

$$W = \lim_{\lambda \to 0} \sum_{i=1}^{n} F(x_i) \Delta S_i$$
 (3)

From the commonality of these two expressions, the definition of definite integral can be introduced.

3.2 Get Mathematical Modeling Thinking in Extracurricular Development

Extracurricular development is based on the purpose of algorithm construction, focusing on the use of what is learned and the characteristics of extracurricular development; application experiment is an experiment focusing on solving practical problems. Comprehensive modeling experiment is the core part of mathematical experiment, and it is cultivated on the basis of basic experiment. Students' initiative, exploration, application, cooperation and innovation. In the first half of the semester, mainly based on basic experiments, extracurricular development, and applied experiments; in the second half of the semester, on the basis of the previous experiments, the students gradually transitioned to comprehensive experiments, and students took 3-5 people as a group, using the time inside and outside the class to give Design topics, purposeful review, learning related knowledge and literature materials, through the team's collaborative experiments, discussions and other forms, using experimental methods to analyze problems, establish mathematical models, solve models, and analyze results, further summarize and conject new results. Under the guidance of the teacher, write the experimental report, and finally complete the design process of the subject through the defense. This new model enables students to learn, explore, and discover mathematical laws and applications in experiments [5]. It not only consolidates the understanding of theoretical knowledge, but also cultivates the initiative, innovation, and teamwork spirit of learning, and enhances students' independent thinking. The ability to solve practical problems by using mathematical thinking methods to achieve the goal of cultivating innovative talents.

Example 3. Application of power series

Here is a combination of mathematical problems that explain an application of power series. Through this example, the analogy modeling idea is embodied and the problem transformation consciousness is cultivated. There are currently 2n letters A, 2n letters B and 2n letters C; find the number of different ways to select 3n letters from among them.

Use factor $I+x+x^2+...+x^{2n}$ to indicate the choice of a letter: 1 means select 0, x means select 1, x^2 means select 2, Let $f(x)=(I+x+x^2+...+x^{2n})^3$, obviously the coefficient of x^{3n} is the number of different combinations of 3n letters (think why?).

Solving the idea Because of
$$f(x) = (I + x + x^2 + ... + x^{2n})^3 = \left(\frac{I - x^{2n+1}}{I - x}\right)^3 \frac{I}{(I - x)^3}$$
,

we want to expand this function into a power series to find the coefficient of χ^{3n} .

This requires expanding
$$(1-x^{2n+1})^3$$
 and expanding $\frac{1}{(1-x)^3}$ to a power series,

which can be multiplied to analyze the coefficients of χ^{3n} . Solve the calculation after class. In this example, there are two aspects to be aware of. The first is the process of turning a practical problem into a mathematical problem; the second is the application of a power series (the problem of determining the number of combinations).

4. Conclusion

Mathematical modeling thoughts should be fully embodied in any part of teaching, and fully mobilize students' enthusiasm. We can start from the following aspects to improve the cultivation of advanced mathematical modeling thinking.

Firstly, introduce the modeling ideas correctly. When introducing modeling ideas, you should use novel and lively and interesting methods, and explain to students the importance of higher mathematics and stimulate students' learning initiative. Encourage students to ask practical questions of their own interest, establish appropriate mathematical models, guide students to combine theoretical and practical issues, actively find solutions to problems, and find and optimize solutions by discussing and communicating with each other [6-7].

Secondly, apply modeling ideas to formula derivation. By applying modeling ideas to formula derivation and learning, students can effectively deepen their understanding and memory of formulas, strengthen learning effects, and enable students to quickly master various formulas. Secondly, the learning and derivation of formulas is rather boring. Teachers should improve the existing teaching methods, enhance the flexibility and vividness of teaching, and create a harmonious and harmonious classroom atmosphere, so that students can quickly and easily grasp relevant knowledge, thus effectively improving teaching efficiency.

Finally, apply the modeling ideas to the examples. The example explanation is a very important part of higher mathematics teaching. The example integrates important formulas and classic problem-solving ideas. Applying mathematical modeling ideas to the example analysis is to combine the formula with the actual problem. In addition, at the end of each chapter, teachers should actively guide and encourage students to conduct in-depth analysis and exploration of examples, which can help students find the correct solution ideas.

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