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Research on Vibration Characteristics of the Curved Pipes Conveying Fluid Based on Dynamic Stiffness Method

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Abstract: *Fluid transmission pipelines, especially curved pipes, are widely used in industrial systems. Vibration characteristics of pipes are highly correlated with the reliability and safety of industrial systems. Therefore, in this paper, a new dynamic stiffness method is proposed to solve the above vibration characteristics of the curved pipes conveying fluid. The dynamic stiffness method can be used to calculate the vibration characteristics of the pipes conveying fluid under arbitrary boundary conditions, By comparing the results of finite element method with those of this method, the correctness of this method is verified. Finally, the vibration characteristics of the pipeline at different angles are calculated by this method. The results show that with the increase of θ value of radian angle in the pipes conveying fluid, both the frequency and critical velocity of in-plane and out-of-plane of the corresponding order decrease, and the natural frequency decreases greatly when θ is small, while the natural frequency decreases little when θ is large.*

Keywords: *Curved Pipes Conveying Fluid, Angles, Fluid-Solid Coupling Vibration, Dynamic Stiffness Method, Arbitrary Support, Vibration Characteristics*

1. Introduction

Fluid transmission pipelines are widely used in aircraft, ship and ocean fields. For example, the role of the ship cooling water system is to transport cooling water to the equipment that needs cooling through pipes to meet the cooling demand of the equipment; Fuel, hydraulic oil and lubricating oil systems in aircraft are of vital importance to the safe flight of aircraft, whose role is to satisfy the use of various hydraulic actuators on aircraft[1]. When the pipeline system is in operation, the fluid interacts with the structure in the pipeline, resulting in vibration of the pipeline system[2]~[4].For the pipeline system, on the one hand, excessive vibration will produce large vibration noise, which will affect the concealment of ships, the comfort of aircraft and automobile; on the other hand, it may cause pipeline damage, so that the system failure, affecting the safety of aircraft and ships. Therefore, it is necessary to comprehensively analyze the vibration characteristics of the pipeline system in order to improve the safety and reliability of the pipeline system [3].

Piping systems generally include straight pipes conveying fluid and curved pipes conveying fluid. At present, most of the fluid-solid coupling vibration of pipelines is mainly considered in the vibration of straight pipes conveying fluid, and its research is relatively mature. For the study of curved pipes conveying fluid, Misra[8]~[9] et al. proposed the non-telescopic theory and the telescopic theory in the calculation of curved pipes conveying fluid, and calculated the influence of pipeline velocity on the natural frequency of curved pipes conveying fluid and the critical velocity for instability by using the finite element method. Han Tao [10] considered curved pipes as a basic unit to study the vibration characteristics of complex pipelines.

In the above studies, the influence of boundary conditions of curved pipe should be considered, and different curved pipe dynamic models or different Galerkin discrete forms are given under different boundary conditions, thus increasing the complexity of modeling and derivation. The traditional finite element method or transfer matrix method is faced with two problems in solving the above problems: first, it cannot be extended to the pipeline system with complex spatial direction; second, the pipeline supports that can be analyzed are limited to the common rigid supports, and the elastic supports are rarely involved. Therefore, most of the studies are limited to the straight pipes conveying fluid or the curved pipes conveying fluid with rigid supports.

In order to solve the boundary consistency problem in the above research and derivation process, a dynamic stiffness method is proposed to solve the vibration problem of curved pipes conveying fluid under arbitrary supporting boundary conditions at specified positions, and this method is used to analyze the vibration of curved pipes with common angles (90°-180°) in industry and ships. Firstly, the 4-DOF dynamics model of straight pipes conveying fluid was established by Euler-Bernoulli beam model. Secondly, Galerkin discrete method of improved Fourier series was used to simulate the harmonic response equation of straight pipe element under arbitrary boundary conditions, and the response dynamic stiffness matrix of complex straight pipes conveying fluid was obtained by local coordinate transformation and element assembly. Then the correctness of the method is verified by simulation of semicircular curved pipes conveying fluid and finite element method. Finally, the relation between the angles between the two supporting points and the center of the curved pipe and the vibration of the curved pipes conveying fluid is studied.

2. Solution of Dynamic Characteristics of Curved Pipes Conveying Fluid

2.1 Straight Pipe Linear Model Derivation

Euler-Bernoulli beam model was used to analyze the straight pipes conveying fluid. In order to deal with boundary conditions at both ends of different straight pipe in the traditional Euler-Bernoulli beam model, linear springs are used to replace the supporting surfaces at both ends of the pipe, and springs with different stiffness were used to simulate different types of pipe boundary conditions. Fig. 1 shows the dynamics solution model of straight pipes conveying fluid and related physical parameters of pipe fluid:

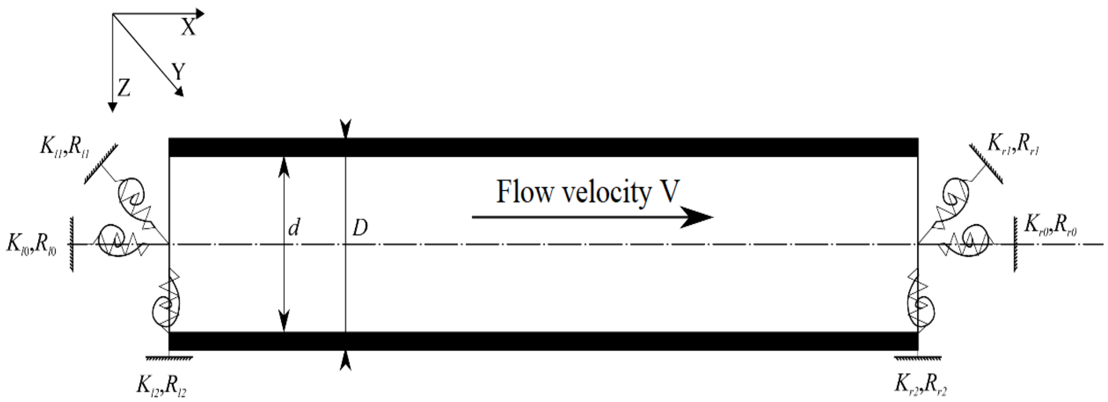

Figure 1: Schematic diagram of straight pipe conveying fluid model calculation

Suppose there is a point in the center line of the pipe whose displacement **Ψ** is represented by the displacement vector:

$$\mathbf{\Psi}=\{u,v,w,\phi,\theta,\varphi\}^{\mathrm{T}}=\{\psi_1,\psi_2,\psi_3,\psi_4,\psi_5,\psi_6\}^{\mathrm{T}} \tag{1}$$

In the formula, u、v and w represent the corresponding axial X displacement, lateral Y displacement and vertical Z displacement at this point in a unit pipeline in the local coordinate system respectively. Φ, θ and φ are the angular displacements in the three directions of the cross section at that point in the center line. The above model is a beam model of unit length.

The lateral and vertical displacements correspond to the angular displacements are as follows:

$$\theta=w';\varphi=v' \tag{2}$$

According to the extended Hamilton principle, the controlling differential equation of Euler-Bernoulli beam pipeline vibration is obtained in the following form:

$$\int_{t_1}^{t_2}\delta(T-U)\mathrm{d}t+\int_{t_1}^{t_2}\delta W\mathrm{d}t-\int_{t_1}^{t_2}\left[m_f V\left(\dot{\boldsymbol{R}}+V\boldsymbol{\tau}\right)\cdot\delta\boldsymbol{\tau}\right]_0^l\mathrm{d}t=0 \tag{3}$$

In the formula, T and U represent the kinetic and potential energy of the pipeline, δW is the virtual work done by the external force, m_f is the corresponding unit fluid mass, τ is the unit normal vector of the section of the Euler Bernoulli beam, *R* is the displacement vector, *V* is the flow velocity of the fluid in the pipe, and *l* is the unit pipe length, which can be set as 1m.

Assuming that the load exists only at both ends of the pipe , and the dead weight and external pressure are neglected, the differential equation can be solved by discretization with Galerkin method. In the solution, the displacement needs to be discretized. According to Equation (2), the Euler-Bernoulli beam takes into account the motion in four directions (u、v、w and ϕ). In this paper, a discrete improved Fourier series method is adopted, which regards the displacement in each direction as a linear superposition of a series of basis functions:

$$\begin{aligned} u(x,t) &= \sum_{i=0}^{N+2}\varphi_i(x)\vartheta_i^u(t) = \bar{\mathbf{r}}^{\mathrm{T}}\mathbf{q}_{\mathrm{u}},\ \phi(x,t) = \sum_{i=0}^{N+2}\varphi_i(x)\vartheta_i^{\phi}(t) = \bar{\mathbf{r}}^{\mathrm{T}}\mathbf{q}_{\phi}, \\ v(x,t) &= \sum_{i=0}^{N+4}\varphi_i(x)\vartheta_i^v(t) = \mathbf{r}^{\mathrm{T}}\mathbf{q}_{\mathrm{v}},\ w(x,t) = \sum_{i=0}^{N+4}\varphi_i(x)\vartheta_i^w(t) = \mathbf{r}^{\mathrm{T}}\mathbf{q}_{\mathrm{w}}. \end{aligned} \tag{4}$$

The transposition basis function φ_i in Equation (4) represents the combination of functions of equation (5) :

$$\begin{cases} \varphi_i(x) = \cos\left(\dfrac{i\pi x}{l}\right) & (i \le N) \\ \varphi_i(x) = f_{i-N}(x) & (i > N) \end{cases} \tag{5}$$

Where N is the number of basis functions of the series, and the definition of the boundary function f of the Fourier series is referred to Reference[11]. The linear governing equation of vector coordinate **q** can be obtained by using the variational method:

$$\mathbf{M}\ddot{\mathbf{q}} + \mathbf{C}\dot{\mathbf{q}} + \mathbf{K}\mathbf{q} = \mathbf{f} \tag{6}$$

Laplace transform is used to represent the relationship between the response of the pipeline system and the excitation in the frequency domain, and Equation (6) can be written as follows:

$$\begin{Bmatrix} \mathbf{q}_{\mathrm{u}}(s) \\ \mathbf{q}_{\mathrm{v}}(s) \\ \mathbf{q}_{\mathrm{w}}(s) \\ \mathbf{q}_{\phi}(s) \end{Bmatrix} = \begin{bmatrix} \mathbf{h}_{\mathrm{u}}(s) & 0 & 0 & 0 \\ 0 & \mathbf{h}_{\mathrm{v}}(s) & 0 & 0 \\ 0 & 0 & \mathbf{h}_{\mathrm{w}}(s) & 0 \\ 0 & 0 & 0 & \mathbf{h}_{\phi}(s) \end{bmatrix} \begin{Bmatrix} \mathbf{f}_{\mathrm{u}}(s) \\ \mathbf{f}_{\mathrm{v}}(s) \\ \mathbf{f}_{\mathrm{w}}(s) \\ \mathbf{f}_{\phi}(s) \end{Bmatrix} \tag{7}$$

In the frequency domain transformation, the nonlinear term is not considered and there is no displacement coupling in each direction. Combined with Equation (4), the harmonic response equation with 12 degrees of freedom at both ends of the straight pipe model can be obtained, and its harmonic response displacement **ψ**(s) is related to **h**(s) as follows:

$$\boldsymbol{\psi}(s) = \mathbf{h}(s)\mathbf{F}(s) \tag{8}$$

In the formula:

$$\boldsymbol{\psi}(s) = \left\{\bar{\mathbf{r}}_0^{\mathrm{T}}\mathbf{q}_{\mathrm{u}}, \mathbf{r}_0^{\mathrm{T}}\mathbf{q}_{\mathrm{v}}, \mathbf{r}_0^{\mathrm{T}}\mathbf{q}_{\mathrm{w}}, \bar{\mathbf{r}}_0^{\mathrm{T}}\mathbf{q}_{\phi}, \mathbf{r}_0'^{\mathrm{T}}\mathbf{q}_{\mathrm{v}}, \mathbf{r}_0'^{\mathrm{T}}\mathbf{q}_{\mathrm{w}}, \bar{\mathbf{r}}_1^{\mathrm{T}}\mathbf{q}_{\mathrm{u}}, \mathbf{r}_1^{\mathrm{T}}\mathbf{q}_{\mathrm{v}}, \mathbf{r}_1^{\mathrm{T}}\mathbf{q}_{\mathrm{w}}, \bar{\mathbf{r}}_1^{\mathrm{T}}\mathbf{q}_{\phi}, \mathbf{r}_1'^{\mathrm{T}}\mathbf{q}_{\mathrm{v}}, \mathbf{r}_1'^{\mathrm{T}}\mathbf{q}_{\mathrm{w}}\right\}^{\mathrm{T}} \tag{8}$$

h(s) is the harmonic response matrix, representing the pipeline response function, which is defined as follows. The displacement at the *i*th degree of freedom caused by the unit force or moment at the *j*th degree of freedom can be represented by the value of the *i*th row and *j* column in the matrix.

The frequency response function **h**(s) is considered damping as a non-singular matrix, and the above frequency response function and dynamic stiffness matrix are reciprocal:

$$\mathbf{F}(s) = \mathbf{z}(\mathbf{s})\boldsymbol{\psi}(\mathbf{s}); \mathbf{z}(\mathbf{s}) = \mathbf{h}(\mathbf{s})^{-1} \tag{9}$$

Where, **z**(**s**) is the dynamic stiffness matrix. The dynamic stiffness matrix can be used to describe the pipeline response considering damping. Different from FEM method, the analysis frequency of dynamic stiffness method can be determined according to the actual situation, and the dynamic characteristics of high frequency band of pipe conveying fluid can be obtained.

2.2 Dynamics Solution of Curved Pipes Conveying Fluid

The shape of fluid pipeline is highly correlated with the field space of industrial system, and there are not only straight pipe segments, but also various curved pipe segments with complex shapes. It is convenient to adopt the above analysis method for dynamics analysis. The reason is that FRF response matrix of straight pipe can be transformed into a combined FRF matrix of the whole element by coordinate transformation. For a pipe with complex shape, it can be divided into several straight pipe segment units (pipe unit) and then assembled separately to obtain the overall FRF matrix.

Firstly, coordinate transformation is carried out. Transformation matrix T can be defined through the global geometric coordinates , and the response matrix of pipe unit is transformed into the response matrix $\mathbf{H}_i$ in the overall coordinate system. Then the whole unit response matrix **H** can be assembled by means of pipe unit assembly. The piping system arbitrarily arranged in the overall coordinate system can be decomposed into *N* pipe units, and the response matrix $\mathbf{H}_i$ of each pipe unit can be expressed in the following form:

$$\mathbf{H}_i = \begin{bmatrix} \overline{\mathbf{h}}_{00}^i & \overline{\mathbf{h}}_{01}^i \\ \overline{\mathbf{h}}_{10}^i & \overline{\mathbf{h}}_{11}^i \end{bmatrix}, i = 1, ..., N \tag{10}$$

In the formula, the superscript is the serial number of pipe unit in the pipeline system, the subscript '0' represents the left end of the pipeline, and '1' represents the right end. Finally, the dynamic stiffness matrix of each pipe unit was obtained:

$$\mathbf{Z}_i = \begin{bmatrix} \mathbf{z}_{00}^i & \mathbf{z}_{01}^i \\ \mathbf{z}_{10}^i & \mathbf{z}_{11}^i \end{bmatrix}, i = 1, ..., N \tag{11}$$

According to the dynamic stiffness matrices of each pipe unit, the total dynamic stiffness matrix **Z** of the pipeline system was obtained by merging them:

$$\mathbf{Z} = \sum_{i=1}^{n} \left(\begin{matrix} & p & q \\ & \begin{bmatrix} \overline{\mathbf{z}}_{00}^i & \overline{\mathbf{z}}_{01}^i \\ \overline{\mathbf{z}}_{10}^i & \overline{\mathbf{z}}_{11}^i \end{bmatrix} & \begin{matrix} p \\ q \end{matrix} \end{matrix} \right) \tag{12}$$

In the formula, 'p' and 'q' represent the number of rows and columns corresponding to the dynamic stiffness matrix of each pipeline element in the whole system. Finally, the overall FRF response matrix can be obtained from the above combined dynamic stiffness matrix, as shown in Equation (13). The overall corresponding matrix can be obtained to analyze the vibration characteristics of curved pipe:

$$\mathbf{H} = \mathbf{Z}^{-1} \tag{10}$$

3. Calculation and Verification of Curved Pipes Conveying Fluid

In this paper, vibration of semicircular curved pipe exists in two directions: the plane direction where the axis is located (XZ plane as shown in Fig. 2) and the plane direction perpendicular to the curved pipe axis. It is assumed that the angle between the two supporting points and the curved pipe center line is θ, then for the semicircular curved pipe, θ=180 degrees. Fig. 2 shows the structure diagram of the semicircular curved pipe conveying fluid model. In order to verify the accuracy of this calculation model, the parameters in Mirsa[8] paper are used for calculation and comparison. The specific parameters are as follows: Water density ρ_f=1000kg/m^3, pipeline density ρ_p=7900kg/m^3, curved pipe geometric radius *R*=0.7m, outer diameter *D*=100mm, inner diameter *d*=94mm, Young's modulus *E*=200GPa, shear modulus *G*=76.9GPa. By using the method in Section 1.2, the curved pipe is decomposed into *N* straight pipe elements, and the convergence of *N* is shown as follows. For the convenience of comparison, the dimensionless results are used to represent the calculation results. Formula(14) defines its dimensionless frequency and speed:

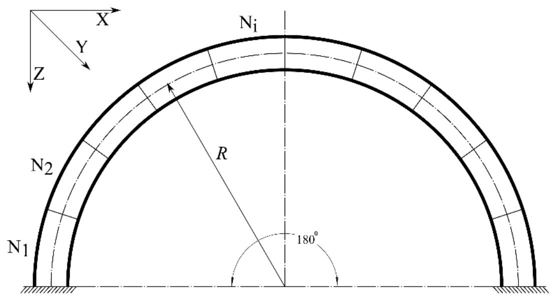

Fig. 2: Schematic diagram of analysis model of semicircular curved pipe conveying fluid (θ=180 °)

$$\omega^* = \omega R^2 \sqrt{\frac{m_p + m_f}{EI}} \quad , \quad V^* = VR\sqrt{\frac{m_f}{EI}} \tag{11}$$

In the above formula, *EI* is the curvature stiffness,

m_f is the liquid mass, m_p is the curvature mass, *V* is the flow rate in the pipe, ω is the dimensional frequency.

In addition, in order to simulate the fixed boundary conditions, the stiffness of the supporting spring at the head and tail of the pipeline is set as 10^8 in the dynamic stiffness matrix. If the pipeline is supported by elastic support, the dynamic stiffness of the support can be measured through the test and substituted into the dynamic stiffness matrix.

Table 1 calculates the dimensionless frequency convergence results of the curved pipe in the static state filled with liquid:

Table 1: Dimensionless frequency convergence of the curved pipe in the static state filled with liquid

N	In-plane frequency			Out-of-plane frequency		
	ω1	ω2	ω3	ω1	ω2	ω3
2	6.2518	9.0376	20.1160	1.6844	6.5434	9.4911
5	4.4378	9.6855	17.7513	1.8788	5.3124	11.0136
8	4.4054	9.6531	17.9456	1.8464	5.2800	11.0784
10	4.4054	9.6531	17.9456	1.8464	5.2800	11.0460
Mirsa[8]	4.39	9.64	17.95	1.83	5.28	11.10

It can be seen that, with the increase of component *N*, the modes in both directions of semicircular curved pipe both converge to the finite element results of Mirsa[8], indicating that the proposed method can simulate the dynamics characteristics of complex pipelines. Calculation resources and calculation accuracy are balanced, the number of units is taken as 10, and the error of the result is less than 1%.

4. Study on Vibration Characteristics of Curved Pips Conveying Fluid with Different Angles

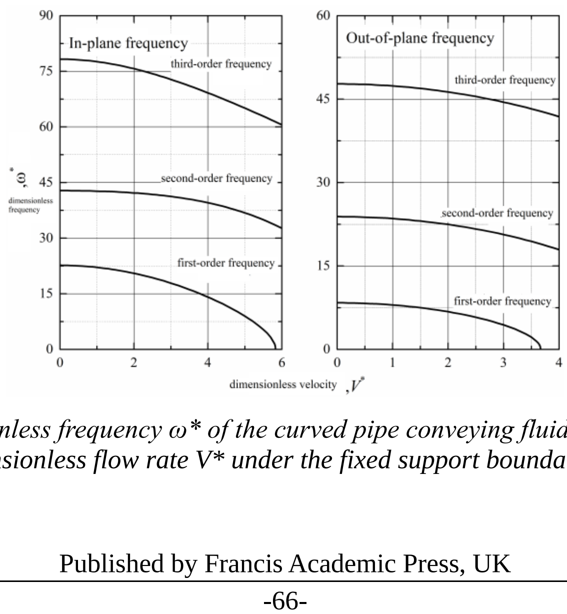

Figure 3: The dimensionless frequency ω of the curved pipe conveying fluid at θ=90 degrees varies with the dimensionless flow rate V* under the fixed support boundary at both ends*

In Section 3, the correctness of the proposed dynamic stiffness matrix function in solving the vibration modes of the curved pipe with flow input θ=180 degrees is verified. In this section, the vibration characteristics of the curved pipe with θ at other angles (90~180) are studied. Fig.3 shows the dimensionless frequency ω* of the curved pipe conveying fluid at θ=90 degree varies with the dimensionless flow rate V* under the fixed support boundary at both ends. It can be seen from the figure that the dimensionless frequencies of the corresponding orders at θ=90 °are larger than those at θ=180 degree, and the first-order critical flow velocities both in and out of the plane are larger than those at θ=180 degree. This is because the stiffness of the current-carrying pipeline when θ=90 degrees is much greater than that when θ=180 degrees, while the pipeline mass is less than that when θ=180 degrees. According to the frequency calculation formula, the correctness of the above calculation can be verified.

According to the above conclusions, when θ=135 degrees, the dimensionless frequency and first-order critical flow rate of the corresponding order are both larger than those of θ=180 degrees, but smaller than those of θ=90 degrees. Fig. 4 shows that the dimensionless frequency ω* of curved pipe conveying fluid at θ=135 °varies with the dimensionless flow rate V*, which verifies the correctness of the above conclusions.

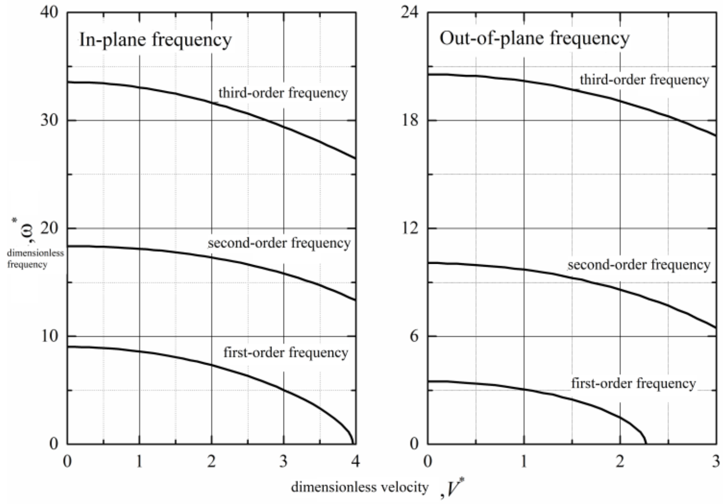

Figure 4: The dimensionless frequency ω of curved pipe conveying fluid at θ=135 ° varies with the dimensionless flow rate V* under the fixed boundary at both ends*

Curved pipe θ is roughly from 90 degrees to 180 degrees in industry. The dynamic stiffness method can be used to calculate the relation between the in-plane and out-of-plane critical dimensionless velocity and θ of the above-mentioned current-carrying curved pipes. As shown in figure 5, with the increase of value of θ, the first order dimensionless critical velocity decreases, and the drop is larger when θ is small, but smaller when θ is large. For a type of curved pipe conveying fluid with large flow velocity, small pipe diameter and thin wall thickness, if θ is large, considering the stability and vibration characteristics of the pipe, supporting conditions can be added in the pipe to enhance the stability of the piping system. However, as for the intermediate support of pipeline system, it is difficult to adopt rigid support because of the limited installation conditions and application scenarios. Most of them adopt elastic support. As shown in Fig.6, an elastic support condition with vertical support stiffness of 58000N/m and lateral support stiffness of 14260N/m were added in the middle of the pipeline to increase the first-order dimensionless critical velocity, thus increasing the stability of the pipeline and reducing the vibration of the pipeline.

The pipeline vibration frequency after increasing support conditions changes with the flow velocity is shown in figure 7, as you can see, all in-plane and out-of-plane frequencies are larger than those without intermediate support conditions. And the critical dimensionless flow velocity corresponding to the first order frequency in the plane increases from 3.00 to 4.21, and the critical dimensionless flow velocity corresponding to the first order frequency outside the plane increases from 1.58 to 2.40. The stability of the piping system has been greatly improved.

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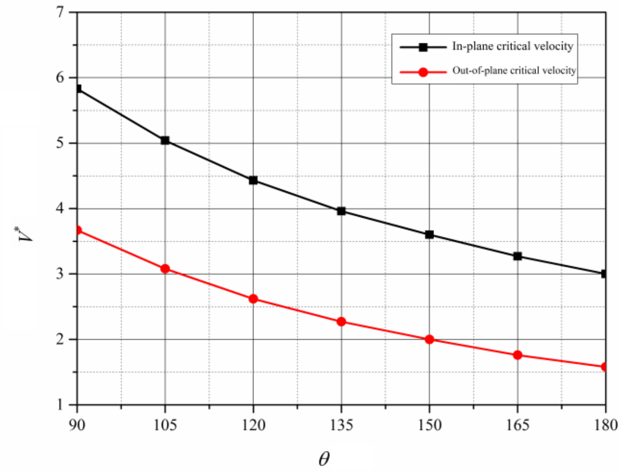

Figure 5: Relationship between the first-order in - plane and out-of-plane critical dimensionless velocity and θ of curved pipe conveying fluid

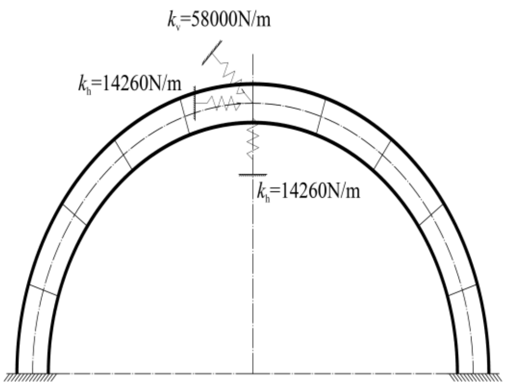

Figure 6: Schematic diagram of analysis model of semicircular curved pipe conveying fluid with middle support (θ=180 °)

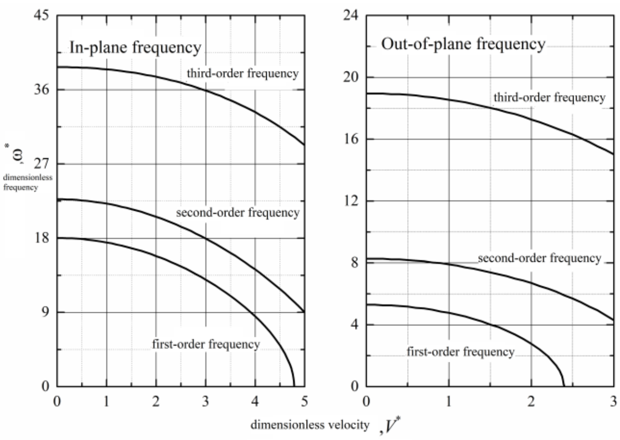

Figure 7: The dimensionless frequency ω varies with the dimensionless flow velocity V* of the curved pipe conveying fluid with middle support at θ=180 °*

Similarly, it can also be used to calculate and analyze the curved pipe conveying fluid under other supporting conditions, including the dynamic stiffness characteristics of the elastic support which can be measured according to the actual pipe support. The dynamic stiffness method proposed in this paper can be used to calculate the vibration characteristics of a curved pipe conveying fluid under the above arbitrary supporting conditions.

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5. Conclusion

Straight pipe elements were combined into curved pipe conveying fluid by the dynamic stiffness method, and any supporting boundary conditions could be simulated. The vibration calculation method of curved pipe conveying fluid based on the dynamic stiffness method was established. Through comparison and verification, and the following conclusions were obtained:

(1) The calculation results of the proposed new dynamic stiffness method are in good agreement with those of Misra finite element method, and the calculation error is less than 5%.

(2) With the increase of radian Angle θ, the frequency and critical flow velocity both in and out of the plane of the corresponding order in the curved pipe conveying fluid decrease, and the first-order dimensionless critical flow velocity decreases, and the decrease is larger when θ is small and smaller when θ is large.

(3) When the stability of curved pipe conveying fluid can not meet the requirements, the stability of the pipeline system can be increased by adding intermediate supports. The calculation method proposed in this paper can be used to solve the vibration characteristics problem the pipeline system under arbitrary elastic supports.

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