ISSN 2616-5805 Vol. 3, Issue 1: 47-54, DOI: 10.25236/AJMS.2022.030107

Mathematical modeling and dynamic trading strategies for gold and bitcoin

Yan Chenge[1,#,*], Wu Zebin[1,#], Gan Haoyu[1,#], Zhu Ziwen[2,#]

[1]*School of Science and Engineering, the Chinese University of Hong Kong, Shenzhen, 518172, China*
[2]*School of Management and Economics, the Chinese University of Hong Kong, Shenzhen, 518172, China*
**Corresponding author: 119020063@link.cuhk.edu.cn*
[#]*These authors contributed equally.*

Abstract: *In recent years, the gold-bitcoin market and corresponding trading strategies have received more scholarly attentions. Predicting gold and bitcoin prices from historical data is a specific stream in this academic area. Many scholars have used financial methods or statistical methods to construct trading models. However, one of the limitations is that few previous studies combined both financial and statistical methods. Therefore, the present study aims to build a mathematical model that predicts price dynamics of gold and bitcoin and utilizes some connections between finance and statistics. To achieve this goal, some financial indicators were computed and Holt-Winters' Model was applied. The research result shows that a trading strategy can be developed with the help of our proposed model and trading shrink ratio, which functions as the risk controller. The sensitivity test indicates that the proposed model has little sensitivity towards commission fees, which means that the model can be widely used in similar situations. In general, this study outlines an analytical approach to evaluate profits in gold-bitcoin market. Traders can generate considerable profits from the proposed trading strategy.*

Keywords: *bitcoin-gold trading; mathematical models; Holt-Winters; dynamic strategies*

1. Introduction

Trading strategies that involve bitcoin and other financial assets have received much scholarly attention in recent years. Scholars have already made remarkable contributions in this academic area, such as studying price dynamics of trading strategies, estimating profits (Ahn, Conrad & Dittmar, 2003) [1] and adjusting risks of strategies. Generally, scholars attempted to build mathematical models that capture and predict the volatility of prices, which can be used to construct trading strategies. The purposes of these trading strategies are various, including maximizing profits, minimizing risks or reaching market equilibrium.

Up till now, scholars have not reached an agreement on research methods. Some people think that a finance-based approach is more reasonable because there is some financial or economic theory behind each step in building the mathematical model (e.g. Farmer & Joshi, 2002; Feuerriegel & Prendinger, 2006) [2]. To be precise, the finance-based approach is supported by financial theory, which ensures that the derivation of the model is sound. However, it is usually less quantitative than statistics-based methods, and its prediction accuracy is relatively low. In contrast, statistics-based approaches focus more on accurate predictions. Various statistical models can be built to suit different situations (e.g. Bouri et al., 2020; Cuoco and Isaenko, 2008)[3,4]. This model is hard to prove because it lacks theoretical support. In addition, it may cause overfitting problems in some cases. Although these previous studies have developed excellent models and trading strategies, they still have some limitations that could be improved upon.

The present study focuses on trading strategies of cash, gold and bitcoin, which is a specific stream of the academic area introduced above. We collected the historical data of gold and bitcoin from September 11th, 2016 to September 10th, 2021. Without the loss of generality, it is assumed that we have 1000 units of cash at the beginning and corresponding transaction costs are 1% and 2% for gold and bitcoin respectively. The purpose of the study is to construct the best trading strategy according to the above information.

ISSN 2616-5805 Vol. 3, Issue 1: 47-54, DOI: 10.25236/AJMS.2022.030107

2. Model building

2.1 Data processing and computing financial indicators

We used two datasets which display historical prices of gold and bitcoin from September 11th, 2016 to September 10th, 2021. Bitcoin is traded every day but gold is only traded when the markets are open. Both datasets have some missing data, which are less than 5% of the sample size. According to Meucci (2005)[5], if the missing data constitutes less than 5% of total financial observations, these missing data can only affect prediction results in tiny scale. Therefore, these missing data was directly deleted in the processing procedure.

Our model aims to combine some financial knowledge and statistical techniques, which bridges the gap that previous studies only focused on finance-based methods or statistics-based methods. To achieve this goal, some financial indicators were computed in data processing procedure, which can trace out performance of financial assets (O'Hara et al., 2000)[6]. These financial indicators would be used in the later statistical analysis procedure. In total, we considered five indicators that described the financial behaviors of gold and bitcoin from different aspects, including

Single moving average (SMA), Double exponential moving average (DEMA), moving average convergence and divergence index (MACD), Relative strength index (RSI), Momentum index.

The five financial indicators listed above were prepared for the statistical analysis. Up till this step, we actually finished data processing and transformed the original data into some of these financial indicators. Figure 1 shows an overview of the method.

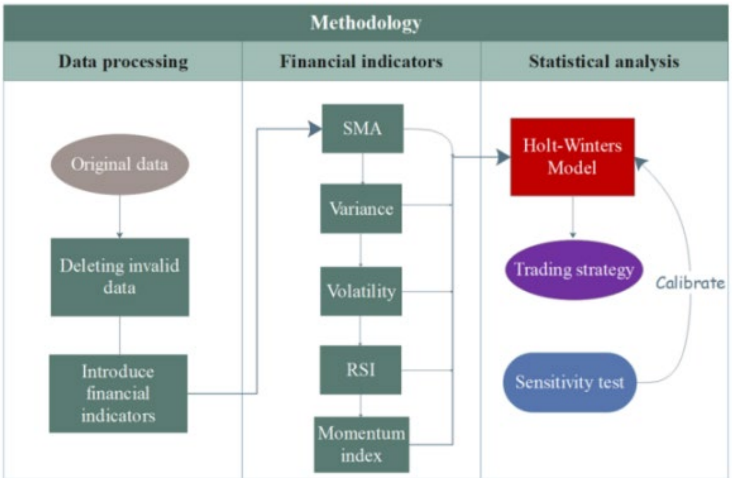

Figure 1: Outline of methodology

2.2 Statistical analysis and model construction

In this section, the detailed steps will be shown for building non-seasonal Holt-Winters' Model. According to Holt (1957) and Winter (1960)[7,8], the Holt-Winters non-seasonal method comprises the forecast equation and three smoothing equations — one for the level e_t, one for the trend β, and one for the non-seasonal component "s", with three corresponding smoothing parameters α β and γ. Both Holt-Winters' additive method and multiplication method were used for predicting price dynamics. The major logics are given by:

$$\begin{aligned} e_t &= \alpha y_t + (1-\alpha)(e_{t-1} + b_{t-1}) \\ b_t &= \beta(e_t - e_{t-1}) + (1-\beta) b_{t-1} \\ \hat{y}_{T+h}\big| &\leq T = e_T + h b_T \end{aligned} \tag{1}$$

In summary, the time series is described as a moving level, which is continuously adapting over time, driven by a similarly adapting, time-dependent slope. The choice of using weighted averages amounts at defining the "scheme" defining the components. After these steps, we predicted future prices of a certain day from the past historical data. These predictions will be used in the following sections for decision making and building trading strategy.

ISSN 2616-5805 Vol. 3, Issue 1: 47-54, DOI: 10.25236/AJMS.2022.030107

2.3 Accuracy back-casting and trading strategy

Based on the prediction form Holt-Winters' Model, this section explains the accuracy back-casting procedure and the logic of building trading strategy. Real data of gold and bitcoin were used for back-casting the accuracy of Holt-Winters' Model. The back-casting focused on three financial indicators: Momentum Index, Daily Price Volatility and Relative Strength Index (RSI).

Figure 2 displays the whole back-casting procedure. We estimated the accuracy rate of one financial indicator for a certain day. This accuracy rate plays important role when building trading strategy.

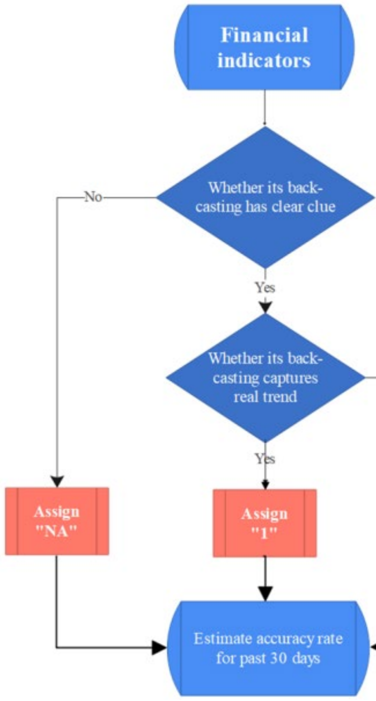

Figure 2: The back-casting procedure

To simplify the decision-making procedure, we selected only one financial indicator out of three. We added the cubic power to the accuracy to increase the penalty of incorrect prediction. However, only considering the results from statistical analysis is a limitation. To avoid this, one of the financial indicators should be included. The Relative Strength Index (RSI) was selected to compute trading shrinking ratio jointly. In this case, we transformed the original representation as follows.

When the Holt-Winters' prediction is opposite of RSI behavior:

$$r_0 = \frac{r_1 - r_1 r_2}{r_1 + r_2 - 2r_1 r_2} \tag{2}$$

When the Holt-Winters' prediction is consistent with RSI behavior:

$$r_0 = \frac{r_1 r_2}{2r_1 r_2 - r_1 - r_2 + 1} \tag{3}$$

In the two equations above, the cubic of r_0 is the adjusted trading shrink ratio after considering the accuracy rate of RSI. The r_1 is the accuracy rate of Holt-Winters' algorithm and r_2 is the accuracy rate of RSI. Our trading strategy was developed by these techniques.

3. Results analysis

3.1 Result of financial indicators

As mentioned in the previous section, five financial indicators were computed to capture the price

ISSN 2616-5805 Vol. 3, Issue 1: 47-54, DOI: 10.25236/AJMS.2022.030107

dynamics, including Simple moving average (SMA), Price Volatility, Extreme Difference, Variance, Relative Strength Index (RSI) and Momentum Index. The result of computing is shown in this section. These indicators were used to construct Holt-Winters Model.

Table 1: Daily RSI for gold (a sample)

	Date <chr>	Gold Daily RSI <dbl>
1	9/22/16	64.22868
2	9/23/16	66.72613
3	9/26/16	54.43351
4	9/27/16	60.77835
5	9/28/16	58.73844
6	9/29/16	57.12984

Table 2: Daily RSI for bitcoin (a sample)

	Date <chr>	Bitcoin Daily RSI <dbl>
1	9/19/16	26.83457
2	9/20/16	28.88818
3	9/21/16	23.76967
4	9/22/16	22.61684
5	9/23/16	41.10168
6	9/24/16	40.09598

Table 1 and 2 display the daily RSIs for gold and bitcoin respectively (only a smaple). For example, the corresponding RSI for gold is 64.2287 on September 22nd, 2016. The two diagrams of RSI are shown in Figures 3 and 4.

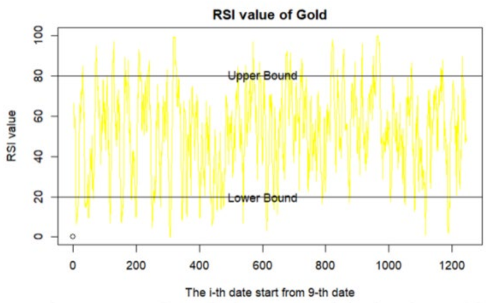

Figure 3: The relative strength index for gold

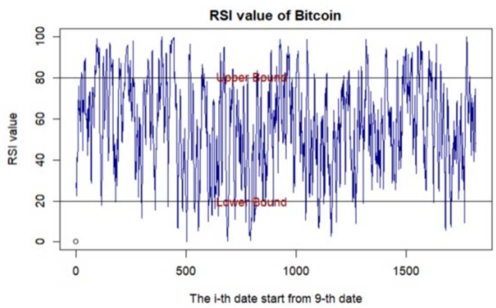

Figure 4: The relative strength index for bitcoin

As can be seen in Figure 3 and 4, the proportion of RSI that exceeds the upper and lower bound of gold is larger than that of bitcoin. This result echoes with the arguments of Brennan et al (1997)[9], which suggests that bitcoin is relatively more volatile than gold. Thus, traders should consider gold as long-term investment and bitcoin for short-term investment.

To calculate other financial indicators, we first computed the daily volatility and extreme difference for these two assets. The following Table 3 and Table 4 are the samples of the result for daily price volatility.

Table 3: Daily volatility for gold (a sample)

	Date <chr>	Gold Daily Volatility <dbl>
1	9/13/16	-0.0007171976
2	9/14/16	-0.0014354248
3	9/15/16	-0.0082844713
4	9/16/16	-0.0018690876
5	9/19/16	0.0049680896
6	9/20/16	-0.0007985702

ISSN 2616-5805 Vol. 3, Issue 1: 47-54, DOI: 10.25236/AJMS.2022.030107

Table 4: Daily volatility for bitcoin (a sample)

	Date <chr>	Bitcoin Daily Volatility <dbl>
1	9/12/16	-0.019271294
2	9/13/16	0.002050290
3	9/14/16	-0.003437439
4	9/15/16	0.002562334
5	9/16/16	-0.002080671
6	9/17/16	-0.003398401

Figures 5 and 6 show the daily price fluctuations of gold and bitcoin, respectively. As you can see, the price of bitcoin is generally more volatile than the price of gold. At times, bitcoin prices have fallen nearly 40%. In contrast, gold's volatility is around 10 per cent.

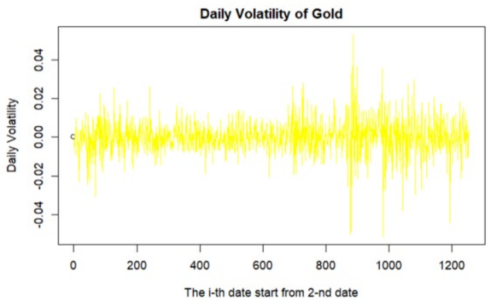

Figure 5: Daily volatility for gold

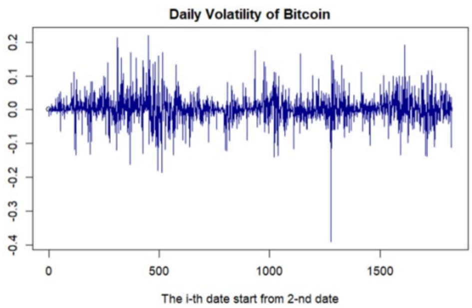

Figure 6: Daily volatility for bitcoin

Table 5 lists the extreme difference and variance of gold and bitcoin prices. The extreme difference of gold price (941.45) is much lower than that of bitcoin (62960.36). As for price variance, the result is similar to that of extreme difference. These results set foundations for the computation of SMA and Momentum ndex.

Table 5: Extreme difference and variance of daily prices

	Price Extreme Difference <dbl>	Price Variance <dbl>
Gold	941.45	62146.41
Bitcoin	62960.36	197230892.03

The average prices displayed in Table 6 and 7 were used to computing simple moving averages in Figure 7 and 8. In Figure 7, the green dot line is the average gold price for the past 13 weeks and the yellow line is the actual daily gold price. In Figure 8, the red dot line is the average daily prices for bitcoin and the blue dashed line is the actual price. These "13 weeks" and "9 days" are the parameters for each simple moving average. It can be seen at the late observation period, the difference of gold actual price and its past averages tends to be larger than bitcoin.

Table 6: The average gold prices for computing SMA

	The i-th date start from 91-st day <int>	The Past 13 Weeks Average Gold Price <dbl>
1	1	1233.187
2	2	1232.002
3	3	1230.588
4	4	1229.137
5	5	1227.753
6	6	1226.484

Table 7: The average bitcoin prices for computing SMA

	The i-th date start from 9-th day <int>	The Past 9 days Average Bitcoin Price <dbl>
1	1	611.0400
2	2	609.5967
3	3	608.3978
4	4	606.8978
5	5	605.2600
6	6	604.5378

ISSN 2616-5805 Vol. 3, Issue 1: 47-54, DOI: 10.25236/AJMS.2022.030107

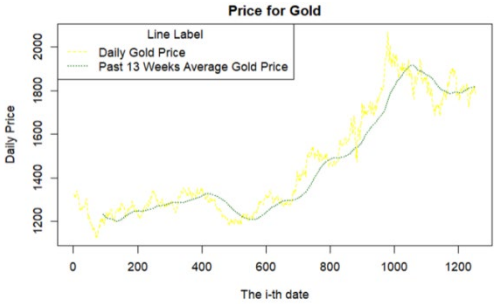

Figure 7: Daily volatility for gold

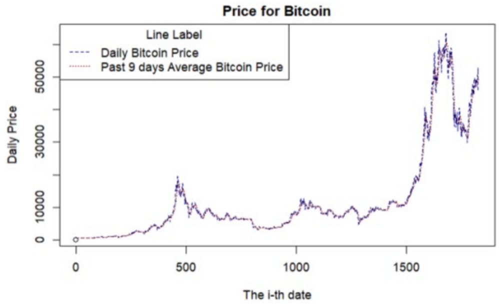

Figure 8: Daily volatility for bitcoin

By similar methods, momentum indices for gold and bitcoin were computed to dig out more financial information. The calculation results are shown in Figures 9 and 10. The Y-axis in the figure is the momentum exponent. Obviously, bitcoin has a much larger momentum index than gold. The momentum index usually reaches a top before the actual price. We will estimate prices based on momentum indicators in the Holt-Winters model below.

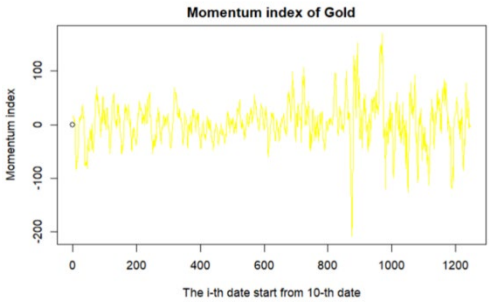

Figure 9: Momentum index for gold

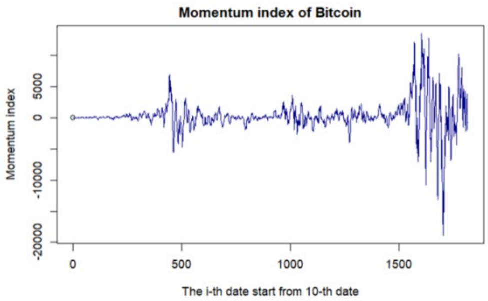

Figure 10: Momentum index for bitcoin

3.2 Result of Holt-Winters' Model predictions

Holt-Winters' Model was used to predict the daily prices of gold and bitcoin. The results of the Holt-Winters model for gold are shown in Figures 11 and 12. The actual price trend was also plotted for comparison.

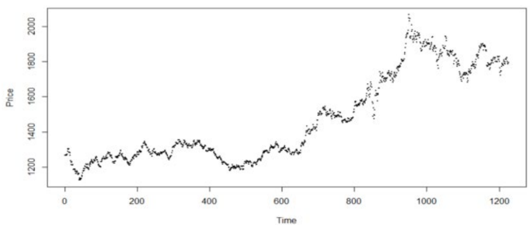

Figure 11: The trend of gold actual prices

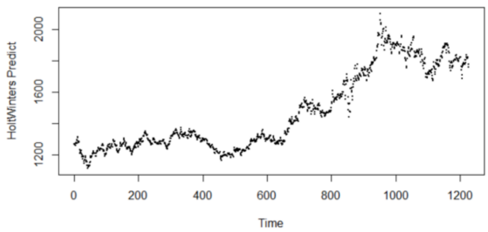

Figure 12: Forecast price trend

As can be seen in Figure 11 and 12, the predictions from Holt-Winters' Model are very similar to the actual trend. This echoes with our argument in Section 2 that non-seasonal Holt-Winters' is an appropriate model for dynamic prices. Similarly, the following Figure 13 and 14 trace out the actual prices and predicted prices for bitcoin.

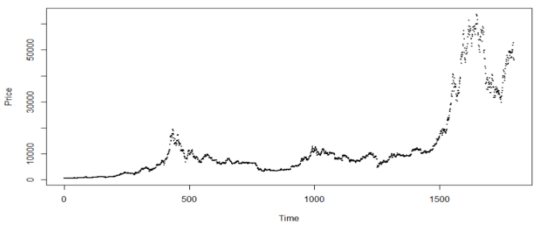

Figure 13: The trend of bitcoin actual prices

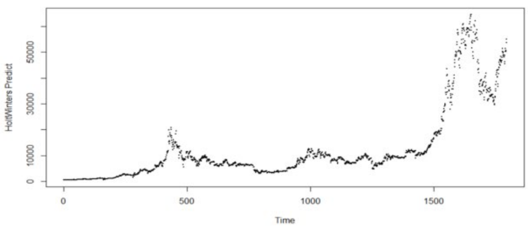

Figure 14: The trend of bitcoin predicted prices

ISSN 2616-5805 Vol. 3, Issue 1: 47-54, DOI: 10.25236/AJMS.2022.030107

3.3 Result of back-casting for three financial indicators

The back-casting procedure was conducted for three financial indicators: the momentum index, the relative strength index (RSI) and simple moving average (SMA). For each trading day that needs back-casting, the data of 30 trading days before that day were used in back-casting.

Table 8: Back-casting for momentum index of gold

	Date <chr>	Daily Price Volatility <dbl>	Pertinent Momentum Index <dbl>	Prediction Result (TRUE/FALSE/NULL) <dbl>
1241	9/3/21	0.006151554	44.65	0
1242	9/6/21	-0.001151505	19.60	1
1243	9/7/21	-0.010677426	-6.30	0
1244	9/8/21	-0.008961518	-2.70	0
1245	9/9/21	0.001259798	1.65	0
1246	9/10/21	0.003550958	-3.90	1

Table 9: Back casting for momentum index of bitcoin

	Date <chr>	Daily Price Volatility <dbl>	Pertinent Momentum Index <dbl>	Prediction Result (TRUE/FALSE/NULL) <dbl>
1812	9/5/21	-0.001757758	2984.58	1
1813	9/6/21	0.036471983	2712.20	0
1814	9/7/21	0.017546001	3779.75	0
1815	9/8/21	-0.111399386	-1997.61	0
1816	9/9/21	-0.015612112	-996.39	0
1817	9/10/21	0.006300352	-787.18	1

Table 8 and 9 display the back-casting results for momentum index of gold and bitcoin respectively. The right-most column shows the prediction results. Here, "0" represents that the prediction from Holt-Winters' Model fails to capture the actual volatility. "1" is assigned if the prediction captures the actual volatility successfully. "NA" means that there is no clear clue for judging the prediction. For example, in Table 8, the prediction result on September 3rd, 2021 is "0". This suggests that prediction of Holt-Winters' cannot reflect the real prices. Since we have 30-days back-casting period for each trading day, the accuracy rate can be calculated, which can be computed as the number of correct predictions (the number of "1") divided by 30. This accuracy rate will be used to make trading strategy in the next section.

Similar to momentum index, the back-casting for simple moving average (daily average price) indicator follows the same steps. The results are shown in Table 10 and 11.

Table 10: Back-casting for average daily price of gold

	Date <chr>	Daily Price Volatility <dbl>	Daily Price <dbl>	Pertinent Past Day Average <dbl>	Prediction Result (TRUE/FALSE/NULL) <dbl>
1160	9/3/21	0.006151554	1823.70	1816.767	1
1161	9/6/21	-0.001151505	1821.60	1817.179	0
1162	9/7/21	-0.010677426	1802.15	1817.508	1
1163	9/8/21	-0.008961518	1786.00	1817.764	1
1164	9/9/21	0.001259798	1788.25	1817.991	0
1165	9/10/21	0.003550958	1794.60	1817.956	0

Table 11: Back-casting for average daily price of bitcoin

	Date <chr>	Daily Price Volatility <dbl>	Daily Price <dbl>	Pertinent Past Day Average <dbl>	Prediction Result (TRUE/FALSE/NULL) <dbl>
1813	9/5/21	-0.001757758	49947.38	48796.27	0
1814	9/6/21	0.036471983	51769.06	49097.62	1
1815	9/7/21	0.017546001	52677.40	49517.60	1
1816	9/8/21	-0.111399386	46809.17	49295.64	1
1817	9/9/21	-0.015612112	46078.38	49184.93	1
1818	9/10/21	0.006300352	46368.69	49097.46	0

As shown in Table 10 and 11, the back-casting result on September 6th, 2021 is "1". This indicates that the prediction result from Holt-Winters' Model on that day can explain the actual price dynamics. The corresponding accuracy rate is 66.6%.

The same back-casting procedure was repeated for relative strength index of gold and bitcoin, we observed a lot of NA, which suggests that there is no strong evidence to judge the accuracy of Holt-Winters' Model on that certain day. This result does not mean the accuracy of Holt-Winters' Model is zero, but means no judgment can be made because of little evidence.

As Table 12 shows, the accuracy for RSI, momentum index and daily average prices are 90.18%, 34.75% and 75.89% respectively. As a result, we chose RSI to formulate the trading shrink ratio.

Table 12: Back-casting for RSI of bitcoin

	RSI <dbl>	Momentum Index <dbl>	Past Days Average Price <dbl>
Average Correctness for Index	0.9018088	0.3475098	0.7589485

1 row

ISSN 2616-5805 Vol. 3, Issue 1: 47-54, DOI: 10.25236/AJMS.2022.030107

4. Conclusion

The present study outlines a mathematical model and trading strategy that maximize the profits in gold-bitcoin trading. The only data used were the historical prices of gold and bitcoin. To avoid some limitations in previous study, both financial methods and statistical analysis were applied in this study. In financial aspect, indicators like relative strength index (RSI), momentum index and simple moving average (SMA) were computed. Trading shrink ratio was also calculated to control the trading volume. In statistical aspect, non-seasonal Holt-Winters' Model was applied to predict price dynamics. A trading strategy was developed combing both financial and statistical information. The result indicates that traders can make huge profit by using our strategy. In addition, the sensitivity test suggests that the change of commission fees has little impact on our model.

References

[1] Ahn, D. H, Conrad, J, & Dittmar, R. F. (2003). Risk adjustment and trading strategies[J]. The Review of Financial Studies, 16(2): 459-485.
[2] Farmer, J. D., & Joshi, S. (2002). The price dynamics of common trading strategies. Journal of Economic Behavior & Organization, 49(2), 149-171.
[3] Bouri, E, Shahzad, S. J. H, Roubaud, D, Kristoufek, L, & Lucey, B. (2020). Bitcoin, gold, and commodities as safe havens for stocks: New insight through wavelet analysis[J]. The Quarterly Review of Economics and Finance, 77: 156-164.
[4] Cuoco, D., He, H., & Isaenko, S. (2008). Optimal dynamic trading strategies with risk limits. Operations Research, 56(2), 358-368
[5] Meucci, A. (2005). Risk and asset allocation (Vol. 1). New York: Springer.
[6] O'Hara, H. T., Lazdowski, C., Moldovean, C., & Samuelson, S. T. (2000). Financial indicators of stock price performance. American Business Review, 18(1), 90.
[7] Holt, C. E. (1957). Forecasting seasonals and trends by exponentially weighted averages (O.N.R. Memorandum No. 52). Carnegie Institute of Technology, Pittsburgh USA.
[8] Winters, P. R. (1960). Forecasting sales by exponentially weighted moving averages. Management Science, 6(3), 324–342.
[9] Brennan, M. J, Schwartz, E. S, & Lagnado, R. (1997). Strategic asset allocation[J]. Journal of Economic dynamics and Control, 21(8-9): 1377-1403.