ISSN 2616-5767 Vol.6, Issue 2: 29-39, DOI: 10.25236/AJETS.2023.060205

A rapid approach on predicting the positioning error in ball-screw motion systems

Zhang He

College of Mechanical and Engineering, Hunan University of Science and Technology, Xiangtan, China

Abstract: *In this paper, a rapid approach on predicting the positioning error with high precision and resolution through measuring the errors of transmission and power in ball-screw motion systems is proposed. The positioning error is considered as the superposition of the transmission error of screw-nut pair and the power error of the motor. The influences on the positioning error of assembly deviation and the deformation of the screw are analyzed according to the space geometry. Given the periodic characteristics of Fourier series, the comprehensive functions can be predicted by fragments. Thus, the method for predicting positioning errors by measuring a few values of transmission errors of screw-nut and that of motor errors is presented. A micro ball-screw motion system is measured to validate the method in the case study. The transmission error of the screw-nut was fitted into linear and trigonometric functions, and the motor error is fitted into sinusoidal curves. The two expressions at the same initial phase are superposed to predict the positioning errors. The coincidence between the measured results of positioning errors and the predicting spline proves the validity and precision of the rapid approach.*

Keywords: *Positioning error model; Error analysis; Ball-screw motion system; Transmission error; Prediction*

1. Introduction

The ball-screw motion system (BMS) is an essential component widely applied in machine tools and its motion errors affect machine accuracy directly. Positioning accuracy of BMSs is a significant indicator characterizing the machining quality, yet it will be undermined by geometric errors of machine elements. Since positioning errors are systematic, repeatable and measurable, error compensation is an effective technique to reduce the positioning error and improve the accuracy.

There are mainly two ways to compensate for errors in BMSs, divided into two categories mainly according to whether there is feedback. One method is adopting closed-loop compensation, which is applied to BMSs with high-precision sensors for detecting real-time positioning errors. A micro tool positioning error measurement and auto-compensation method utilizing machine vision technology with 2 CCDs were proposed by Wang[1]. The on-line asynchronous compensation methods for static/quasi-static error caused by machine geometry were proposed in Shen's research[2]. An adaptive compensation strategy for quasi-static error correction in intrinsic machines is proposed by Barakat[3]. Nevertheless, because of the high cost, complexity and instability, another method with the semi-closed loop is extensively used in industrial manufacturing.

Positioning error data is a prerequisite for positioning error compensation in semi-closed loop systems, and the compensation data of positioning error is obtained by point-by-point measurement in traditional practice. The operating resolution of BMSs can reach micron level and the stroke can be tens of millimeters, hence it takes thousands of measurements to get the total off-line data, which is time-consuming. To solve this problem, the positioning error model needs to be established to predict the values of positioning errors at each position. A model to predict the transmission accuracy of BMSs considering the manufacturing errors and installation errors is proposed in Wang's study[4]. The analysis in the study shows that the eccentricity error is the dominant factor leading to the periodic fluctuation of the transmission error of BMSs. Kono[5] described a systematic method to model and compensate geometric errors of machine tools. Geometric errors were separated from other errors in the frequency domain by using the Fourier series. The positioning error is mainly affected by lead error, assembly error and motor error in Tang's study[6]. Yue[7] presents a methodology to compensate profile errors by modifying tool path in corner milling process. Liang[8] proposes a novel compensation method that includes both position and posture errors of the tool center point of a five-axis machine tool, and both kinds of errors are compensated in an off-line way that compares the coordinates and angles point by

ISSN 2616-5767 Vol.6, Issue 2: 29-39, DOI: 10.25236/AJETS.2023.060205

point through iterations. These studies proposed error models for off-line compensation for geometric errors mainly through numerous measurements, whereas the positioning error model lack rapid, detailed and systematic development in BMSs.

In this paper, a rapid approach on predicting the positioning error of BMSs with high precision and resolution is proposed. The positioning error is divided into power and transmission error according to the propagation of error in BMSs. In order to obtain the characteristics of the transmission error including assembly deviation and deformation of the screw, the kinematic mechanism is analyzed and expressed based on space geometry. Based on the error patterns of crucial errors including deformation error, assembly deviation, and power instability, the rapid approach to predict positioning error through measuring errors of transmission and power of BMSs with superposition is established. The method can provide precise data for error compensation and guidance for designing BMSs by measuring the geometric errors of machine elements.

2. Positioning error analysis in ball screw systems

2.1. Machine configuration

In the area of engineering application, BMSs are widely utilized in much high-accuracy equipment such as chip mounter, precision CNC machine tool, femtosecond laser manufacturing process, etc. Compared with other gearings, BMSs with effective transmission are highly accurate and stable. Fig. 1 illustrates a typical configuration of BMS driven by a leadscrew with a motor. It is composed of motor, coupler, ball screw, guideway and worktable, etc. and is universally used for manufacturing systems aiming at high-precision positioning. The motion mechanism of the BMS is that the screw is rotated by the motor shaft relative to the base, and the nut is rotated by balls relative to the screw. Meanwhile, the nut is translated in the y-direction.

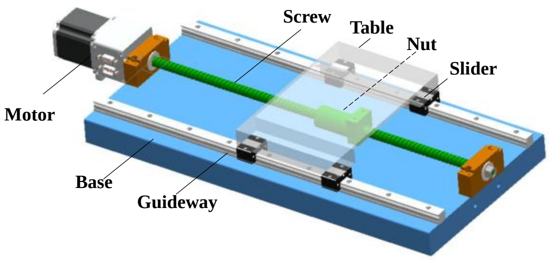


Figure 1: Schematic for a typical configuration of BMS.

2.2. Analysis of positioning error

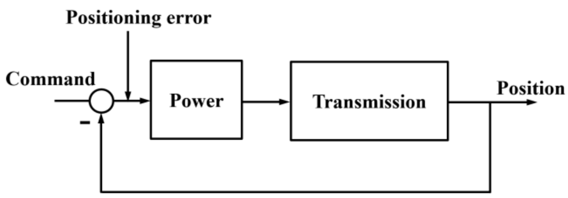


Figure 2: The closed-loop diagram of the positioning error of BMSs.

Figure 2 is the closed-loop diagram of the positioning error of BMSs. The input of system is the command of the target position, and the actual position is disturbed by the power and transmission error. The positioning error is the deviation of the actual and target position. Since the positioning error is composed of power and transmission error, the two major error components are analyzed in this part.

2.2.1. Analysis of transmission error

Assembly deviation of screw Figure 3 is the schematic for the installation of a lead screw that is widely adopted in the BMSs. The screw shaft is supported by two duplicated bearings at its two terminals, and the nut is connected with the screw with balls in it. Meanwhile, the nut is fixed with a table and slider constrained by the linear guideways to avoid rotating with the screw shaft.

ISSN 2616-5767 Vol.6, Issue 2: 29-39, DOI: 10.25236/AJETS.2023.060205

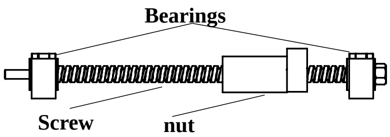
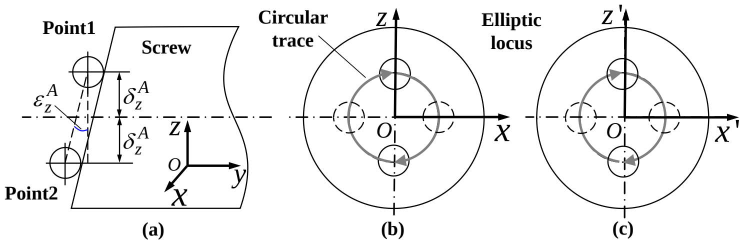


Figure 3: Schematic for the installation of screw-nut pair in the BMS.

The bearings at both ends put axis force on screw and can be equivalent to point contact as Figure 4(a). During the process of installation and adjustment of the screw, the dislocation between the contact point and axis of screw shaft and the nonorthogonality between the Y-axis and the axis of screw. Two coordinate systems are defined for analyzing the assembly error clearly. Figure 4(b) shows the y-direction view from the left side of the screw which is a circular trace. The x'-and z'-direction are the coordinates in normal view from the interface in which the trace of contact point is an elliptic locus in Figure 4(c). Since the contact point has no size, the axial displacement of the contact point is that of the screw shaft. When the screw shaft rotates through one cycle, the point will travel along the elliptic locus in the interface shown in Figure 4(c). Thus, the elliptic locus can be expressed as Eq. (1).



(a) front view (b) Y-direction view of the circular trace from the left side of the screw (c) normal view of the interface, in which there is an elliptic locus.

Figure 4: Schematic for assembly error of screw shaft.

$$(\frac{x'}{\delta_z^A})^2 + (\frac{z'}{\delta_z^A / \cos\varepsilon_z^A})^2 = 1 \quad (1)$$

where δ_z^A is the deviation between the contact point and the axis of screw in the z-direction; ε_z^A is the rotary deviation between the interface plane and the normal plane of the rotation axis of the screw shaft. The $\delta_z^A / \cos\varepsilon_z^A$ and δ_z^A denote half of the length of the major and minor axis of the elliptic in the interface plane, respectively, as shown in Figure 4(c).

Therefore, when the screw rotates with angular rate ϖ, the parametric equation of the spatial trace of the contact point in the world coordinate system can be expressed as Eq. (2). From the equation, it can be deduced that the y-direction displacement error varies with the rotation of screw and the pattern of the positioning error caused by y-direction displacement error of screw is a sinusoid as shown in Figure 5. The wavelength and the amplitude of the assembly error are 2π and $-\delta_z^A \cdot \tan\varepsilon_z^A$ which reflecting the installing deviation of the screw.

$$\begin{cases} \delta x = \cos\varpi t \\ \delta z = \sin\varpi t \\ \delta_1(\varpi t) = -\delta_z^A \cdot \sin\varpi t \cdot \tan\varepsilon_z^A \end{cases} \quad (2)$$

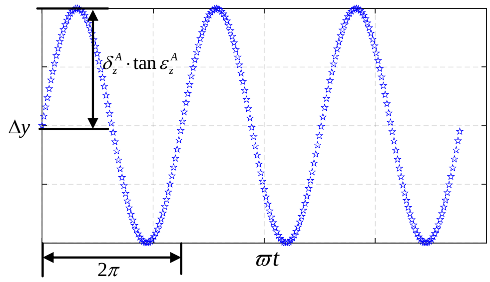


Figure 5: Component of the assembly error of screw depending on displacement ϖt .

ISSN 2616-5767 Vol.6, Issue 2: 29-39, DOI: 10.25236/AJETS.2023.060205

Deformation error the screw is supported by duplicate bearings at both ends, hence the screw bends downward forced by gravity. Assume that the uniform load acting on the screw is *q*, the deformation of screw is simplified as Figure 6; the deflection of the screw is as Eq. (3).

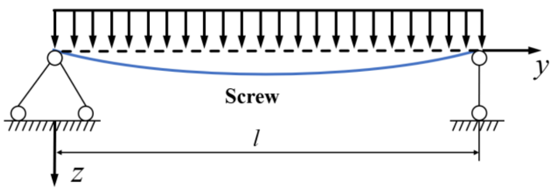


Figure 6: The deformation of screw forced by the uniform load of gravity.

$$\omega(y)=\frac{1}{EI}(\frac{ql}{12}y^3-\frac{ql^3}{24}y-\frac{q}{24}y^4) \tag{3}$$

where q is the gravity of the ball screw per unit length; E is the elastic modules; I is the polar moment of inertia; when the nominal displacement is y_i, that is the distance on distorted screw; assume the displacement in the horizontal direction is y_a. The equation between the actual and ideal displacement is

$$y_i=\int_0^{y_a}\sqrt{1+[\omega'(y)]^2}dy \tag{4}$$

The positioning error caused by the deformation of screw is

$$\delta_2(y)=y_a-y_i \tag{5}$$

2.2.2. Analysis of power error

The rotational errors of motor result from the limited machine accuracy of the rotor and stator. The simulation model of a two-phase hybrid stepper motor open-loop control system based on the mathematical model of the stepper motor illustrates that the angular error of stepper motor is high-frequency periodic fluctuation[9], as shown in Figure 7 and Eq.(6).

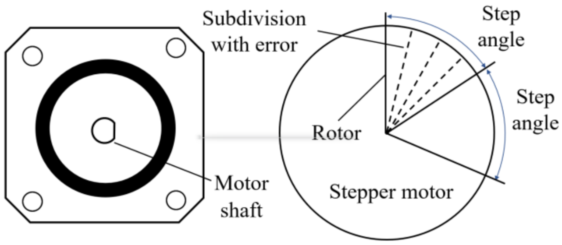


Figure 7: The schematic for the subdivision of stepper motor and the rotation error following a step cycle.

$$\delta_3(y)=\delta_3(y+\varphi\cdot\frac{P_h}{2\pi}) \tag{6}$$

where φ is the step angle of the motor; P_h is the lead of the ball screw.

2.2.3. Positioning errors prediction in the rapid method

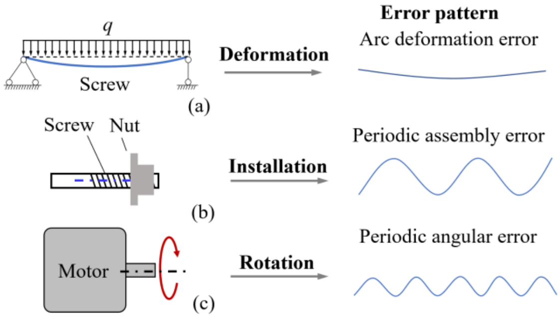
The positioning error curve presents a random pattern which results from the mentioned errors conjointly. In practice, any random curve can be described through a series of Fourier. Similarly, the positioning error curve can be predicted in the same way as Eq. (7). Some scholars chose the first harmonic of Fourier to represent the transmission error curves. Guo[10] ignores the higher harmonics to describe the periodic component of positioning error curve in BMS, shown as Eq. (8).

$$f(y)=\frac{4}{\pi}\sum_{n=1,3,5...}^{\infty}\sin(\frac{2n\pi y}{\lambda}) \tag{7}$$

ISSN 2616-5767 Vol.6, Issue 2: 29-39, DOI: 10.25236/AJETS.2023.060205

$$f(y)=\frac{4}{\pi}\sin(\frac{2n\pi y}{\lambda}) \tag{8}$$

In view of all types of errors in BMS, a systematic method for predicting positioning errors with much higher resolution and precision is needed. Specifically, the deformation error presenting arc pattern is attributed to the gravity, as shown in Fig. 8(a). The periodic fluctuations with a wavelength equal to the lead of screw-nut pair is deduced by the installation according to section 2.3, as shown in Fig. 8(b). The periodic fluctuations with a wavelength equal to the step angle of motor is displayed as shown in Fig. 8(c).



(a) Positioning error attributed to deformation forced by gravity (b) Assembly error of screw-nut pair (c) Rotational deviation of motor.

Figure 8: Schematic for three major categories of error components of positioning error in BMS

Given the high coupling of the assembly and deformation error of screw-nut pair, the transmission error including the two error components is defined in the new method to predict positioning error. The transmission error of the ball screw refers to the deviation between the theoretical and actual displacements of the nut, which can be obtained through dynamic measurement rapidly and accurately.

The mechanism of positioning error induced by the transmission and power error is shown in Fig. 9. Thus, a systematic and rapid method to establish positioning error model by obtaining the transmission error of screw-nut and rotational error of motor is proposed, based on the relationship between the two error components and positioning errors of BMS.

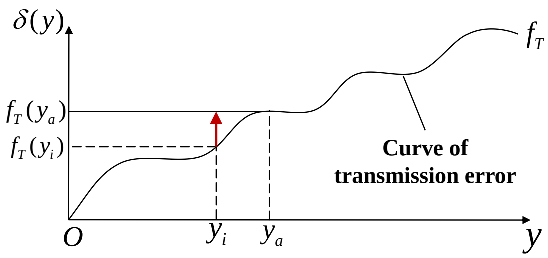


Figure 9: Schematic for positioning error influenced by transmission error of ball-screw and rotational deviation of motor conjointly.

(1) Measure errors of machine elements. Since the motor error is generally minuscule, a high-precision encoder is generally utilized to measure the deviation of rotational angular. A 16-bits optical encoder with resolution of 0.005 °is applied in this paper. In addition, an adjustable centering fixture and laser interferometer with resolution of 10nm are adopted cooperatively to determine the transmission errors of screw-nut pair. The measurement should be started at the initial end of screw.

(2) Calculate the rotational deviation of motor and the equivalent actual displacement y_a through Eq. (9) and transmission deviation of ball-screw. Fit the rotational deviation curves of motor marked as $\varepsilon(y)$ and transmission deviation function of ball-screw marked as $f_T(y)$ with precise characteristic functions according to Fig.9.

$$y_a = y_i + \varepsilon \cdot 2\pi / p \tag{9}$$

$$\delta(y_i) = f_T(y_a) \tag{10}$$

Where y_i is the ideal position; p is the pitch of the ball screw; $\varepsilon(y)$ is the rotational deviation

ISSN 2616-5767 Vol.6, Issue 2: 29-39, DOI: 10.25236/AJETS.2023.060205

function of motor; f_T is the actual displacement among axial direction.

(3) Calculate and predict positioning error. Considering the real positioning error curve contains various characteristics, Eq. (10) is applied to describe the positioning error $\delta(y)$ referring to the transmission deviation and rotational deviation of motor shown in Figure.

(4) Validation of prediction. In order to verify the practicability and precision of the new approach, the positioning errors should be measured by a laser interferometer. Assemble the measured motor and screw-nut rationally. Measure the positioning error from the start end of the BMS to guarantee the transmission part of the BMS is the same as its screw-nut. All steps in the new method are illustrated in Fig. 10.

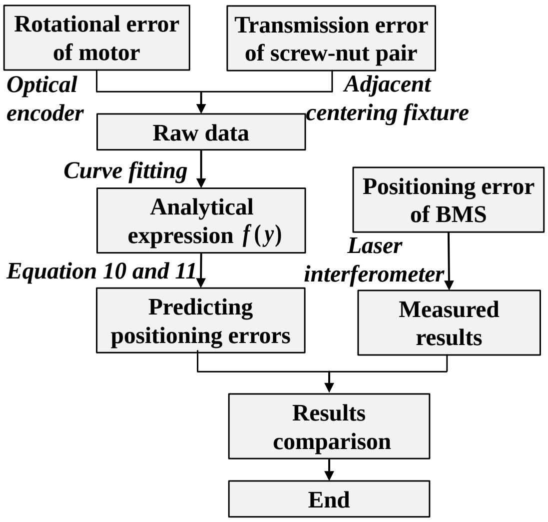


Figure 10: The specific process of the positioning error prediction in the new method.

Compared with the conventional method, the new method predicts the positioning error with a superposed function including crucial geometric errors instead of simple trigonometric function. Furthermore, the positioning error data at every position can be calculated and predicted instead of point-by-point measurement. This approach is more accurate and efficient than conventional method, which can better predict the positioning error at full stroke. Moreover, positioning error can be measured by precise devices, the measured results can reveal the characteristics of error sources. Based on this approach, through obtaining the rotation angles of motor and transmission accuracy of ball-screw nut pair, the positioning error can be calculated during design stage. The new approach is beneficial for improving accuracy through error compensation and understanding how the positioning errors be influenced by geometric errors of machine elements in BMS.

3. Case study

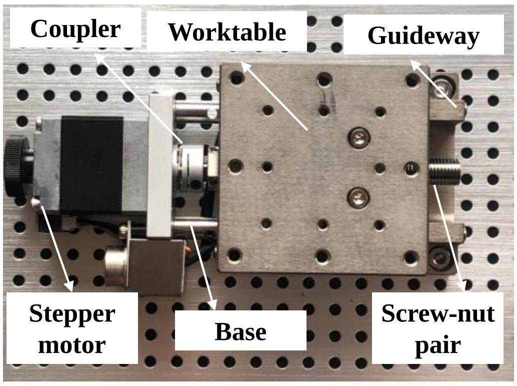


Figure 11: Image of the studied micro BMS.

Fig. 11 shows a micro BMS with linear guideways, ball-screw nut pair, stepper motor, worktable and other parts widely applied in engineering field. Based on section 2, the positioning error is influenced by

ISSN 2616-5767 Vol.6, Issue 2: 29-39, DOI: 10.25236/AJETS.2023.060205

transmission error of ball-screw and the rotational deviation of stepper motor. To validate the proposed predicting positioning error by determining the errors of ball-screw and stepper motor, a series of experiments and data processing are conducted in this section.

3.1. Measurement of transmission error and curve fitting

In the industrial field, transmission error is usually obtained by dynamic measuring methods. The transmission error is the translational deviation among axial direction thus the displacements are measured by high-precision sensors. And the rotational angle should be measured simultaneously to plot the transmission error curve depending on position.

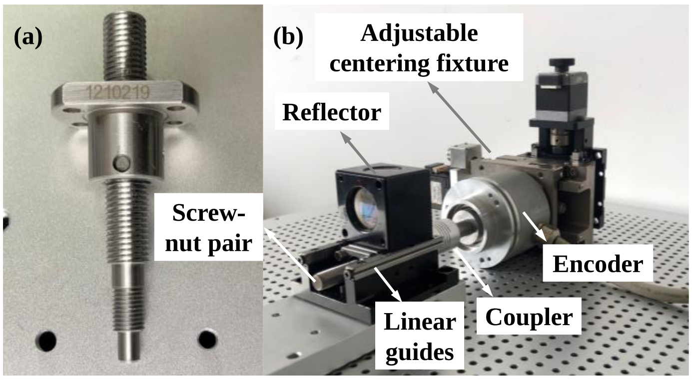


Figure 12: (a) The screw-nut pair (b) The installation for measuring the transmission error with an adjustable centering fixture.

The C3 screw-nut pair with 1mm pitch and 5mm stroke is adopted in the micro BMS as shown in Fig.12 (a). The accuracy and the leading efficiency of the screw is very high, which is widely applied in various motion systems. In the new approach, an adjustable centering fixture is adopted to clamp the screw-nut pair shown in Fig.12 (b). The rotational angle of the screw is measured by the optical encoder and the displacement of the nut is measured through a laser interferometer. Since the pitch of the screw is 1mm, the transmission error is measured with steps of 0.1mm and the total number of measured points is 51.

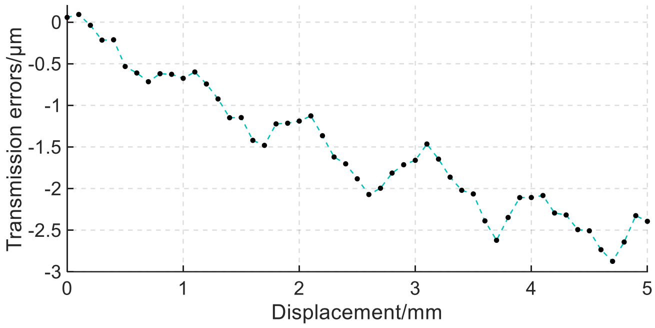
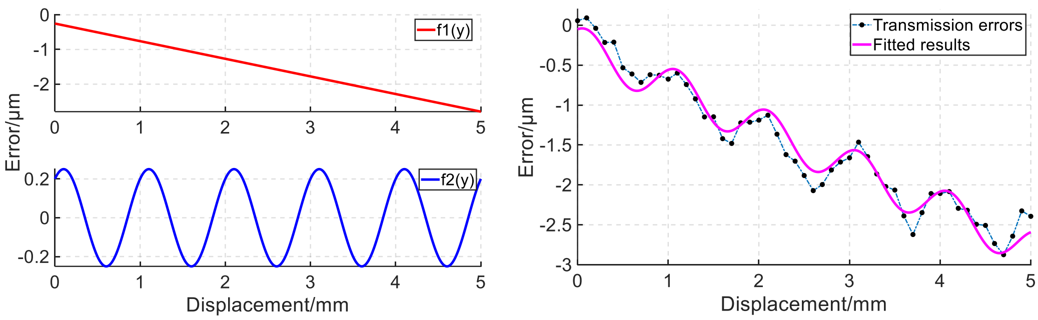


Figure 13: The measured results of transmission of screw-nut with an interval of 0.1mm at the displacement of 5mm.

Fig. 13 plots the measured values of the transmission error of screw-nut pair with an interval of 0.1μm at the total displacement of 5mm, whose number of measured points is 51. The two characteristics of transmission errors can be analyzed from Fig. 13. Since the deformation error of the screw is a low-frequency function and the screw-nut in this study is small-size, the deformation error is fitted with a linear function in approximate treatment for efficiency. The linear trend (function $f_1(y)$ in Fig.14 (a)) reflects the deformation error of screw and the other is a fluctuation (function $f_2(y)$ in Fig.14 (b)) reflecting the assembly of the screw-nut pair. The fitted results of the measured transmission errors with the superposition of the linear and trigonometric functions are shown in Fig.14 (b).

$$f(y) = f_1(y) + f_2(y),\ f_1(y) = a_1 y + b_1,\ f_2(y) = a_2\cos(\omega_2 y) + b_2\sin(\omega_2 y)$$
$$a_1 = -0.5084, b_1 = -0.2515, a_2 = 0.2005, b_2 = 0.1504, \omega_2 = 6.2832$$

ISSN 2616-5767 Vol.6, Issue 2: 29-39, DOI: 10.25236/AJETS.2023.060205



(a) *(b)*

Figure 14: (a) Error pattern of transmission errors of screw-nut pair (b) Measured results of transmission errors and fitted curve.

3.2. Measurement of motor error and curve fitting

In Fig.15, the stepper motor with step angle of 0.72° is adopted in the micro BMS. Since the crucial influence on positioning error of motor is the rotational deviation, the rotational error should be considered in the prediction of positioning error of the BMS. The 16-bits high-precision optical encoder is utilized to determine the rotational deviation of motor and the installation for the measurement is shown in Fig.15. The motor shaft is jointed with the encoder shaft through a coupler, the coaxiality of the three parts is guaranteed after adjustment to eliminate the measuring error.

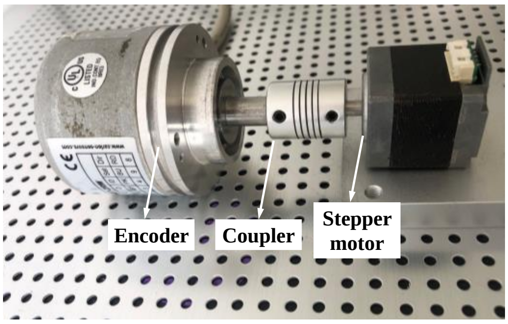


Figure 15: The installation for measuring the rotational deviation of motor with 16-bits high-precision optical encoder.

$$f_3(y) = a_3 \cos(\omega_3 y) + b_3 \sin(\omega_3 y) + c_3$$
$$a_3 = 0.09491, b_3 = -0.1333, c_3 = -0.05796, \omega_3 = 0.0628$$

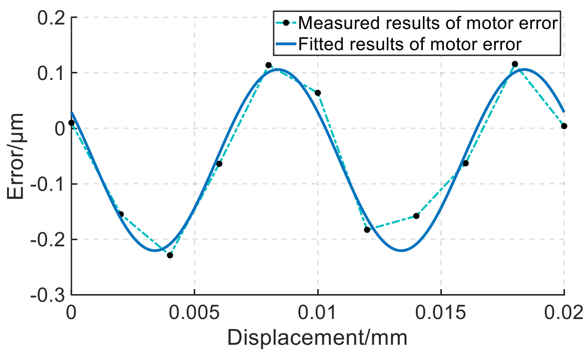


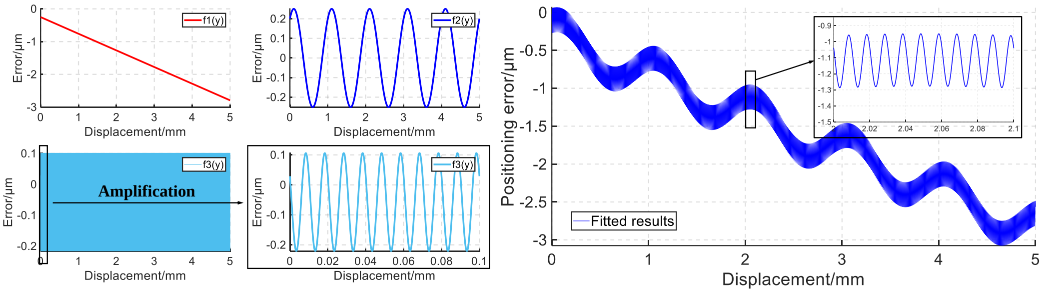
Figure 16: Measured results of motor errors and fitted curve.

The step of motor is set as 0.72° , equivalent to 0.002mm of displacement, and the number of the measured points is 11. It can be clearly seen from Fig.16 that the error pattern of the motor error is the first term of Fourier series and the fitted curve is shown as the solid spline. The characteristic of the motor error matches the machining accuracy of the rotor and the stator of the stepper motor.

ISSN 2616-5767 Vol.6, Issue 2: 29-39, DOI: 10.25236/AJETS.2023.060205

3.3. Calculation and measurement of positioning error

Based on the fitting expressions of transmission errors of screw-nut pair (function $f_1(y)$ and $f_2(y)$) and rotational deviation of stepper motor marked as function $f_3(y)$ shown in Fig.17 (a), the fitted superposition of the three error components at the same positions and displacements are calculated to predict the positioning errors of the BMS shown in Fig.17 (b) .



(a) *(b)*

Figure 17: (a) The error pattern of predicting positioning errors in the BMS (b) The calculated predicting positioning errors in the BMS.

To validate the availability of the predicting function of the new approach, the positioning errors in the BMS should be measured simultaneously to compare with the calculated values of positioning errors in Figure(b). Therefore, a laser interferometer with 10nm resolution is applied to determine the positioning errors, shown in Fig.18. The step of the positioning error measurement is 0.002mm and the full stroke is 5mm, thus 2501 validating points are obtained. The comparison between calculated and measured results is shown in Fig.19.

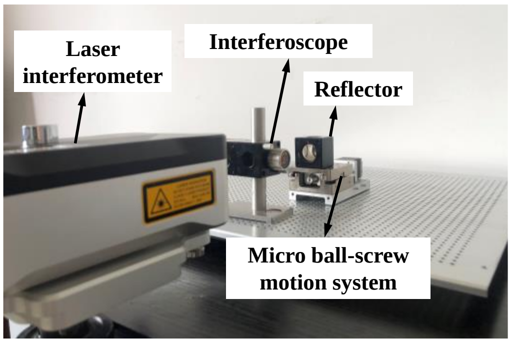


Figure 18: The installation for positioning error measurement of the micro BMS.

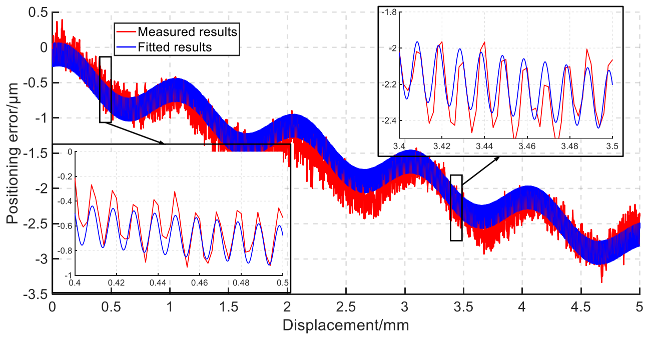


Figure 19: Comparison between the measured and predicted positioning errors and their partial enlarged details.

Fig.19 suggests the calculated predicting spline agrees well with the measured results of positioning errors of the BMS with R-square equal to 0.9657 approaching 1 highly. The general trend of the positioning error values varies from 0μm to −3μm, which is highly consistent with the predicting spline. Additionally, the wavelength and amplitude of assembly and motor error are both small, consequently the general trend of predicting spline agrees well with the measured results. The positioning error is determined partly by deformation error of screw, which can guide the design of the BMS through increasing the stiffness of the screw-nut for smaller deformation and higher accuracy.

Furthermore, there are fluctuations with 1mm wavelength in the measured results of positioning error, which is coincident with the predicting function basically. The assembly errors of screw-nut pair are transferred to axial deviations leading to periodic fluctuation, and the positioning error of the micro BMS is influenced by deformation error and periodic fluctuation conjointly. Since the theoretical predicting function is simulated in the ideal situation (some assumptions stated in section 2), there are some variations existing in peaks and valleys in the fluctuations with 1mm wavelength. It is because the assembly error is attributed to not only the installation of screw-nut but several other elements such as guides and motor. The precision of the general trend validated the predicting method preliminarily, which can reveal the characteristic of assembly and geometric error of screw.

In partial enlarged details in Fig.19, the wave range of measured positioning error values is about 0.5μm, which is similar to the predicting spline. Though the wavelength and amplitude of motor error are smaller than those of transmission errors, the high-frequency oscillations of motor affect the positioning error of BMS partly, which are non-negligible especially in micro movements. Since the physical dimension of the rotor and stator in stepper motor is limited by machining accuracy, the rotational deviations have a random discrepancy slightly. Therefore, some differences exist in peaks and valleys in the fluctuations with micro wavelength. Based on the traditional linear and trigonometric characterization of positioning error, motor errors are involved to describe the positioning error curve from macro to micro movements more accurately and systematically. This provides a method to establish a positioning error model for compensation with high resolution and precision in BMSs.

4. Conclusions

This paper proposed a rapid method for predicting positioning errors for error compensation with high accuracy and resolution. Compared with conventional method using a trigonometric to characterize the positioning error, the new approach adopts precise devices to obtain the transmission errors of screw-nut pair and the rotational deviation of stepper motor and measured results cans be fitted precisely, based on which the positioning error of BMS can be predicted with corresponding expressions.

(1) The analysis demonstrates crucial error components of BMS are the low- frequency deformation error of the screw, the assembly error of the screw-nut pair with the wavelength equal to the lead, and the high-frequency rotational error of power with the wavelength equal to the step angle.

(2) Considering the periodic characteristics of the three crucial error components, thousands of positioning error data at each position can be predicted through measuring a small quantity of points. The goodness of fit is 0.9657 and it validates the new method greatly improves the measurement efficiency with high accuracy. The rapid approach can provide data basis for error compensation and explanation of error mechanism for configuration optimization in BMSs.

References

[1] S.-M. Wang, J.-J. Lin, Z.-Z. Ye, S. Tsooj, C.-C. Wang (2014) A micro cutter auto-alignment system with on-machine positioning error measurement and compensation methods. Int J Precis Eng Man 15: 177-182.

[2] H. Shen, J. Fu, Y. He, X. Yao (2012) On-line Asynchronous Compensation Methods for static/quasi-static error implemented on CNC machine tools. Int J Mach Tool Manu 60: 14-26.

[3] N.A. Barakat, A.D. Spence, M.A. Elbestawi (2000) Adaptive compensation of quasi-static errors for an intrinsic machine. Int J Mach Tool Manu 40: 2267-2291.

[4] K. Wang, C.-G. Zhou, Y. Ou, H.-T. Feng (2021) Investigation of the transmission accuracy of ball screw considering errors and preloading level. Int J Adv Manuf Tech 118: 3917-3932.

[5] D. Kono, A. Matsubara, I. Yamaji, T. Fujita (2008) High-precision machining by measurement and compensation of motion error. Int J Mach Tool Manu 48: 1103-1110.

[6] H. Tang, J.-a. Duan, Q. Zhao (2017) A systematic approach on analyzing the relationship between straightness & angular errors and guideway surface in precise linear stage. Int J Mach Tool Manu 120: 12-19.

[7] C. Yue, X. Liu, Y. Ding, S.Y. Liang (2018) Off-line error compensation in corner milling process. Proc Inst Mech Eng, Part B 232: 1172-1181.

[8] R. Liang, Z. Wang, W. Chen, W. Ye (2021) Accuracy improvement for RLLLR five-axis machine tools: A posture and position compensation method for geometric errors. J Mater Process Technol 71: 724-

ISSN 2616-5767 Vol.6, Issue 2: 29-39, DOI: 10.25236/AJETS.2023.060205

733.
[9] D. Zhang, J. Wang, L. Qian, J. Yi (2019) Stepper motor open-loop control system modeling and control strategy optimization. Arch Electr Eng 63-75.
[10] S. Guo, J. Yang, G. Qiao, X. Mei (2022) Assembly deviation modelling to predict and trace the geometric accuracy of the precision motion system of a CNC machine tool. Mech Mach Theory 169: 104687.