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Application of Weibull Analysis in the Optimization of Civil Aviation Aircraft Engineering Reliability Management

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Abstract: *Reliability management is one of the basic work to protect the safety of civil aviation. At present, more and more domestic passenger planes are invested, but due to the low availability of data and limited number of aircraft, there are few studies on reliability management of domestic civil aviation aircraft. In this paper, the three-parameter Weibull distribution is used to analyze the small sample problem, and Take reliability management of pitot tube of domestic ARJ21 fleet as the research object, and the application of Weibull in the reliability management optimization of civil aviation aircraft engineering is explored.*

Keywords: *Reliability management; Weibull analysis; Civil aviation*

1. Introduction

The operation reliability of civil aircraft equipment plays an important role in ensuring flight safety, reducing maintenance costs and improving operation efficiency. Reliability management, also known as life/time to failure analysis, is a specific application of statistics to study failures and their probabilities [1]. At present, there are two important branches of reliability evaluation methods worth studying: I. fuzzy reliability; II. Imprecise reliability. Many classical reliability methods and models not only assume that the probability and probability distribution are completely determined or completely known, but also assume that the components of the equipment are independent of each other, that is, all variables describing the reliability behavior are statistically independent, or at least the correlations between the components are precisely known. But in practice these conditions are too difficult to fully meet. In order to quantitatively evaluate the reliability of aircraft system, it is necessary to establish the distribution model of aircraft system failure time.

Although the utilization of domestic airliners in civil aviation is increasing, domestic research lacks a stable model for conducting reliability research. Firstly, there is insufficient collection of reliability data. Foreign manufacturers consider aircraft design and global fleet usage data as highly confidential trade secrets and do not disclose them to the public.Secondly, there are significant challenges in collecting usage and maintenance data from domestic airlines, which can only be treated as a small sample analysis problem. The three-parameter Weibull distribution is widely used in reliability engineering due to its strong applicability and high precision in assessing small sample reliability. It plays a crucial role in the management of civil aircraft's reliability [2].

Advanced Regional Jet for 21st Century 1(ARJ21) is a Chinese aircraft developed in accordance with international standards and possessing independent intellectual property rights [3]. It primarily serves the purpose of meeting radial route demands from central cities to surrounding small and medium-sized cities, and was officially delivered in November 2015. The successful development and production of the ARJ21 bridges the gap within China's civil aviation industry, establishing a strong foundation for subsequent serial development and large-scale production of domestic civil aircraft. This paper takes the reliability management of pitot tubes within the domestic ARJ21 fleet as an example, applying the CPOGSA algorithm process to solve the Weibull model. Through this approach, we aim to explore how Weibull can be utilized in optimizing reliability management for civil aircraft engineering while providing valuable insights for future reference.

2. ARJ21 fleet pitot tubes right deletion data analysis

The initial fault data presented in this study were obtained from 56 pitot tubes installed in the ARJ21

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fleet of a domestic airline. These data have been organized and are listed in Table 1 based on the chronological order of their usage until the end of data collection. By utilizing reliability data specific to the pitot tube used in the ARJ21 fleet, we have established a life rule for these attachments. The "Status" column within Table 1 indicates the current condition of each attachment: F denotes a fault, indicating that the attachment failed during collection and sampling, resulting in complete data; S represents cessation, signifying that the attachment was still operational during collection and sampling, leading to excluded or deleted data.

Table 1: ARJ21 fleet pitot tube failure time data

No.	Status	Usage time/h
1	F	6921
2	S	6817
3	S	7298
4	S	7539
5	S	7750
6	F	8123
7	F	8151
…	…	...
51	F	21947
52	F	21468
53	S	23985
54	F	24341
55	F	24393
56	F	24652

Among the 56 pieces of data, 29 are complete while the remaining 27 indicate that the pitot tube remained non-failed at the end of data collection. The exact time of failure cannot be determined due to observation limitations and data that may fail after observation time is referred to as right-censored data. In some cases, full data may not be available for reliability phase assessment purposes or because equipment has not yet failed. Deletion mechanisms are based on fixed years, failures or arbitrary points in time. Parameter estimation between complete and censored data only differs in median value calculation where component faults need to be adjusted for ranking purposes. When analyzing probability distribution, censored and complete data should not be considered together but rather their impact on each other must also be taken into account. Therefore, it is necessary to process right-censored data and show its impact on complete dataset's distribution form through median rank adjustment in order to obtain accurate distribution type [4]. Follow these steps for rank adjustment.

(1) Arrange all data in ascending chronological order and label the 29 complete data separately.

(2) Adjust the rank of the complete data using an incremental formula.

$$\frac{\mathbf{r}^{*}\mathbf{j}+n+\mathbf{1}}{\mathbf{r}+\mathbf{1}}$$

Where j is the adjusted rank of the previous data, r is reverse rank is the data in total samples, and n is the total number of data samples.

(3) Benard approximation algorithm was used to estimate the median rank

$$\hat{F}(n_i) = \frac{i-0.3}{n+0.4} \tag{1}$$

Where, i is the data sequence number, n is the total number of data samples, and $\hat{F}(n_i)$ is the median rank of the i th data. The 29 complete data after adjustment are shown in Table 2

Table 2: Complete data of adjusted failure time

Status	Time/h	Adjusted No.	Median rank
A	6921	1.0000	0.0111
A	8123	2.0544	0.0280
A	8151	3.0877	0.0449
A	21947	5.1512	0.0787
A	21468	6.5179	0.0969
A	23985	7.4436	0.1150
...	...	...	...
A	24341	49.2940	0.7754
A	24393	52.5571	0.8288
A	24652	55.8926	0.8821
A	24741	59.3652	0.9355

(4) Take failure time as the X-axis and median rank as the Y-axis to draw a graph of the cumulative

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ineffectiveness function $\hat{F}(n_i)$, as shown in Figure 1.

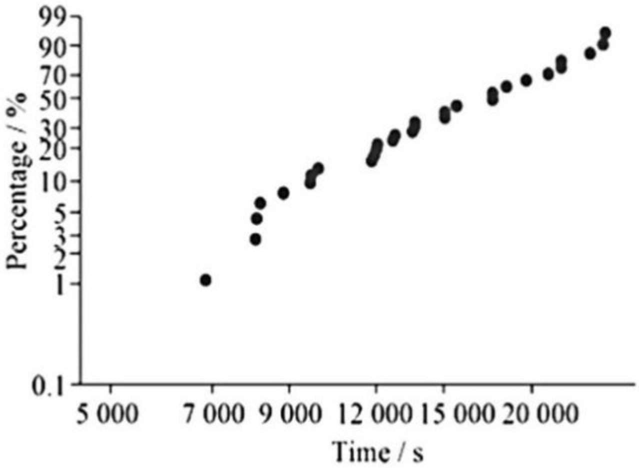

Figure 1: Analysis of pitot tube failure data

The obtained data was analyzed using Minitab for evaluating the fitting of data distributions. Specifically, Minitab was employed to compare the goodness-of-fit of Weibull, three-parameter Weibull, minimum extreme value, and normal distribution models. Two parameters, namely A_D value and P-value, were utilized in Minitab to assess the degree of fit. The A_D value represents the weighted sum of squared variances between the observed points and the best linear fit according to the formula.

$$\boldsymbol{A}_{\mathrm{D}} = -\sum_{i=1}^{n}\left(\frac{2i-1}{n}\right)\ln[F_0(t_i)R_0(t_{n+1-i})] - n \tag{2}$$

By definition, a smaller A_D value indicates a higher degree of data fitting. P-value represents the probability that the statistical summary (e.g., difference between means of two sample groups) is equal to or greater than the observed data in a probability model. A P-value below the chosen significance level (0.05) signifies an invalid hypothesis, while a larger value suggests stronger data fitting. Evaluation results demonstrate that the three-parameter Weibull distribution exhibits minimal A_D and maximal P-value, indicating better consistency with the data.

Table 3: Fitting evaluation of different distribution models

Distribution	A_D	P
Exponential	5.580	< 0.004
2 parameter exponential	1.132	0.049
Minimum extremum	1.158	< 0.009
Normal	0.541	0.092
Logistic	0.578	0.084
Weibull	0.526	0.185
Gamma	0.361	> 0.250
Maximum extremum	0.336	> 0.250
Logistic Logarithmic Logistic	0.331	> 0.250
3 Logistic 3 parameter logarithmic Logistic	0.327	*
Gamma 3 parameter Gamma	0.321	*
3 parameter lognormal	0.301	*
Box-Cox transformation	0.285	0.602
Lognormal	0.285	0.602
Weibull 3 parameters	0.261	> 0.500

3. Solution of three-parameter Weibull distribution model

The cumulative distribution function F and the probability density function f for the three-parameter Weibull distribution are as follows

$$\begin{aligned} F(t_i;\beta,\theta,\eta) &= 1 - \mathrm{e}^{-\left(\frac{t_i-\eta}{\theta}\right)^{\beta}} \\ f(t_i;\beta,\theta,\eta) &= \frac{\beta}{\theta}\left(\frac{t_i-\eta}{\theta}\right)^{\beta-1}\mathrm{e}^{-\left(\frac{t_i-\eta}{\theta}\right)^{\beta}} \end{aligned} \tag{3}$$

Where, β is the shape parameter; θ is the scale parameter; δ is the positional parameter. Where θ > 0, β > 0, 0 < η < t_i < ∞.

Since the GSA algorithm has good traversal ability and can fully guarantee the diversity of the population, combining the GSA algorithm with the CPSO algorithm, the speed and position update formula of the algorithm can be expressed as

$$\begin{cases} V_i^d(t+1) = K \times \begin{pmatrix} \omega(t) \times V_i^d(t) + a_i^d(t) + \\ c_1 r_1\left(a_i^d(t) - V_i^d(t)\right) + \\ c_2 r_1\left(g_{\text{best}} - V_i^d(t)\right) \end{pmatrix} \\ X_i^d(t+1) = X_i^d(t) + V_i^d(t+1) \end{cases} \tag{4}$$

The estimation of Weibull distribution parameters using CPSOGSA algorithm can be reduced to the following optimization problems:

$$\begin{cases} minf(a,b) = min\sum_{i=1}^{n} \left[F(t_i,\eta,\beta) - F_i\right]^2 \\ s,t.\eta > 0, \beta > 0 \end{cases} \tag{5}$$

Where t_i is the failure time, η is the scale parameter, and β is the shape parameter.

$$F(t_i,\eta,\beta) = 1 - \exp\left[-\left(\frac{t}{\eta}\right)^{\beta}\right]$$
$$F_i = \frac{i-0.3}{n+0.4} \tag{6}$$

Its fitness function is

$$f(i) = \sqrt{\frac{1}{n}\sum_{i=1}^{n} \left(\mathrm{F}(t_i,\eta,\beta) - F_i\right)^2} \tag{7}$$

Normalized Root Mean Square Error (NRMSE) of the measured data was selected as the evaluation criterion of the results.

$$\text{NRMSE} = \sqrt{\frac{\sum_{i=1}^{n}(y_i - f_i)^2}{\sum_{i=1}^{n} y_i^2}} \tag{8}$$

Wherey_i is the estimated failure rate obtained by the estimated parameters, and $f_i$is the actual failure rate obtained by the average rank method.

The CPSOGSA algorithm is used to solve the reliability parameter estimation problem studied in this paper. The main steps are as follows:

Step 1 involves initializing the population, followed by Step 2 which calculates the fitness value of each particle using equation (7). In Step 3, the velocity and position of particles are updated based on equation (4). Subsequently, in Step 4, both individual optimal and global optimal values of the population are updated along with the optimal fitness value. If the termination conditions for iteration are met, the process is concluded and outputs the global optimal solution. Otherwise, continue iterative calculations from Step 2 to Step 4 until meeting termination condition. The CPSOGSA algorithm yields shape parameter β = 0.8113 and scale parameter η =1452.6183 for Weibull model.

A genetic algorithm was implemented using Matlab programming to solve the problem of finding the optimal solution for a two-parameter equation within a specified range. The number of individuals (M) was set to 100, with constraints on β ranging from 2 to 2.6 and δ ranging from 5000 to 5500. The algorithm iterated for 200 times, resulting in an optimal solution of β = 2.17, δ = 5416.20, θ = 13,137.42, and ln Lmax= -321.43. Table 4 presents a comparison between the parameter values obtained by our method and the least square method [5], using A_D value as the evaluation criterion. According to formula (2), a smaller A_D value indicates better data fitting accuracy. Our algorithm achieved a lower A_D value compared to the least square method, demonstrating its superior precision.

Table 4: Parameter values of final solution results

Algorithm	β	δ	θ	A_D
Genetic	2.17	5416.20	13137.42	1.82
Least squares	2.08	5206.63	13716.41	2.01

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4. Optimization analysis of fleet pitot tube management

Based on the aforementioned analysis results, it can be concluded that the life distribution of the airpitot tube in this fleet conforms to a three-parameter Weibull distribution with right omission. The failure rate function curve and reliability function curve are plotted using Minitab software, as depicted in FIG.2 and FIG. 3. Considering $\beta = 2.17 \in (1, 4)$, it is observed that the failure rate increases over time; thus, enhancing the overall fleet reliability can be achieved by calculating service time based on a specific reliability level to formulate an appropriate maintenance plan.

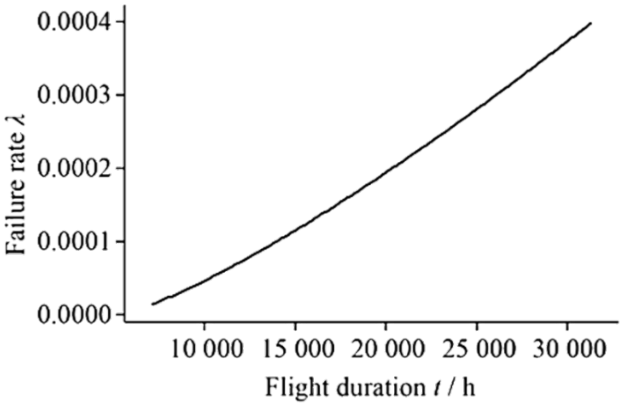

Figure 2: Failure rate function curve

The ARJ21 aircraft of an airline company is considered to be an aging aircraft, with a service life of approximately 8 years and a daily utilization rate of the fleet at around 6 hours per day, resulting in a total operational time close to 17,000 hours. By conducting calculations, it can be inferred that when t = 17,000 hours, the failure probability amounts to 0.584618. The reliability R(17000) is calculated as 0.3915, indicating that at t = 17000 hours, nearly six out of every ten pitot tubes are expected to fail; clearly falling short of the required reliability standards. (1) After conducting calculations, it can be inferred that the reliability R (8000) = 0.8615 at t = 7,000 h and the reliability R (10000) = 0.9128 at t = 10,000 h. At t = 12,000 h, the reliability R (12000) is determined to be 0.8084. Assuming a required reliability of R = 0.8000, the calculation yields an approximate value of t ≈ 12,056 h; similarly, for a required reliability of R = 0.9000, the calculation results in an approximate value of t ≈10,118 h. Furthermore, when aiming for a desired reliability level of R=0.9500, a calculated value of approximately t≈8,882h can be obtained. It is evident that reducing the lifespan effectively enhances the fleet pitot tube's reliability performance. However， considering that this fleet has been operational for an extended period， the condition with daily utilization rate around six hours would allow utilizing a reliability level of R=0.800， which corresponds to approximately 6.65 years. Taking into account cost and climate factors, the following scheme is proposed:

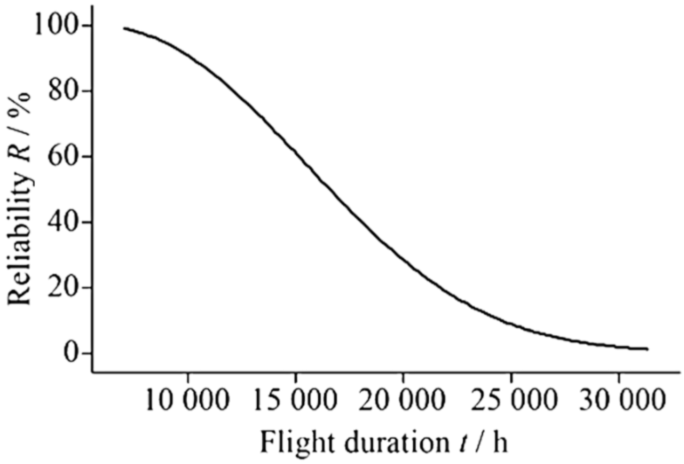

Figure 3: Reliability function curve

(1) For the pitot tube with unclear maintenance records or a short operational lifespan, the maintenance management will adhere to a time limit of 10,118 flight hours.

(2) In the case of an airpitot tube with clear maintenance records or extensive operational usage, it should be replaced in batches before the upcoming rainy season. After replacement, a time limit of 12,056 flight hours shall be implemented for control purposes.

(3) From the moment the aircraft lands until the post-flight inspection is completed and the aircraft is released, a period of approximately 2 to 6 hours elapses. During this time, the pitot tube remains exposed to external elements. To prevent contamination or blockage by foreign objects such as mosquitoes entering the airspeed tube, it is necessary to employ specialized protective covers on the pitot tube during post-flight maintenance. These covers should be removed during pre-flight inspection. Additionally, it is essential for the airline's aviation materials department to maintain an adequate supply of spare parts for timely replacement in case any faults are detected, thereby minimizing maintenance costs.

5. Conclusion

The Weibull distribution is employed in this study to analyze the operational reliability of the aircraft system. This method can also be utilized for evaluating the reliability of aircraft equipmentsand engines. The evaluation results can serve as a direct basis for reliability monitoring, enabling dynamic management and effective control of both the aircraft maintenance program and the airline's maintenance program. This ensures that they remain accurate, comprehensive, efficient, practical, and provides valuable insights for scientific maintenance practices and new aircraft development.

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