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Research on Teaching Mathematical Theorem Based on DELC Deep Learning Route-Taking "Pythagorean Theorem" as an Example

Yuxin Liu[1,a], Jianhua Wu[1,*]

[1]*School of Mathematical Sciences, Jinan University, Jinan, China*
[a]*liuyuxin_lucky@163.com*
[*]*Corresponding author: sms_wujh@ujn.edu.cn*

Abstract: *Mathematical theorems are an important sufficient condition for effective problem-solving. This is because it not only helps learners build a systematic knowledge network but also significantly improves and develops logical thinking skills. The Pythagorean theorem is fundamental in junior high school mathematics, playing a crucial role in solving geometric problems. The new standard of mathematics advocates "deep learning", and the study of the deep learning route and conceptual teaching, more, but less, combined with theorem teaching. Research on teaching math theorems via DELC deep learning routes is explored, and seven strategies are proposed: alignment with standards and curriculum, pre-assessment, fostering a positive learning culture, preparing and activating prior knowledge, acquiring new knowledge, processing knowledge in depth, and evaluating student learning. These strategies are derived from an in-depth exploration of deep learning routes, with corresponding teaching approaches developed. SOLO classification theory is utilized to assess students' knowledge application and predict their learning of math theorems (e.g., the collinear theorem), while criteria for evaluating deep learning are also established. Finally, based on the above deep learning route, a specific teaching design for the collinear theorem is proposed, which is expected to provide corresponding support for the theorem teaching of first-line junior high school teachers.*

Keywords: *Deep Learning; Deep Learning Route; Theorem Teaching; Collinear Theorem*

1. Background of the study

(1) Teaching Math Theorems and the Pythagorean Theorem

The core mission of teachers in teaching mathematical theorems is to lead students to recognize the background of the theorems, to clarify the conditions and conclusions of the theorems, to master the skills of proving them, to clarify their application contexts, and to be able to apply them practically to solve problems, and to understand the interrelationships between the theorems and to construct a systematic framework of theorems; Mathematical theorems are very important, and, first of all, they are the cornerstones of the system of mathematical knowledge. Similar to numerous theorems in Euclidean geometry, individuals leverage these theorems to reinforce geometric knowledge—such as understanding figure properties, positional relationships, and theoretical foundations—enabling logical reasoning to tackle more intricate geometric problems.

(2) Implementing the requirements of curriculum standards and nurturing the needs of talents

In the "Compulsory Teaching Mathematics Curriculum Standards (2022 Edition)", students need to experience the process of proving geometric propositions, and understand the essence of the idea of logical reasoning in mathematics; Simultaneously, the teaching of geometric graphics and their properties is conducted through real-life or mathematical contexts to guide students in perceiving mathematical knowledge[1]. This coincides with the idea of deep learning to learn, students need to build their knowledge system on their own.

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2. An investigation of teaching strategies for DELC deep learning routes

2.1 Teaching routes

Designing Standards and Curriculum - Pre-Assessment - Creating a Positive Learning Culture - Preparing and Activating Prior Knowledge - Acquiring New Knowledge - Processing Knowledge in Depth - Evaluating Student Learning[2];

2.2 Research ideas

Figure 1 below shows the research idea of this paper, which is divided into theoretical research and practical application.

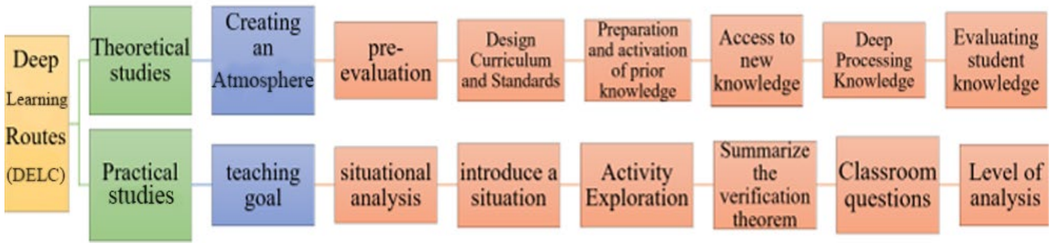

Figure 1 Instructional Design Ideas

2.3 Process Design

Process 1: Design Standards and Curriculum

Teachers always start with the curriculum design for each lesson according to the Compulsory Education Curriculum Standards, and design the lesson through the instructional objectives. In the classroom, teachers must always teach with the goal in mind that they want their students to achieve and that students will gain a great deal from the class. Teaching objectives typically encompass three aspects: knowledge acquisition and skill enhancement, learning process and method application, and the cultivation of affective attitudes and values, which are interrelated and work together to promote students' development.

Process 2: Pre-assessment

Deep learning builds on prior foundational knowledge, where old knowledge is linked to new knowledge, and learning should be meaningful[2]. Therefore, teachers should conduct a pre-assessment of students between lessons, which is also known as the analysis of the learning situation, to understand the background knowledge of students, the ability level of students and the students' attitude and interest in learning knowledge, through the understanding of the above, we can understand that there are significant individual differences between students, and through tailored teaching can help students to better grasp the new knowledge. Because we are going to learn the Pythagorean Theorem, we can let the students do a test paper before the lesson, including whether they understand the origin of the Pythagorean Theorem and the knowledge related to the Pythagorean Theorem that they have already learned, such as the nature of isosceles triangles, the nature of right triangles, and the relationship between three sides of triangles.

Process 3: Creating a positive learning culture (lively, supportive, and belonging)

Students need to maintain a moderate emotional balance to keep learning going smoothly. Negative emotions, such as boredom and apathy, are not conducive to student learning. On the contrary, the ideal state of learning is energized, relaxed, and keenly inquisitive[2]. Therefore, it is necessary to create a relaxed and pleasant classroom atmosphere, which makes it possible for students to let their brains focus on learning new knowledge in the classroom. For example, teachers should keep smiling, be enthusiastic, and communicate with students in a gentle and friendly manner. At the class’s onset, greet the classmates cordially, so that the students can feel the teacher's warmth and draw the distance between teachers and students closer. Use diverse teaching, so that students can easily accept mathematical knowledge, learning the hook and strand theorem at the beginning, you can adopt the introduction of the history and culture of mathematics to give students an in-depth understanding of the hook and strand theorem, such

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as the hook and strand theorem named there can be originated in the early Chinese period of the Western Zhou Dynasty[3]; in the middle of the teaching, you can create a suitable life situation or mathematical situation to let the student explore the hook and strand theorem and understanding of the hook and strand theorem.

Process 4: Preparation and activation of prior knowledge

Deep learning is meaningful learning, learning that establishes a substantial connection with learning the original knowledge experience[4]. From the perspective of knowledge understanding, it is not rote memorization, but understanding the internal logic and meaning of knowledge; from the perspective of practical application, meaningful learning allows students to transfer the knowledge they have learned to new situations; from the perspective of emotional experience to examine the learning process, it allows students to create a sense of fulfillment and achievement through the process of learning. Deep learning focuses on students' experiences[4]. For example, in the teaching of the Pythagorean theorem, the teacher understands the current state of students' knowledge by asking questions during the class; the students discover the Pythagorean theorem through student discovery and discussion. Before the lecture, the teacher can divide the students in the class into groups, each group is assigned a task (about the knowledge learned in this lesson), and the teacher puts the power to each group[2], these tasks are divided by the group members to go to an independent inquiry, so that each student can shine in the classroom.

Process 5: Acquiring new knowledge

A distinctive feature of deep learning that distinguishes it from shallow learning is [4]: deep learning focuses on students' comprehensible learning and memorization of knowledge, and some experts even believe that deep learning is a kind of comprehension-based learning, in which conceptual knowledge can be internalized as their own only after it is understood by students. Therefore, it is important for teachers to talk about the nature of the theorem thoroughly in the process of students' knowledge acquisition. Teachers generally go through four stages in explaining theorems: the conjecture-exploration-discovery-expression process.

For example, for the discovery of the collinear theorem, people, in the practice of production and life for a long period of time, have the initial observation of the relationship between the three sides of a right triangle, and began to guess whether the lengths of the three sides of all right-angled triangles follow a specific quantitative relationship; the Pythagorean school of thought, through the study of the number of squares (the number of squares), the use of right-angled triangles with the side as the side of the length of the square constructed, and he began to try to analyze the relationship between the areas of these squares ; through continued attempts and deeper analysis, he observed that the sum of the areas of the two squares formed by the two right-angled sides of a right triangle is exactly equal to the sum of the areas of the squares formed by the hypotenuse; and finally, he made the expression that in a right triangle, the sum of the squares of the two right-angled sides is equal to the square of the hypotenuse. If the lengths of the right-angled sides are a,b, and the hypotenuse is c, $a^2 + b^2 = c^2$.

Process 6: Deep Processing Knowledge

Deep learning focuses on the transfer and application of knowledge. Realizing the transfer and application of learned knowledge is one of the goal achievement features of deep learning[4]. In the learning of theorems, it is necessary to understand the nature of the theorem and deeply comprehend the connotation of the theorem. At the same time, it is necessary to establish a connection between the newly learned theorem and the previously learned knowledge. Students can practice diversified exercises and do some targeted practice problems, including basic direct application of theorems and comprehensive interdisciplinary topics. Finally, the theorems can be put into real-life scenarios.

Process 7: Evaluating what is learned

Students should learn to self-reflection, reflect on their mastery of theorem knowledge in the classroom, and evaluate their level of theorem learning appropriately to adjust their learning status, SOLO classification theory is a qualitative evaluation method, and its core feature lies in the division of students' learning outcomes through hierarchical descriptions, which are specifically divided into five hierarchical levels[4]. According to the SOLO classification theory, the deep learning level and surface learning level, the following relationship is summarized, and students can evaluate themselves.

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Table 1 SOLO classification theory and math theorem learning level, and deep learning relationship

Math Theorem Level Descriptions	SOLO Structure	Learning Levels
The student has little or no knowledge of the content of the theorem; the student has only heard the name of the theorem or has isolated false impressions of the theorem.	Front Structure Level	Shallow Learning
The student can recognize one key point in the theorem.	Single Point Structure Level	
The student can recognize more than one key point in the theorem, e.g., the conditions of the theorem, the conclusion, and the individual parameters in the equation.	Multi-point structure level	
The student can appreciate the relevance of the theorem to other knowledge. In complex problems faced, students can relate the content of theorems to other knowledge to form integrated applications.	Associative Structure Level	deep learning
Students can make abstract generalizations about theorems and distill the general principles behind them; students are able to extend theorems and apply them to complex life or mathematical situations, or make creative improvements to them.	Abstract Extended Structure Level	

Table 1 above is a hierarchical classification of math theorem learning level and deep learning relationship using the SOLO theory.

In theorem classroom, teachers can use the learning levels listed in the above table to determine what level of mathematical theorem students are at and whether they have reached deep learning. The following is an example of the SOLO categorization level for the collinear theorem.

Table 2 SOLO analysis of learning results of the Pythagorean theorem

Learning Level	Learning Outcomes	The level of SOLO categorization theory at which
No Learning	The student lacks a basic understanding of the collinear theorem and does not understand the theorem about right triangles.	Front Structure Level
Shallow Learning	The student knows that the Pythagorean Theorem is probably a formula about one of the three sides of a right triangle, but remembers only part of the formula or has a vague impression of it.	Single Point Structure Level
Shallow learning	The student can accurately state the formula for the Pythagorean Theorem and explain its meaning.	Multi-point structure level
Deep Learning	Students are not only familiar with the content of the Pythagorean Theorem, but they can also relate it to other geometric knowledge and experience the ideas of equations and combining numbers and shapes.	Associative Structure Level
Deep Learning	Students are able to understand the Pythagorean theorem in the abstract and understand that it is a special geometric measure relationship. Apply ideas of combining numbers and shapes, and equations to solve problems.	Abstract Extended Structure Level

Table 2 above shows the hierarchical classification of the learning level of the collinear theorem using the SOLO theory.

3. Case Study on the Implementation of Deep Learning

Process 1: Design of Teaching Objectives

[Textbook analysis]

The collinear theorem from the human education version of the eighth-grade book chapter 17 of this chapter collinear theorem class content, is the previous chapter of the right triangle or angle relationship has been a preliminary study based on a more accurate study of right triangles between the three sides of

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the quantitative relationship between the embodiment of the research ideas from qualitative to quantitative. The Pythagorean theorem is an important property of triangles, which plays a fundamental role in the later study of trigonometric functions, quadrilaterals, circles, and other geometric content.

By guiding students to prove the Collinear Theorem using strategies such as the "special-to-general" approach, area calculations via inclusion-exclusion principles, and number-shape integration methods, this approach equips students with novel techniques and perspectives for addressing real-world problems.

The collinear theorem builds a bridge between geometry and algebra, is one of the most important theorems in elementary geometry, has an important position in the study of geometry, and is essential to promote the depth of students' study of geometry.

[Teaching Objectives]

Knowledge and Skills: Through the teacher's creation of situations, life, and cultural examples in class, students can understand the source of the collinear theorem and experience the process of proving the collinear theorem. Master the collinear theorem and apply it to cope with practical problems.

Process and Method: Students can construct graphs to prove the Pythagorean Theorem through the method of complementarity, experience the ideas of equations and the Pythagorean Theorem, and improve students' ability of abstract generalization and reasoning.

Affective Attitude and Values: Through exploring the process of proving the collinear theorem, we can experience the spirit of perseverance and exploration. Through the use of teaching software in the classroom, students can feel the joy of math class, and at the same time, experience the math culture of different cultural backgrounds.

Process 2: Pre-assessment

[Analysis of Learning Situation]

Before learning the Pythagorean Theorem, in terms of knowledge, students have rigorously proved the Triangle Trigonometry and Theorem through activities such as measurements, puzzles, and paper folding, and students have mastered the basic properties and operations of triangles. Students may not be aware of the special relationship between the three sides of a right triangle, but they have learned the property of mutual reciprocity of acute angles of right triangles and explored the determination of congruence of right triangles through operations such as ruler and graphing. From a cognitive perspective, junior high school students are transitioning from imagery-based to abstract thinking. Utilizing graphics can aid their understanding of the Collinear Theorem, foster reasoning skills, and accumulate practical experience, thereby laying the groundwork for exploring and proving the theorem in this lesson. In terms of learning interest, new and interesting puzzles and practical activities can stimulate their desire to explore.

Process 3: Creating a positive cultural atmosphere (creating a situation)

Teacher: Students, today we will explain the hook and strand theorem. The origin of the hook and strand theorem can be traced back to the period of ancient China's Dayu's rule of water, and abroad is the Pythagorean school of ancient Greece. The principle of Dayu's water rule can be explained by the Zhou Thigh Calculating Classic[5]. A related video is also been released.

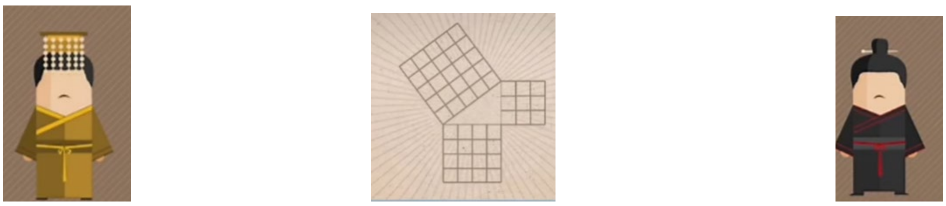

Figure 2 Teaching Video Material

Teacher: From the video just now (Figure 2), what related theorems can the students discover?

Students: We can find out that hook three, stock four, string five.

Teacher: Right, the theorem of hooks and strands is mentioned in Zhou Thighs Arithmetic; however, no strict proof is given, and a strict and detailed proof will be given in this lesson.

[Design intent]: By watching the related video of the hook and strand theorem, students can understand the origin of the hook and strand theorem in China, feel the mathematical culture, and at the

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same time arouse students' interest, so that the math classroom is no longer boring and uninteresting.

Process 4: Preparation and Activation of Prior Knowledge

Teacher: Before class, your teacher had everyone prepare four squares of paper (three of the same color with sides of 8 cm and one of the same color with sides of 6 cm) and a straightedge. First, the students took out two squares with sides of 8 cm and had the students put together a large square[6].

Students: students worked independently and summarized the task (by connecting the diagonals of the squares and then cutting the squares along the object, ending up with four isosceles right triangles, and then piecing together a large square by using the diagonal sides of the isosceles right triangles as the right-angled edges as in Figure 3 below)

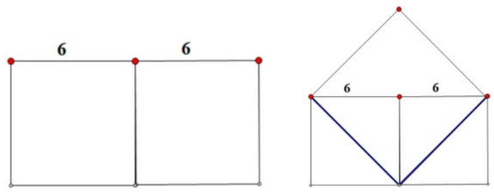


Figure 3 Patchwork of large squares

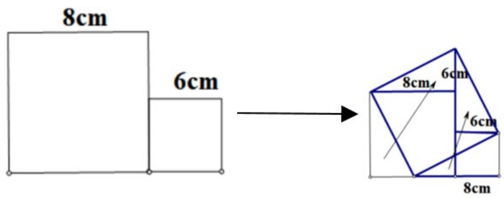


Figure 4 Cutting and piecing of squares

Teacher Activity 1: So the area of the pieced large square is:

$$S_{\text{chevron}} = 4 \times \frac{1}{2} \times 6 \times 6 = 72$$

So at this point, can we put together a large square with a side length of 6 and a side length of 8? Students work in groups to investigate. Students, can you find the similarities and differences between this task and the previous one?

Students: The similarity is two squares put together to form a large square; the difference is two squares with different side lengths.

Teacher activity 2: we just put together a large square can be found. To be more specific, the first is to find the diagonal of the two identical squares as a collateral edge for splicing. As a result, can we also find two identical collateral edges for splicing when it comes to this question; classmates what are the characteristics of the shapes we cut?

Students: All cut into the same right triangle.

Teacher: Similar to the above cutting, this cutting will not cut into the same right angled triangle. How to cut and where is the cutting point??

Students: Work in small groups to investigate and make a square splice, as illustrated in Figure 4 below.

Teacher: The cutting point is found to be the same at 6 cm of a square with a side length of 8 cm as it is for a square with a side length of 6 cm.

[Design intent]: Through the above two activities, students are allowed to work independently and in groups to investigate the formation process of the Pythagorean Theorem, and at the same time, they are able to establish a connection with their previous knowledge (knowledge about right triangles) in the process of investigation.

Process 5: Acquisition of new knowledge

Teacher Activity: Through the above two activities, students can find out what the area of the large square is that is put together. What are the lengths of the sides of the large square?

Student: Since the area of a square is the square of the side length, the area of the large square for the first activity is

$$S_{\mathrm{chevron}} = \frac{1}{2} \times 4 \times 6 \times 6 = 6^2 + 6^2 = (6\sqrt{2})^2 \mathrm{cm}^2$$

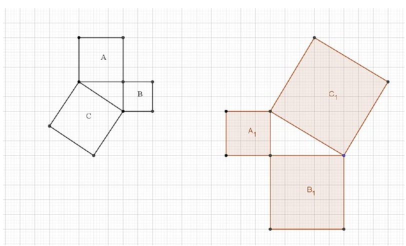
And the side length is $6\sqrt{2}cm$.

The area of the second large square is $S_{chevron} = 6^2 + 8^2 = 100 = 10^2 cm^2$, and the side length is $10cm$.

Teacher: As illustrated by our analyses, the sides of the large square put together are the hypotenuse of the triangle, in line with the above formula can be the square of the hypotenuse is equal to the sum of the squares of the two right-angled sides, so if the lengths of the two sides of a right triangle are a and b, and the hypotenuse is c, the expression is $a^2 + b^2 = c^2$.

Teacher activity: Let's carry out the following verification, observe the following figures, students can you find the relationship between squares A and B and square C? What method was used?

Student: You can find $S_A + S_B = S_C$, using the Pythagorean Theorem and verifying it by counting the squares. Figure 5 below.



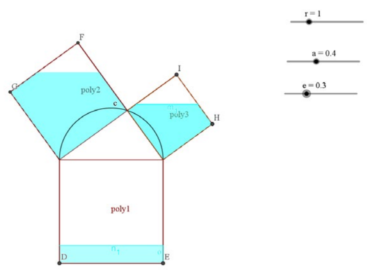
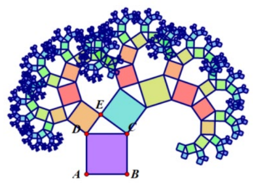


Figure 5 Checkerboard Experiment. *Figure 6 Watering-down Experiment.*



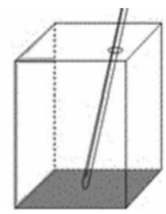


Figure 7 The Pythagorean Tree. *Figure 8 Pictures of Example Problems*

Teacher: So, students think about the following: can you continue to accumulate small squares on squares A, B?

Students: Yes.

Teacher: The teacher uses GeoGebra software to demonstrate the watering down experiment of the Pythagorean Theorem to visually explain Figure 5 above. Figure 6 below.

Teacher: Using a Geometry Drawing Board, draw the diagram below as the Pythagorean Tree. Figure 7 below.

[Design intent] Through the teaching activities of Process 4, students are guided to understand the Pythagorean Theorem through area, while being able to connect with previous knowledge (multiplication, properties of triangles) for meaningful learning, and deeper learning through students' own inquiry and thinking to understand the inner logic of the Pythagorean Theorem.

Process 6: In-depth processing knowledge

Teacher Activities: Consider a rectangular beverage carton with external dimensions of 5 cm (length), 4 cm (width), and 12 cm (height). A straw is inserted through a circular hole, extending 1 cm perpendicularly into the adjacent sides (length and width). To design an optimally comfortable and efficient straw, the exposed length of the straw after insertion should range between 3 cm and 5 cm for ease of drinking. What is the allowable range for the straw's total length? As shown in Figure 8 below.

Students: students think about answering.

[Design Intent] Through the life situation type of problem, not only can students further understand the hook and strand theorem, because of the requirements of the standard, the context of the teaching of students is more and more important, so it can also strengthen students' reading ability.

Teacher's Activity: On a calm lake, there is a small island (which can be regarded as a point) that is 10 meters above the surface of the lake, and a small boat sails from the south shore of the lake in a due east direction. When the boat is at point A, the island is observed to be located 30°northwest of the boat. Subsequently, the boat continues to sail east for 20 meters to point B. At this point, the boat observes the island again and finds that it is located 60° northwest of the boat. Q: How far is the boat from the island at this point? (This question can be solved by linking trigonometric functions with the Pythagorean theorem.)

[Design intent] By practicing different topics, students can better understand the Pythagorean Theorem, and the solution of the above topics can be linked to the knowledge learned before, which promotes the transfer of knowledge and in-depth learning.

Process 7: Evaluating Student Knowledge

Based on the SOLO classification theory mentioned above, five levels have been classified, and based on the assignments and tests at the end of the class, the teacher can determine what level the students are at, analyze the level they are at to the students, so that the students can understand what level they are at and which level they are working towards. Teachers can analyze the overall class level and reflect on their teaching.

4. Conclusion and recommendations

Firstly, each process of Deep Learning Route (DELC) is analyzed in detail, and the analysis is accompanied by the corresponding teaching strategies. Subsequently, a teaching design using the Deep Learning Route (DELC) to take the Pythagorean Theorem as an example is given. It is hoped that it can provide corresponding references for first-line teachers in junior high school.

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