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Whale Optimization Algorithm Based on Skew Tent Chaotic Map and Nonlinear Strategy

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Abstract: *To solve the problems that whale optimization algorithm is easy to fall into local optimization and slow convergence speed, and the improved whale optimization algorithm is proposed. Firstly, the algorithm uses Skew Tent chaotic map to initialize the whale population, improve the diversity of the original whale population and make the individual position distribution of whales more uniform; Secondly, the nonlinear convergence factor based on inverse incomplete* Γ *function is used to balance the global exploration and local development ability of whale algorithm. Through the simulation experiments of 8 benchmark functions, from the perspective of mean square deviation and average value, the convergence speed and optimization accuracy of the improved whale optimization algorithm are significantly higher than those of the traditional whale optimization algorithm.*

Keywords: *Whale optimization algorithm, Skew Tent chaotic map, nonlinear convergence factor, inverse incomplete* Γ *function*

1. Introduction

The swarm intelligence optimization algorithm obtains the optimal solution by iteratively updating the population. Representative swarm intelligence optimization algorithms include particle swarm optimization algorithm (PSO) [1]; genetic algorithm (GA) [2]; ant colony optimization algorithm (ACO) [3]; firefly algorithm (FA) [4]; Bat Algorithm (BA) [5].

Whale optimization algorithm (WOA) is a new swarm intelligence optimization algorithm proposed by Professor [6] Mirjalili[6] in 2016. The WOA seeks the optimal solution by simulating the predation behavior of humpback whales. The WOA has the advantages of simple structure and few parameters and has been applied in many research fields, such as photovoltaic cell parameter estimation [7], prediction of water resource demand [8], vehicle communication network [9] etc. However, similar to other swarm intelligence optimization algorithms, this algorithm still has the problems of slow convergence and is easy to fall into local optimization.

To solve the shortcomings of traditional whale optimization algorithms such as low optimization accuracy and slow convergence speed, this study proposes an improved whale optimization algorithm (STNWOA), which uses Skew Tent chaotic map to initialize whale populations to obtain better diversity Initial population; use a nonlinear convergence factor based on the inverse incomplete Γ function to balance the capabilities of global exploration and local development.

2. Whale optimization algorithm

The WOA simulates the predation behavior of humpback whales. According to the characteristics of whale predation behavior, the algorithm is mainly divided into three stages: encircling prey, bubble-net attacking, and searching prey. In traditional WOA, assuming that the population of whales is N and the spatial dimension is Dim, the position of each whale in the Dim-dimensional space can be expressed as $X_i = (X_i^1, X_i^2, ..., X_i^{Dim}), i = 1,2, ..., N$, the current optimal position is the position of the prey.

2.1. Encircling prey

At the current stage, the whales do not know the location of their prey, and the whales obtain information about the location of their prey through cooperation. The whale closest to the prey is the current optimal whale, and other whale individuals will approach the current optimal whale position to surround the prey. The mathematical model at the current stage is:

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$$D = \left|C \cdot X_{gbest}^{t} - X^{t}\right| \tag{1}$$

$$X^{t+1} = X_{gbest}^{t} - A \cdot D \tag{2}$$

Where t is the current number of iterations, X_{gbest}^{t} is the current optimal whale position, X^{t} is the position of the individual whale, and A and C are coefficient vectors. The expressions of A and C are:

$$A = 2 \cdot a \cdot r - a \tag{3}$$

$$C = 2 \cdot r \tag{4}$$

Where a linearly decrease from 2 to 0 as t increases, and r is a random number of $[0, 1]$.

2.2. Bubble-net attacking

At the current stage, the whale exhales bubbles around the prey at the same time, encircles the prey, and then continues to exhale the bubbles in a circle, narrowing the range of the bubble circle, and enclosing the prey in a small area. The whale rushes up from the bottom of the bubble circle to catch prey. In this stage, the individual whale first calculates the distance to the current optimal whale position, and then approaches the prey position in a spiral trajectory. The mathematical model is as follows:

$$D = \left|C \cdot X_{gbest}^{t} - X^{t}\right| \tag{5}$$

$$X^{t+1} = X_{gbest}^{t} + D' \cdot e^{bl} \cdot cos(2\pi l) \tag{6}$$

Where b is a constant coefficient and l is a random number in $[-1,1]$.

In the WOA algorithm, whales realize whale bubble attack behavior through spiral update position and contraction encirclement. The synchronization update of the two mechanisms is adjusted by the probability p. Its expression is as follows:

$$X^{t+1} = \begin{cases} X_{gbest}^{t} - A \cdot D & p < 0.5 \\ X_{gbest}^{t} + D' \cdot e^{bl} \cdot \cos(2\pi l) & p \geq 0.5 \end{cases} \tag{7}$$

Where $p \epsilon [0, 1]$.

2.3. Searching prey

At this stage, when $|A| > 1$, the whale will select a whale individual in the current population to perform a global search to prevent the population from falling into a locally optimal solution. Its expression is as follows:

$$D = |C \cdot X_{rand}^{t} - X^{t}| \tag{8}$$

$$X^{t+1} = X_{rand}^{t} - A \cdot D \tag{9}$$

Where X_{rand}^{t} represents the position of a random individual whale in the current population.

3. Improved Whale 0ptimization Algorithm

3.1. Skew Tent chaotic map to initialize whale population

Studies have shown that the quality of the initial population will affect the performance of the algorithm [10]. The population generated by the traditional whale optimization algorithm is random, and the diversity of the population cannot be guaranteed. The diversity of the initial population will improve the performance of the algorithm [11]. The chaotic map has good randomness and ergodicity [12]. Initializing the positions of individual whales using chaotic maps can improve the performance of the algorithm.

In this study, the Skew Tent chaotic map [13] is used to initialize the population, so that the population has good diversity. The expression is as follows:

$$x_{i+1} = \begin{cases} x_i/\varphi & 0 < x_i < \varphi \\ (1 - x_i)/(1 - \varphi) & \varphi < x_i < 1 \end{cases} \tag{10}$$

When $\varphi \epsilon (0, 1)$and $x \epsilon [0, 1]$, the system (10) is in chaos.

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3.2. Nonlinear convergence factor

In the traditional whale optimization algorithm, the parameter a decreases linearly from 2 to 0. It can be seen from Eq. (3) that the parameter A changes with the change of the parameter a, and the coordination between the global exploration and local development of the WOA algorithm depends on the parameter A. In other words, the coordination of the global exploration and local development of the WOA algorithm depends on the parameter a. In the early stage of the iteration, the WOA algorithm mainly uses global exploration, and it is hoped that the value of the parameter a will be larger and decrease quickly; in the later stage of the iteration, the WOA algorithm mainly uses local development, and it is hoped that the value of the parameter a is small and the decrease is slower. To better achieve the balance between global exploration and local development, this study adopts the method of adjusting the convergence factor based on the inverse incomplete Γ function [14]. The inverse incomplete Γ function has the characteristics of close to the linear decline in the early stage of the iteration and close to an exponential decline in the later stage of the iteration. Studies have shown [15] that the use of the inverse incomplete Γ function to dynamically adjust the convergence factor can better achieve the balance between global exploration and local development, and better avoid algorithm premature. The expression of the nonlinear convergence factor a is as follows:

$$a(t) = a_{initial} + \frac{a_{final} - a_{initial}}{\lambda} \times gammaincinv\left(\lambda, 1 - \frac{t}{Max\_iter}\right) \tag{11}$$

Where $a_{initial}$ and a_{final} are the initial and final values of the convergence factor a, Max_iter is the maximum number of iterations, $\lambda\ (\lambda \geq 0)$ is a random variable, and $\lambda = 0.01$ in this study.

3.3. STNWOA algorithm flow analysis

Combining the proposed Skew Tent chaotic map initialization population and the nonlinear convergence factor based on the inverse incomplete Γ function two optimization strategies to improve the traditional WOA algorithm. The pseudo-code of the STNWOA is shown in Algorithm 1.

Algorithm 1 (STNWOA)

1. Use Eq. (10) to initialize the whale population
2. Initialize related parameters a, A, C, l, p
3. The fitness of each search agent is calculated
4. X_{gbest}^{t} =the search agent with the best fitness
5. While (T<maximum number of iterations)
6. Use Eq. (11) to update a
7. Update A, C, L, and p
8. if(p<0.5)
9. if(|A|<1)
10. Use Eq. (2) to update the current position
11. else if(|A|≥1),
12. Randomly choose a search (X_{rand}^{t}).
13. Use Eq. (9) to update the position of the search agent
14. end if
15. else if(p≥0.5),
16. Use Eq. (6) to spiral update the current position
17. end if
18. end for
19. Check if there is a search agent out of bounds
20. The fitness of each search agent is calculated
21. Update X_{gbest}^{t} if there is a better fitness
22. T=T+1
23. end while
24. return X_{gbest}^{t}

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4. Experiment and discussion

4.1. Benchmark function

To test the performance of the STNWOA algorithm, experiments were performed on the optimization benchmark problem. As shown in Table 1, eight standard benchmark functions are used, where Dim represents the dimension of the function. These functions are divided into two categories: unimodal functions and multimodal reference functions. Among them, F1-F4 represents a unimodal function with a global optimal value. F5-F8 represents a multimodal function with multiple local optimal values.

Table 1: Detailed values of the benchmark function.

Function	Expression	Dim	Range	Optimal value
F1	$f_1(x)=\sum_{i=1}^{n} x_i^2$	30	$x_i \in [-100,100]$	0
F2	$f_2(x)=\sum_{i=1}^{n} \lvert x_i\rvert+\prod_{i=1}^{n}\lvert x_i\rvert$	30	$x_i \in [-10,10]$	0
F3	$f_3(x)=\max\{\lvert x_i\rvert, 1\le i\le D\}$	30	$x_i \in [-100,100]$	0
F4	$f_4(x)=\sum_{i=1}^{n}(\lvert x_i+0.5\rvert)^2$	30	$x_i \in [-100,100]$	0
F5	$f_5(x)=\sum_{i=1}^{n}[x_i^2-10\,cos(2\pi x_i)+10]$	30	$x_i \in [-5.12,5.12]$	0
F6	$f_6(x)=\frac{1}{4000}\sum_{i=1}^{n}(x_i^2)-\prod_{i=1}^{n} cos\left(\frac{x_i}{\sqrt{i}}\right)+1$	30	$x_i \in [-600,600]$	0
F7	$f_7(x)=\frac{\pi}{30}\{10\,sin^2(\pi y_i)+\sum_{i=1}^{n-1}(y_i-1)^2\,[1+10sin^2(\pi y_{i+1})]+(y_n-1)^2\}+\sum_{i=1}^{n}u(x_i,10,100,4)$, $y_i=1+\frac{(x_i+1)}{4}$, $u(x_i,a,k,m)=\begin{cases}k(x_i-a)^m x_i>a\\0-a\le x_i\le a\\k(\text{-x}_i-a)^m x_i<-a\end{cases}$	30	$x_i \in [-32,32]$	0
F8	$f_8(x)=0.1\{sin(3\pi x_i)+\sum_{i=1}^{n-1}(x_i-1)^2[1+sin(3\pi x_{i+1})]+(x_n-1)^2[1+sin^2(2\pi x_{i+1})]\}+\sum_{i=1}^{n}u(x_i,5,100,4)$, $u(x_i,a,k,m)=\begin{cases}k(x_i-a)^m x_i<a\\0-a\le x_i\le a\\k(\text{-x}_i-a)^m x_i\ge a\end{cases}$	30	$x_i \in [-50,50]$	0

4.2. Benchmark function results and data analysis

Compare the STNWOA algorithm proposed in this study with the traditional WOA algorithm and the PSO algorithm to test the effectiveness of the STNWOA algorithm.

To be fair, the experiment of each benchmark function is repeated 30 times. Use the best of 30 runs, the worst of 30 runs, the average of 30 runs, and the standard deviation of 30 runs to evaluate the performance of all the different algorithms. All algorithms were implemented on an ASUS laptop equipped with Intel (R) Core (TM) i5-10210U CPU @ 1.60GHz, 8.0 GB RAM, Windows 10 operating

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system, and MATLAB R2019a.

Use the F1-F4 unimodal benchmark function to test the local development capability of the algorithm. The experimental results are shown in Table 2.

It can be seen from Table 2 that for the four unimodal reference functions F1-F4, the STNWOA algorithm is better than the other two algorithms in terms of best value, worst value, mean and standard deviation. The standard deviation reflects the degree of deviation between the algorithm and the mean, that is, the smaller the standard deviation, the better the stability of the algorithm. This shows that the whale optimization algorithm proposed in this study has better stability and higher convergence accuracy.

From Figure 1-4, it can be found that for F1-F4, the convergence speed of the STNWOA algorithm is better than that of the other four algorithms. This shows that the convergence speed of the STNWOA algorithm has been improved.

Table 2: Experimental results of unimodal benchmark function.

Function	Result	WOA	PSO	STNWOA
F1	Best	4.7626e-90	8.1378e-06	1.9755e-150
	Worst	1.302e-69	0.001345	1.3906e-120
	Mean	4.4302e-71	0.00017114	4.6353e-122
	SD	2.3358e-70	0.00024346	2.4962e-121
F2	Best	1.0614e-56	0.003153	6.1564e-81
	Worst	1.124e-47	0.35126	4.5438e-59
	Mean	3.9043e-49	0.043525	1.5146e-60
	SD	2.016e-48	0.064395	8.1563e-60
F3	Best	0.97056	0.70364	2.4419e-70
	Worst	84.6929	1.6667	3.3568e-57
	Mean	52.0871	1.1515	1.1276e-58
	SD	26.9339	0.25049	6.0241e-58
F4	Best	0.095161	1.7803e-05	6.1681e-08
	Worst	0.92245	0.0031632	1.3806e-05
	Mean	0.43235	0.00023648	1.4052e-06
	SD	0.25471	0.00057153	2.5596e-06

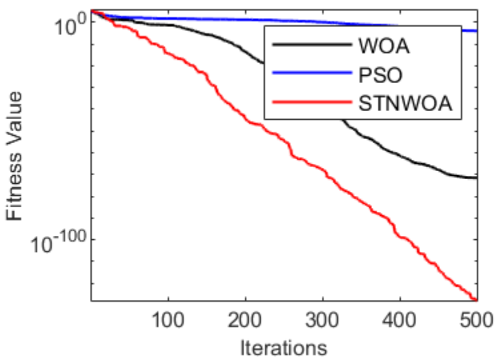

Figure 1: F1 convergence curve chart.

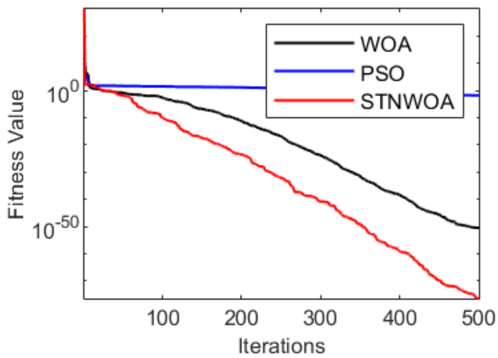

Figure 2: F2 convergence curve chart.

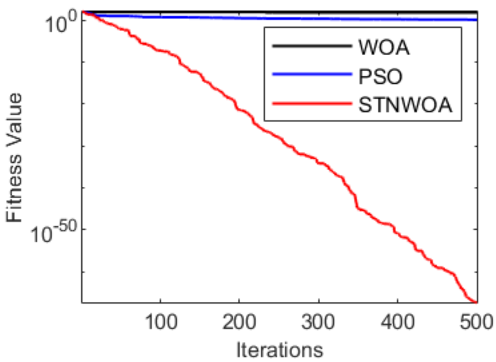

Figure 3: F3 convergence curve chart.

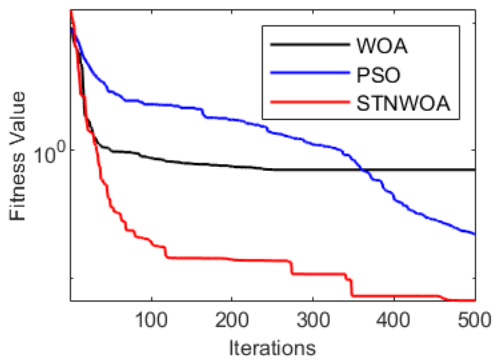

Figure 4: F4 convergence curve chart.

Table 2 shows the experimental results of the multimodal benchmark function. The multimodal

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benchmark function can be used to test the global exploration capability of the algorithm. It can be seen from Table 2 that for F5, the STNWOA algorithm has reached the theoretical optimal value and has good stability; for F6 and F8, the optimal value of the STNWOA algorithm is better than the other two algorithms; for F7, The STNWOA algorithm is superior to other algorithms instability and convergence accuracy.

The experimental results show that the STNWOA algorithm has better stability, higher convergence accuracy, better stability than the other two algorithms, and it has better global exploration capabilities.

Figures 5-8 show the convergence curves of the multimodal reference functions F5-F8. It can be found from the figure that for F5-F8, the convergence speed of the STNWOA algorithm is significantly better than other algorithms. This reflects that the convergence speed of the STNWOA algorithm has been improved.

In summary, no matter from the unimodal function perspective or the multimodal function perspective, STNWOA has better experimental results, with higher convergence accuracy, better stability, and faster convergence speed. It fully reflects the superior performance of its algorithm.

Table 2: Experimental results of multimodal benchmark function.

Function	Result	WOA	PSO	STNWOA
F5	Best	0	23.0085	0
	Worst	1.1369e-13	115.561	0
	Mean	7.5791e-15	54.622	0
	SD	2.8358e-14	17.8347	0
F6	Best	0	8.9809e-07	0
	Worst	0.15916	0.024613	1.0912
	Mean	0.012111	0.007096	0.071995
	SD	0.037497	0.0075014	0.2694
F7	Best	0.0064149	7.3484e-08	3.8162e-09
	Worst	0.050471	0.20732	2.6466e-07
	Mean	0.019931	0.017281	6.9841e-08
	SD	0.01063	0.017281	6.6493e-08
F8	Best	0.10771	3.2337e-06	3.957e-08
	Worst	1.3054	0.098899	0.37362
	Mean	0.52274	0.0095796	0.063344
	SD	0.26231	0.019553	0.088894

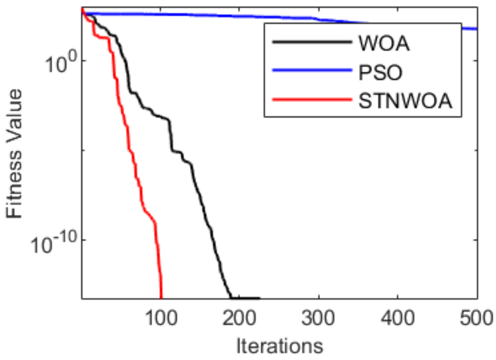

Figure 5: F5 convergence curve chart.

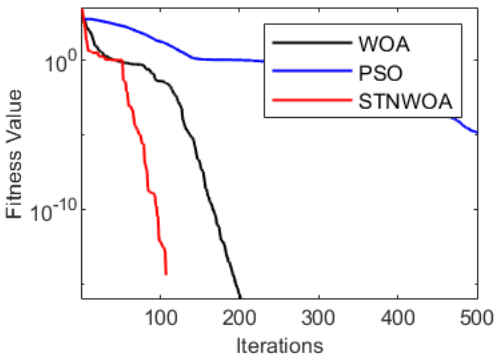

Figure 6: F6 convergence curve chart.

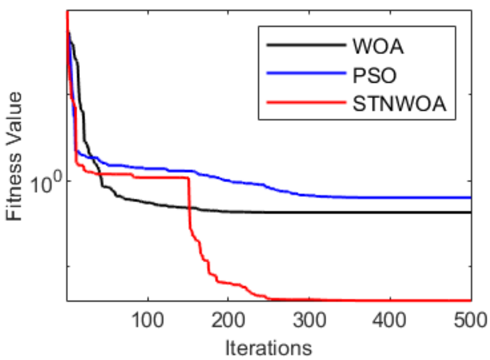

Figure 7: F7 convergence curve chart.

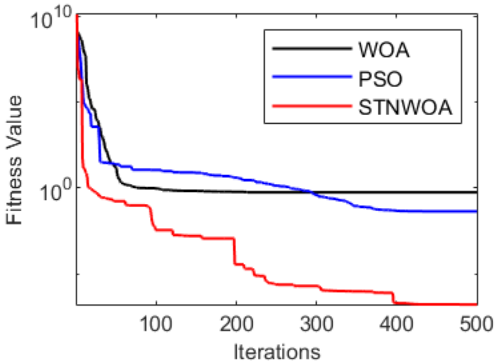

Figure 8: F8 convergence curve chart.

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5. Conclusions

Aiming at the problems of traditional whale optimization algorithms that tend to fall into local optimality and slow convergence speed, this research proposes an improved whale optimization algorithm (STNWOA). First, use Skew Tent chaotic map to initialize the whale population to improve the diversity and algorithm of the original population Second, the use of nonlinear convergence factor to better achieve the balance between global exploration and local development. Finally, through simulation experiments on 8 benchmark functions, the experimental results show that the convergence speed and optimization accuracy of the STNWOA algorithm has been improved to a certain extent, which reflects the superiority of the algorithm.

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