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A Study of Reverse Instructional Design Based on UbD Theory: An Example of "Derivatives"

Qiqi Xiao[1,a,*]

[1]College of Mathematics and Statistics, Henan Normal University, Xinxiang, Henan, 453007, China
[a]2217621675@qq.com
**Corresponding author*

Abstract: *High-quality teaching is the way to implement key competencies, and good instructional design is the foundation for realizing high-quality teaching. Aiming at the traditional teaching design misunderstandings of "activity-oriented teaching design" and "indoctrination learning", combined with the core idea of UbD theory, we constructed a framework of reverse instructional design for high school mathematics based on UbD theory. We carried out a reverse instructional design for the unit "Derivatives" in the high school of People's Education Press Version A (PEP Version A). It provides a reference for mathematics educators to design instruction that focuses on the core mathematical knowledge. The unit of "Derivatives" is used as an example of reverse instructional design, which provides a reference for math educators' instructional design and an idea for the implementation of the key competencies of mathematics.*

Keywords: *UbD, Instructional Design, Derivatives*

1. Introduction

As the new curriculum reform progresses, the goal of education has shifted from being "knowledge-based" to "competency-based", and finding ways to fully realize these key competencies has become an urgent challenge. High-quality teaching is crucial for implementing key competencies, and effective instructional design serves as the foundation for achieving this goal. To address misconceptions in traditional instructional design, such as activity-focused planning and rote learning, Grant Wiggins and Jay McTighe proposed the Understanding by Design (UbD) theory [1]. Unlike traditional instructional design, which follows a linear sequence, UbD theory begins with the desired outcomes and uses assessment as a bridge to guide instructional activities, ensuring that instruction remains centered on students' understanding and ability development. Therefore, reverse instructional design based on UbD theory is an effective approach to implementing key competencies in mathematics.

2. Overview of UbD Theory

2.1 Big Idea

Big ideas are concepts, themes or issues that can connect discrete facts and skills and give them meaning, and are characterized by abstraction and universality, unification and integration, and transferability and durability [1]. In the field of education, the big idea can build a framework for curriculum design and make the content more coherent and systematic. Its value is reflected in two dimensions: first, as a navigational tool for instructional design, it helps teachers establish a macroscopic perspective, guides students to break through the limitations of fragmented knowledge points, and forms a deep understanding of the essential laws of the discipline; second, it plays the role of scaffolding in the cognitive development of students and synchronizes the development of higher-order thinking and complex problem-solving abilities by promoting the systematic weaving of knowledge networks. This two-way empowerment mechanism not only ensures the scientific nature of knowledge transfer but also realizes the migratory nature of ability cultivation, which fully demonstrates the core demand of modern educational paradigm transformation. Among other things, big ideas help to identify desired results and provide direction for teaching and learning activities. At the same time, UbD theory helps the big ideas to be grounded in teaching and learning through the six facets of understanding, essential questions, etc., which in turn enhances students' ability to understand and apply knowledge.

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2.2 Six Facets of Understanding

The immediate aim of education is to develop understanding and application. Mathematics education is not only about imparting basic knowledge and skills, but also about guiding students to a deeper understanding of the underlying knowledge and structure of mathematics. Understanding is a symptom of deep learning. It is the process of cognitive schema reorganization, which enables students to go through the appearance of knowledge, grasp the essence of mathematics, realize the application of knowledge in different contexts, and promote knowledge transfer. And the process of understanding can promote students to structure the knowledge vein independently, thus generating a continuous internal drive. In the era of rapid development of information, comprehension has become an indispensable core competence of individuals. With strong comprehension, individuals can absorb, integrate and utilize new knowledge more efficiently, and when faced with new challenges, they can respond quickly and adaptively, analyze the problems and formulate solutions, thus building a solid foundation for their long-term development. In the teaching of mathematics, more attention should be paid to students' understanding and transfer of knowledge to develop key competencies. The UbD theory proposes “six facets of understanding”, in which students are evaluated from six perspectives, namely: explanation, interpretation, application, perspective, empathy and self-knowledge. These six dimensions can be categorized into three levels, namely: narration (explanation, interpretation), utilization (application, perspective), and development (empathy, self-knowledge), as shown in Table 1[2][3]. These six dimensions are both overlapping and perfectly integrated, judging students' understanding and ability to transfer knowledge from all angles.

Table 1: Six facets of understanding.

Level	Dimension	Implication
Narration	Explanation	Use theories and diagrams to illustrate points of view, systematically analyze data and phenomena, gain perspective on connections and give examples to support them.
	Interpretation	Demonstrate, explain and paraphrase to provide some sense of meaning.
Utilization	Application	Can effectively apply knowledge in new, realistic situations.
	Perspective	Ability to present critical and insightful perspectives.
Development	Empathy	Ability to perceive the emotions and worldviews of others.
	Self-knowledge	The wisdom of knowing your ignorance and how your patterns of thinking and behavior get in the way

2.3 Essential Questions

Problems serve as the starting point of teaching, and problem-oriented approaches help students understand the intrinsic connection between knowledge, construct a knowledge system, and develop their thinking progress. In teaching, students actively explore knowledge based on problem situations, and transfer and apply knowledge continuously. In the context of deepening education reform, the problem-oriented teaching mode has become a key pathway for developing students' higher-order thinking skills and key competencies in the discipline. Problems are more the heart of mathematics, and the core competencies of the mathematics discipline are enhanced in the interaction between students and contexts and problems. Teachers design appropriate contexts and problems, which not only help students understand the essence of mathematics but also promote the formation and development of key competencies in mathematics [4]. “Essential questions” are important questions that point to the core of a subject, are very inspiring and fundamental, and can stimulate learners to explore key concepts, theories, and themes that are not yet understood in their knowledge [5], which can promote the development of students' higher-order thinking and improve their learning ability. The solution of essential questions requires students to fully mobilize their higher-order thinking, which triggers the deeper reconstruction of cognitive structure.

2.4 Evaluation and Feedback

Assessment is a key part of teaching and learning. During instruction, teachers need to think like assessors - what is sufficient evidence to reveal understanding? Do the assessments we do show and differentiate between students who really understand and those who appear to understand? Am I clear about the reasons behind students' mistakes [2]? Therefore, teachers need to design appropriate tasks to assess and test students, and the types of assessment should be varied, including observation and dialogue,

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traditional accompanying tests, performance tasks, and students' self-assessment, among others. Among them, performance tasks are those that test knowledge and abilities in a situation that replicates or simulates the real world and are “tests” where students get the same experience as in real work and life [2]. A typical performance-based task usually involves setting up a real-world contextual problem embedded with cognitive conflicts for students and requires the completion of a virtual practice task within a specific framework. When performing the task, students are expected to synthesize their knowledge, transfer, problem-solving, and creative thinking skills to demonstrate their level of understanding and application of knowledge through practical performance.

2.5 WHERETO Elements

In order to improve students' motivation, enhance learning ability and promote deep learning, teachers have to pay constant attention to students' understanding of what they have learned, and can adopt WHERETO elements in UbD theory to optimize the instructional design. WHERETO elements, as shown in Figure 1, can help teachers to construct and test their instructional design, and to integrate the expected goals and identified assessment evidence into specific instruction, but without fixing the steps and sequence of the unit design. The WHERETO elements, as shown in Figure 1, can help teachers construct and test their instructional designs by integrating intended goals, identified assessment evidence, and specific instruction, without having to fix the steps and sequence of the unit design.

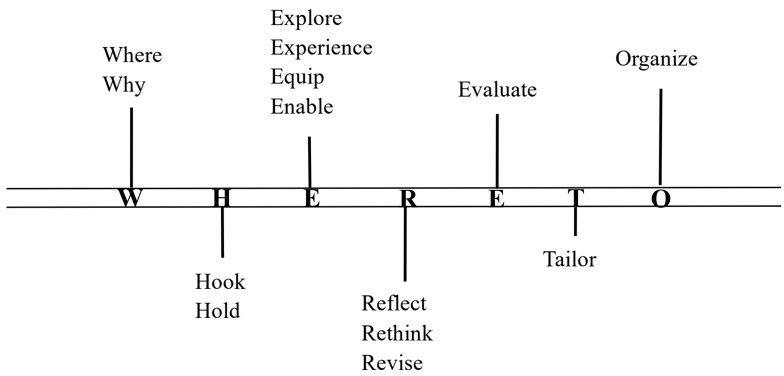

Figure 1: WHERETO elements.

3. A Framework for Reverse Instructional Design Based on UbD Theory

UBD theory divides instructional design into three stages: determining desired results, determining acceptable evidence for assessment, and designing learning experiences, as shown in Figure 2.

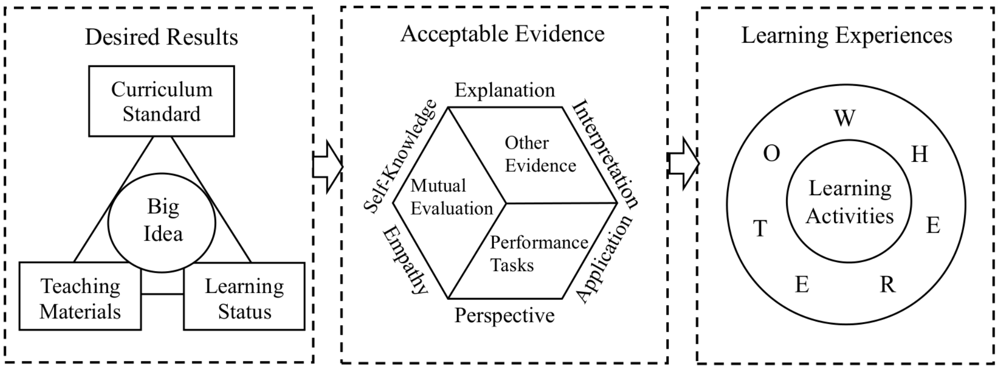

Figure 2: A framework for reverse instructional design based on UbD theory.

3.1 Determining Desired Results

The UbD theory requires teachers to determine what content students need to understand and what knowledge and skills they need to acquire when designing lessons. Therefore, teachers should first thoroughly study the curriculum standards and textbooks, reorganize the unit's knowledge structure, and,

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based on the students' actual situation, determine the big idea for the unit. Then, based on the big idea and the curriculum standards and textbooks, they should determine the essential questions and teaching objectives, thereby determining the expected understandings and the knowledge and skills that students should master.

3.2 Determining Acceptable Evidence for Assessment

Unlike traditional teaching, which first designs teaching activities and then determines assessment evidence, UbD theory advocates the principle of "assessment first," reversing the order of teaching design and requiring assessment evidence to be designed based on desired results. To test whether students have achieved the expected level of understanding, teachers should set up various types of assessment tasks, especially performance tasks, which enable students to learn how to apply knowledge and skills in the real world, improve their innovative awareness and problem-solving abilities, and compensate for the shortcomings of traditional assessment methods. At the same time, the assessment tasks should be combined with the six facets of understanding to comprehensively evaluate students' levels of understanding.

3.3 Designing Learning Experiences Fundamental Questions

Subject knowledge must be conveyed through unit activities [6]. Once teaching objectives and evaluation criteria are clearly defined, teachers should use these as a guide to systematically plan teaching activities and establish an implementation framework that deeply integrates assessment and instruction. According to UbD theory, optimizing teaching activities through the "WHERETO" elements and accurately planning cognitive paths and interactive links can effectively improve teaching effectiveness. On this basis, it is necessary to focus on cultivating students' independent exploration abilities and creating a space for deep thinking by reducing direct intervention by teachers. This kind of goal-oriented, evaluation-based, and strategy-supported teaching design not only helps to substantially improve classroom efficiency but also effectively promotes the integrated development of students' key competencies in mathematics.

4. A Case Study of Backwards Design Based on UbD Theory

Derivatives play a crucial role in high school mathematics education. They not only complement functional knowledge but also lay the foundation for future calculus study. In this unit, students will first encounter the concept of limits. Function limits are key to solving function problems but are highly abstract. Many students only focus on solving problems in this unit without truly understanding its underlying principles. Therefore, this paper designs a teaching plan for the derivatives unit based on the UbD theory, providing a new teaching approach to help students truly understand and master derivatives concepts.

4.1 Determining Desired Results

For the "Derivatives" unit, the curriculum standards require students to gain an intuitive understanding of the concept of derivatives through specific contexts and to grasp the idea of limits; to master the basic rules of derivatives operations and be able to calculate the derivatives of simple functions and simple composite functions; to apply derivatives to study the properties and patterns of simple functions; and to use derivatives to solve simple real-world problems, among other objectives [7]. The knowledge in this unit is arranged in a spiral progression, covering the concept, operations, and applications of derivatives. The logical relationships between the knowledge points are closely intertwined. The knowledge structure of this unit is illustrated in Figure 3. Before studying this unit, students have already learned about simple elementary functions, the monotonicity of functions, and the slope of a line, and have experienced the process of using the "bisection method" to approximate the zeros of a function during the study of function zeros. However, they have not systematically studied limits. Based on an analysis of the curriculum standards, textbooks, and student learning situations, it was found that derivatives are mathematical expressions of instantaneous rates of change, and the study of derivatives concepts should be integrated throughout the learning of derivatives of single-variable functions and their applications. Therefore, the overarching concept for this unit was determined to be "mathematical expressions of instantaneous rates of change."

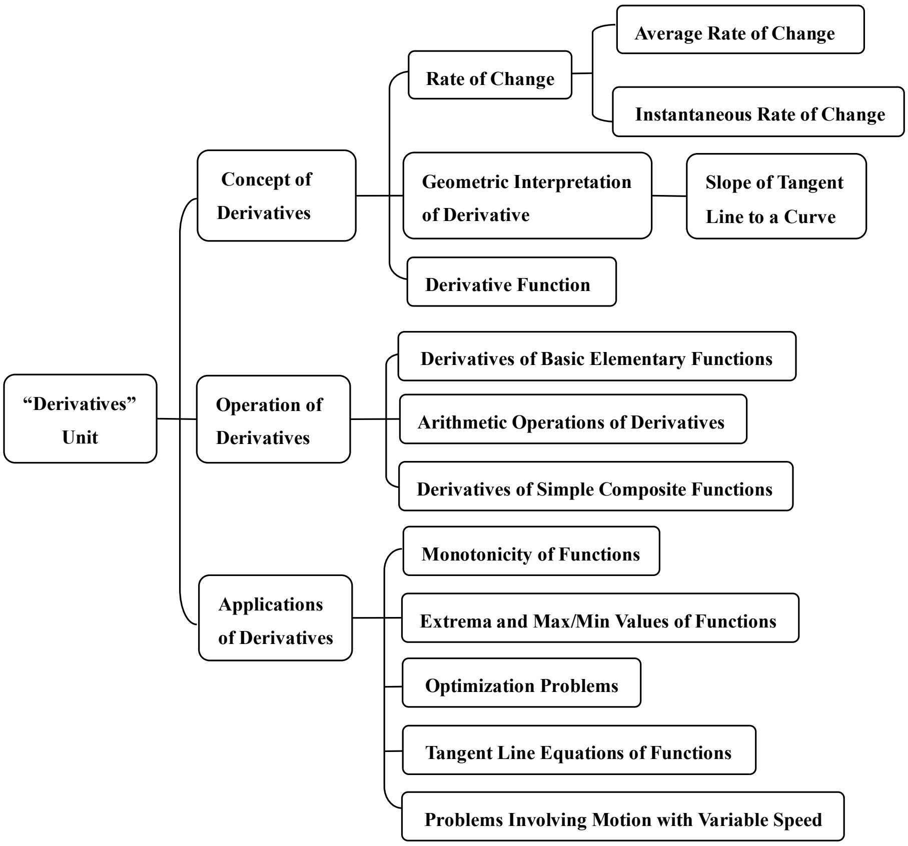

Figure 3: Knowledge framework of the "Derivatives" unit.

To determine desired results, teachers need to consider what the teaching objectives of this unit are. What basic questions need to be considered when studying derivatives? What do you expect students to understand? What knowledge and skills will students acquire? Based on the broad concept of "mathematical expression of instantaneous rate of change," the basic questions of this unit can be determined. Derivatives are a type of operation that uses limits, and their core lies in depicting the local change trends of functions, providing a powerful tool for analyzing the properties of functions and optimizing problems. Teachers should guide students through specific examples to experience the process from average rate of change to instantaneous rate of change, thereby grasping the essence of derivatives; then, by combining function graphs, help students understand the geometric meaning of derivatives and appreciate the integration of numbers and shapes; next, guide students to master the basic operations of derivatives and use them to explore the properties of functions; finally, students should be able to understand through specific examples that derivatives are not only a mathematical tool but also a way of thinking for analyzing dynamic phenomena. Based on this analysis, and in combination with the requirements and teaching objectives for derivatives in the curriculum standards, the desired learning outcomes can be determined as shown in Table 2.

Table 2: Desired results of the "Derivatives" unit design.

<table>
<tr><td colspan="2">Phase 1: Determine Desired Results</td></tr>
<tr><td colspan="2">The objectives are:
1. Understand the concept of derivatives through rich practical contexts;
2. Master the basic operations of derivatives;
3. Use derivatives to study the properties of functions;
4. Use derivatives to solve some practical problems.</td></tr>
<tr><td>What essential questions should we consider?</td><td>What is the expected understanding?</td></tr>
<tr><td>1. How can we abstract the concept of derivatives from real-world problems?
2. How does the geometric meaning of derivatives help us understand the local characteristics of function graphs?</td><td>Students will understand:
1. The essence of derivatives is a mathematical expression of instantaneous rates of change, which is an operation that uses limits.
2. How derivatives reflect the local trend of a</td></tr>
</table>

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3. How can we use derivatives to analyze the properties of functions (monotonicity, extrema, and maximum values)? 4. How can we use derivatives to find optimal solutions when solving real-world problems?	function graph at a certain point. 3. The relationship between derivatives and function properties, and appreciate the idea of moving from the local to the whole, and then from the whole to the local. 4. The status and role of derivatives as an analytical tool in calculus.
What important knowledge and skills will students gain as a result of unit learning?	
Students will learn 1. The definition of derivatives and their geometric significance; 2. Derivatives formulas for basic elementary functions and the four arithmetic rules of derivatives; 3. Methods for calculating derivatives of composite functions; 4. The relationship between derivatives and the monotonicity of functions.	Students will be able to 1. Establish functions based on actual contexts and calculate their derivatives; 2. Use derivatives to study the properties and patterns of simple functions; 3. Apply derivatives to solve simple real-world problems; 4. Approach problems from a dynamic, changing, and infinite variable mathematical perspective, rather than remaining stuck in a static, unchanging, and finite constant mathematical perspective.

4.2 Determining Acceptable Evidence for Assessment

In the derivatives unit, students' learning tasks require them to understand the concept of derivatives, master derivatives operations, apply derivatives to study function properties, and solve practical problems. Based on this, the six facets of understanding in UbD theory are combined with different types of assessment tasks to determine the assessment evidence for the derivatives unit, as shown in Table 3.

Table 3: Assessment evidence design for the unit "Derivatives".

Phase 2: Determining Acceptable Evidence	
What can be used to prove that students have understood what they have learned?	
What essential questions should we consider?	What is the expected understanding?
Performance tasks: 1. Students independently research the history of calculus and write a short essay discussing its origins, important results, main tasks, key events, and contributions to human civilization; Interpretation 2. Students independently design an optimization problem from everyday life, review each other's work in groups, and write reflective journals; Empathy Self-knowledge 3. Suppose you are a beverage manufacturer selling a certain drink in spherical bottles. The manufacturing cost of each bottle is $0.8\pi r^2$ cents, where r is the radius of the bottle (in cm). Given that the manufacturer earns a profit of 0.2 cents for every 1 mL of beverage sold, and the maximum radius of the bottle you can produce is 6 cm, what is the optimal radius of the beverage bottle for maximum profit? In real life, is profit solely influenced by the bottle's radius? Application Perspective	
Based on the desired results of Phase 1, what additional evidence needs to be collected?	
1. Classroom quiz: Derivation of derivatives formulas and operations involving derivatives; Explanation 2. Informal assessment: Post-class exercises for the derivatives unit; Application 3. Classroom Q&A: Explanation of the geometric meaning of derivatives; Explanation 4. Observation and dialogue: Students' interactions, discussions, and responses to questions in class; Application 5. Short test: Given a function f(x), determine its monotonic intervals, extrema, and maximum/minimum values on a closed interval, and plot and compare the graphs of the original function and its derivatives for analysis; Application 6. Unit test: Apply derivatives to solve real-world problems (optimization problems, dynamic problems) in practical contexts. Application	
Student self-evaluation and feedback:	
1. List the difficulties encountered in learning, write a reflection journal and an improvement plan; Self-knowledge 2. Give examples of using derivatives to solve practical problems. Perspective	

4.3 Designing Learning Experiences

At this stage, teaching activities can be designed based on teaching objectives and assessment evidence, incorporating the WHERETO elements to gradually implement assessment tasks. When designing activities, it is necessary to consider what students should do after determining teaching expectations and how to efficiently organize the teaching process. Therefore, the activity arrangements for the "derivatives" unit are shown in Table 4 below (where E represents the first E of the WHERETO elements, and E-2 represents the second E).

Table 4: Learning activities design for the unit "Derivatives".

Phase 3: Designing Learning Experiences
1. Introduce the topic through the physics problem "calculating the instantaneous velocity of variable-speed motion" and combine it with the background of calculus to stimulate students' interest in learning derivatives. H 2. The teacher explains the teaching objectives of this unit to the students and explains the significance of learning derivatives. W 3. Students independently draw the graph of the function, observe the process of the secant line approaching the tangent line, and calculate the average rate of change and instantaneous rate of change to understand the geometric meaning of derivatives; E 4. Organize students to use GeoGebra to draw the graph of the function and dynamically demonstrate the process of the secant line approaching the tangent line; E, O 5. Have students work in groups to calculate the derivatives of basic elementary functions, and compete between groups to reinforce the four arithmetic operations of derivatives and the rules for differentiating composite functions; E, O 6. Students discuss how the derivatives sign reflects the monotonicity of a function and analyze incorrect cases (such as situations where the derivatives does not exist); R 7. Students select real-life scenarios (e.g., maximum profit, shortest path), establish function models, and use derivatives to determine optimal solutions, then write reports and present them; E, T 8. Conduct tiered assessments on derivatives calculations, tangent equation solutions, and extremum applications to evaluate learning outcomes for this unit; E-2 9. Students self-assess their exercise books based on the grading criteria, engage in peer review within groups, and analyze the causes of errors; E-2, R 10. Students create a mind map for this unit, integrating concepts, formulas, application examples, and mathematical ideas; E-2 11. Organize a debate to compare the advantages and disadvantages of geometric intuition and derivatives tools in solving extremum problems, and summarize the value of the integration of numbers and shapes; R, O 12. Students research the history of calculus development, explain how the concept of derivatives broke through the limitations of ancient Greek geometry, and discuss its impact on modern science; E, T 13. Teachers and students jointly review the knowledge framework of derivatives, analyze comprehensive application strategies using actual college entrance exam questions, and encourage students with extra capacity to explore advanced content (such as Taylor series expansions). O, W

5. Teaching Insights

Mathematics education under the new curriculum reform emphasizes the integrity, structure, and consistency of course content, with a focus on the integration of teaching and assessment. The UbD theory has a solid theoretical foundation and is highly practical. This study is based on the UbD theory, using "the mathematical expression of instantaneous rate of change" as the unit's big idea, and reconstructs the teaching framework for the "Derivatives of Single-Variable Functions and Their Applications" unit. This theory reverses the traditional teaching design sequence, starting with the desired learning results, effectively avoiding problems such as unclear teaching objectives and deviations in teaching activity design. Based on various types of assessment evidence, such as performance tasks, it allows students to simulate real-life situations to fully mobilize their knowledge to solve problems, cultivate their understanding and transfer abilities, and develop their key competencies. Finally, learning activities are reasonably designed based on the desired results and assessment evidence, which improves the effectiveness of teaching design and is conducive to the integration of teaching and assessment. The three stages of UbD theory are closely linked and interrelated, and its structured framework is already very mature. Teaching design based on UbD theory enables students to achieve cognitive progression through understanding, and teachers to achieve professional growth through design. This study demonstrates that UbD offers actionable guidance for shifting mathematics education from a

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"knowledge-centered" to a "competency-centered" approach and injects sustained innovative momentum into curriculum reform in the new era.

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