Academic Journal of Computing & Information Science
ISSN 2616-5775 Vol. 9, Issue 4: 9-20, DOI: 10.25236/AJCIS.2026.090402

Finite-time Optimal Control for Uncertain Nonlinear Systems via Adaptive Dynamic Programming

Heping Gu[1,a,*]

[1]*Department of Mathematics and Physics, Sichuan Minzu College, Kangding, China*
[a]*phg3979@163.com*
[*]*Corresponding author*

Abstract: *This paper conducts an investigation into the finite-time optimal control problem of matching uncertain nonlinear systems through adaptive dynamic programming methods. Initially, a novel finite-time control approach is put forward. This approach reformulates the robust control problem as the optimal control problem of the nominal system by devising an appropriate cost function. Moreover, a crucial neural network is constructed to approximate the optimal cost function, and the gradient descent algorithm is employed to train the weights of the neural network. The semi-global practical finite-time stability criteria and the Lyapunov method are utilized to guarantee the convergence of closed-loop systems within a finite time. Ultimately, a simulation example is presented to validate the theoretical analysis.*

Keywords: *Finite-time Stabilization; Adaptive Dynamic Programming; Neural Networks; Uncertain Nonlinear Systems*

1. Introduction

In the 1950s, scholars began to study the robustness of practical control systems. Robustness requires the control system to maintain a certain performance under the inevitable parameter uncertainty. For this reason, the design of robust controller is particularly significant. In 1992, Lin et al[1] pointed out that there is an equivalent relationship between the optimal control of the nominal system and the robust control of the original system. The robust controller design can be completed by constructing the corresponding cost function to solve the optimal control problem of the nominal system. The breakthrough of the above research contributes a new idea to the robust control of uncertain systems. Of course, for the robust controller design of such systems, various control methods were proposed in the early stage. For example, reference [2] proposed a robust control method based on linear matrix inequality. These achievements have laid the foundation for robust control.

Adaptive dynamic programming (ADP) was originally proposed by Werbos, which overcomes the dimension explosion of classical dynamic programming[3]. Specifically, ADP combines the ideas of reinforcement learning and dynamic programming, thus, the numerical approximate solution of Hamilton-Jacobi-Bellman (HJB) equation can be obtained based on the neural network module, further deduced the optimal control strategy for the nonlinear systems[4]. The above works effectively avoid the direct solution of HJB equation, which provides a novel way to study the optimal control problem of practical systems. It is worth noting that ADP has become a hotspot research in the field of optimal control for a long time, which is attributed to the close relationship between ADP method and reinforcement learning field. Therefore, ADP method has attracted the attention of a large number of scholars both in algorithm and application [5-6]. In 2021, Sun et al[7] proposed a cross iterative algorithm based on HJB, and proved the convergence of the algorithm by the classical mathematical analysis methods. Zhao et al. [8] developed an output feedback control method based on ADP for a class of linear systems, and adopted event-trigger technology in the design of the controller to effectively reduce the execution times. In [9], the authors proposed an ADP-based robust tracking controller for a class of nonlinear systems with matched uncertainty, which was implemented by online adaptive online learning technology. Generally, the structure of ADP is composed of critic neural network (CNN) and action neural network, the former is used to approximate cost function and the latter is used to approximate optimal control strategy reported in [10]. However, the affine system breaks this convention and makes action neural network an unnecessary module, which undoubtedly reduces the complexity of algorithm implementation.

For the actual control system, in addition to the demand for robustness, it is interesting to consider that the controlled system is stable in a finite time(FT)[11-12]. Li et al[13] designed an attitude controller with implicit mode estimation, it makes the system globally stable in FT when overcoming the reverse action of the flexible system. In [14], a concept of robust FT passivity is proposed, and the FT stability of switched nonlinear systems is studied. Aiming at the problem of satellite attitude tracking, In [15], The author studied the event triggered finite time stabilization problem of Takagi Sugeno fuzzy systems with time scale delay. A controller that does not rely on power functions or delayed state feedback is proposed by comparing strategies, inequality techniques, and time scale theory. However, the results of FT stability related to ADP method are rare in existing literature. Explicitly, based on the initial iterative constraint method or iterative ADP algorithm, a FT domain optimal controller in the sense of ε is designed to make the system stable in a finite time[16]. By using the value iterative algorithm and quadratic heuristic dynamic programming structure, work [17] designed the FT actuator with saturated input for the nonlinear discrete-time system by the time forward, nevertheless, the Lyapunov stability criterion for FT control systems is not considered.

Based on the above discussion, an ADP-based method for nonlinear systems with matched uncertainty is proposed in this paper. Firstly, the transformation of the robust control problem of the original system is equivalent to the study of the optimal control problem of the nominal system. This transformation is attributed to the construction of the cost function. In the framework of ADP, a single NN is considered to approximate the constructed cost function, and then the solution of the modified HJB equation is obtained. It is worth mentioning that the relationship between cost function and system state is established. Based on the relationship, by using the classical Lyapunov method and FT stability criterion, the original system is the semi-global practical finite-time stability (SGPFS) under the robust optimal control strategy of the nominal system. In addition, the effectiveness of the proposed method is verified by one example.

The main contributions of this paper are summarized as follows:

(1) Different from the asymptotic stability and exponential stability results obtained in works [18,19,20], this paper not only studies ADP method to solve the optimal control problem, but also discusses the FT stability problem of the matched uncertain nonlinear system.

(2) A new FT stability control method is proposed by designing an upper bound function for the cost function.

The remainder of this paper is arranged as follows. In Section 2, the control problem is briefly introduced and transformed. In Section 3, the main results are given, including the construction of CNN and the stability analysis of the system. Section 4 presents a practical example validate the theoretical results. And conclusion is listed in Section 5.

2. Preliminaries

Consider the nonlinear system with matched uncertainty described by

$$\dot{x}(t) = f(x(t)) + g(x(t))(u(t) + v(x(t))), x(0) = x_0, \tag{1}$$

where $x(t) \in \mathbb{R}^n$, $u(t) \in \mathbb{R}^m$ and $v(x(t)) \in \mathbb{R}^m$ are the state, input and uncertain perturbation, respectively. $f(x(t)) \in \mathbb{R}^n$ and $g(x(t)) \in \mathbb{R}^{n \times m}$ are known smooth continuous function, and x_0 is the initial state.

In this paper, we aim at finding an optimal feedback control $u(t)$ to stabilize the nonlinear system (1). However, for the control problem of the matched uncertain systems, the control signal is affected by uncertain nonlinear disturbance, which makes it difficult to design the controller. Therefore, we give two assumptions, which are helpful to transform the original system into a nominal system.

Assumption 1: *$f(x(t)) + g(x(t))u(t)$ is Lipschitz continuous on the set $\Omega \subset \mathbb{R}^n (0 \in \Omega)$. Moreover, $f(0) = 0$ and $v(0) = 0$.*

Assumption 2: *$g(x(t))$ is bounded, i.e., $\|g(x(t))\| \le d_g$, where $d_g > 0$ is a constant. Meanwhile, there exists a known bounded function $\Gamma(x(t)) \in \mathbb{R}^m$ such that $v^{\mathrm{T}}(x(t))\mathcal{R}v(x(t)) \le \Gamma^{\mathrm{T}}(x(t))\mathcal{R}\Gamma(x(t))$, where $\mathcal{R} \in \mathbb{R}^{m \times m}$ is a positive definite matrix and $\Gamma(0) = 0$.*

Consistent with works [9,21], we define the nominal system of system (1) as follows

$$\dot{x}(t) = f(x(t)) + g(x(t))u(t)\,, \tag{2}$$

where the definitions of $x(t)$, $f(x(t))$, $g(x(t))$ and $u(t)$ are the same as system (1). For simplicity, t is omitted for subsequent time-dependent variables.

Remark 1: *It is a common way to solve the robust control problem of the original system by studying the optimal control of the nominal system. It is worth mentioning that work [22] proves the equivalence of the conversion of such problems by condition $v^T(x)\mathcal{R}v(x) \le \Gamma^T(x)\mathcal{R}\Gamma(x)$ in Assumption 2. The literature [23] further shows that the optimal control strategy of nominal systems is effective for the original system. Inspired by these works, the same method is also used to deal with the SGPFS problem of the original system.*

For the optimal control problem of the system (2), we define the cost function as follows

$$\mathcal{J}(x) = \int_0^\infty (\mathcal{A}(x(t),u(t)) + \Gamma^T(x(t))\mathcal{R}\Gamma(x(t)))dt, \tag{3}$$

where $\mathcal{A}(x,u) = x^T\mathcal{Q}x + u^T\mathcal{R}u$, $\mathcal{Q} \in \mathbb{R}^{n\times n}$ is a positive definite matrix.

Further, one can get

$$\mathcal{J}^*(x) = \min_{u\in\Upsilon(\Omega)} \int_0^\infty (\mathcal{A}(x(t),u(t)) + \Gamma^T(x(t))\mathcal{R}\Gamma(x(t)))dt, \tag{4}$$

where $\mathcal{J}^*(x)$ denotes the optimal cost function, $\Upsilon(\Omega)$ indicates an admissible control domain.

When the cost function (3) is continuously differentiable, the Hamiltonian of system (2) can be defined as

$$\mathcal{H}(x,u,\nabla_x\mathcal{J}(x)) = \mathcal{A}(x,u) + \Gamma^{\mathrm{T}}(x)\mathcal{R}\Gamma(x) + (\nabla_x\mathcal{J}(x))^{\mathrm{T}}(f(x) + g(x)u)\,, \tag{5}$$

according to Bellman's optimality principle, the HJB equation can be formulated as

$$0 = \min_{u\in\Upsilon(\Omega)} \mathcal{H}(x,u,\nabla_x\mathcal{J}^*(x)). \tag{6}$$

Take the partial derivative of u^* from the right side of (6), we can get the optimal control policy:

$$u^* = \arg\min_{u\in\Upsilon(\Omega)} \mathcal{H}(x,u,\nabla_x\mathcal{J}^*(x)) = -\frac{1}{2}\mathcal{R}^{-1}g^{\mathrm{T}}(x)\nabla_x\mathcal{J}^*(x), \tag{7}$$

based on (4)-(7), we can rewrite the HJB equation as

$$0 = x^{\mathrm{T}}\mathcal{Q}x + \Gamma^{\mathrm{T}}(x)\mathcal{R}\Gamma(x) + (\nabla_x\mathcal{J}^*(x))^{\mathrm{T}}f(x) - \frac{1}{4}(\nabla_x\mathcal{J}^*(x))^{\mathrm{T}}g(x)\mathcal{R}^{-1}g^{\mathrm{T}}(x)\nabla_x\mathcal{J}^*(x) \tag{8}$$

with $\mathcal{J}^*(0) = 0$.

Therefore, in order to obtain the optimal robust controller of the original system, we need to solve $\nabla_x\mathcal{J}^*(x)$ from (8). And the FT stability of the original system is further studied. Firstly, we introduce the definition of the SGPFS as follows.

Definition 1[24]: *The equilibrium $x \equiv 0$ of the system (1) is SGPFS, if for $x(0) = x_0$, there exist a positive constant ϵ and a set time $t_f(\epsilon, x_0) < \infty$ such that $\|x(t)\| < \epsilon$ for all $t>t_f$.*

Lemma 1[25]: *For the system state variables a_1 and a_2, there exist any positive real numbers b_1, b_2 and b_3 such that*

$$|a_1|^{b_1}|a_2|^{b_2} \le \frac{b_1}{b_1+b_2}b_3|a_1|^{b_1+b_2} + \frac{b_2}{b_1+b_2}b_3^{\frac{-b_1}{b_2}}|a_2|^{b_1+b_2}\ . \tag{9}$$

Lemma 2[26]: *Let $c_i \ge 0$, $i = 1,2,\ldots,N_1$, and $0 < d \le 1$, one has*

$$\left(\sum_{i=1}^{N_1}|c_i|\right)^d \le \sum_{i=1}^{N_1}|c_i|^d \le N_1^{1-d}\left(\sum_{i=1}^{N_1}|c_i|\right)^d. \tag{10}$$

***Lemma 3*[27]*:** For the nonlinear system (1), suppose that there exist a continuously differentiable function* $\mathcal{V}(x)$*, real numbers* $\alpha > 0$*,* $\beta \in (0,1)$ *and* $\omega > 0$ *satisfying*

$$\dot{\mathcal{V}}(x) \le -\alpha(\mathcal{V}(x))^{\beta} + \omega, t \ge 0, \tag{11}$$

then, the zero solution $x \equiv 0$ *of the system (1) is SGPFS. And for any* $0 < \varsigma < 1$*, one has*

$$t_f = \frac{1}{\varsigma\alpha(1-\beta)}\left((\mathcal{V}(x(0)))^{1-\beta} - \left(\frac{\omega}{\alpha(1-\varsigma)}\right)^{\frac{1-\beta}{\beta}}\right). \tag{12}$$

***Remark 2:** Lemma 3 is a new FT stability criterion first proposed in [27]. This criterion makes it possible to design an optimal robust controller based on ADP and to analyze the stability of closed-loop systems with uncertain disturbances in the sense of FT stability.*

***Lemma 4:** There exists an upper bound function for* $\mathcal{J}^*(x)$*, i.e.,* $\dot{\mathcal{J}}^*(x) \le \eta(1+\theta\|x\|^2)$*, where* $0 < \theta < 1$ *is a constant that can be designed and* $\eta \triangleq \mathcal{J}^*(x_0)$ *is a positive constant.*

Proof. Let $\mathcal{J}^*(x)$ defined as in (4) be the Lyapunov function candidate. Based on (6), taking derivative along the trajectory of (2) yields

$$\dot{\mathcal{J}}^*(x) = (\nabla_x \mathcal{J}^*(x))^{\mathrm{T}}(f(x)+g(x)u^*) = -\mathcal{A}(x,u^*) - \Gamma^{\mathrm{T}}(x)\mathcal{R}\Gamma(x) \le 0, \tag{13}$$

we can easily get that $\mathcal{A}(x,u^*) + \Gamma^{\mathrm{T}}(x)\mathcal{R}\Gamma(x) = -\dot{\mathcal{J}}^*(x)$, Notice that (13) implies that the closed-loop system (2) is asymptotically stable, which means that $\lim_{t\to\infty} x(t) = 0$. Now, integrating both sides of (14) from 0 to $+\infty$ yields

$$\mathcal{J}^*(x) \le -(1+\theta\|x\|^2)\int_0^{\infty} \mathrm{d}\mathcal{J}^*(x(t)) = (1+\theta\|x\|^2)\mathcal{J}^*(x_0). \tag{14}$$

This completes the proof.

***Remark 3:** Lemma 4 constructs the inequality relationship between the optimal cost functions* $\mathcal{J}^*(x)$ *and* $\|x\|^2$*, which provides a new idea for using* $\mathcal{J}^*(x)$ *as a Lyapunov candidate function to study the FT stability of nonlinear systems.*

3. Main results

3.1. SGPFS for Nonlinear Systems

***Theorem 1:** Consider the nominal systems (2) with the optimal cost function (4). The optimal control policy is given by (7), if there are real numbers* $\alpha_1 > 0$*,* $\beta \in (0,1)$ *and* $\omega_1 > 0$ *such that (11) hold. Then the system (1) is SGPFS.*

Proof. Consider the Lyapunov function candidate $\mathcal{V}_1(x) = \mathcal{J}^*(x)$. For the difference of $\mathcal{V}_1(x)$ along the solution of $\dot{x} = f(x) + g(x)(u^* + v(x))$, we have

$$\begin{aligned}\dot{\mathcal{V}}_1(x) &= (\nabla_x \mathcal{J}^*(x))^{\mathrm{T}}(f(x)+g(x)(u^*+v(x))) \\ &= (\nabla_x \mathcal{J}^*(x))^{\mathrm{T}}(f(x)+g(x)u^*) + (\nabla_x \mathcal{J}^*(x))^{\mathrm{T}} g(x)v(x).\end{aligned} \tag{15}$$

According to the HJB equation (6) and the optimal controller (7), we find

$$(\nabla_x \mathcal{J}^*(x))^{\mathrm{T}}(f(x)+g(x)u^*) = -\mathcal{A}(x,u^*) - \Gamma^{\mathrm{T}}(x)\mathcal{R}\Gamma(x), \tag{16}$$

$$(\nabla_x \mathcal{J}^*(x))^{\mathrm{T}} g(x) = -2(u^*)^{\mathrm{T}}\mathcal{R}. \tag{17}$$

Together with (15), (16) and (17), we get

$$\begin{aligned}\dot{\mathcal{V}}_1(x) &= -\mathcal{A}(x,u^*) - \Gamma^{\mathrm{T}}(x)\mathcal{R}\Gamma(x) - 2(u^*)^{\mathrm{T}}\mathcal{R}v(x)\\ &= -x^{\mathrm{T}}\mathcal{Q}x - ((u^*)^{\mathrm{T}}\mathcal{R}u^* + 2(u^*)^{\mathrm{T}}\mathcal{R}v(x) + v^{\mathrm{T}}(x)\mathcal{R}v(x)) - (\Gamma^{\mathrm{T}}(x)\mathcal{R}\Gamma(x) - v^{\mathrm{T}}(x)\mathcal{R}v(x))\\ &\le -x^{\mathrm{T}}\mathcal{Q}x.\end{aligned} \tag{18}$$

Now, based on Lemma 1 and 4, let $a_1 = 1$, $a_2 = \eta^{-1}\theta^{-1}\lambda_{min}(\mathcal{Q})\mathcal{J}^*(x)$ and $b_1 = 1-\beta$, $b_2 = \beta$ and $b_3 = \beta^{\frac{\beta}{1-\beta}}$, we have

$$\begin{aligned}\dot{\mathcal{V}}_1(x) &\le -x^{\mathrm{T}}\mathcal{Q}x - \theta^{-1}\lambda_{min}(\mathcal{Q}) + \theta^{-1}\lambda_{min}(\mathcal{Q})\\ &\le -\theta^{-1}\lambda_{min}(\mathcal{Q})(\theta\|x\|^2 + 1) + \theta^{-1}\lambda_{min}(\mathcal{Q})\\ &\le -\eta^{-1}\theta^{-1}\lambda_{min}(\mathcal{Q})\mathcal{J}^*(x) + \theta^{-1}\lambda_{min}(\mathcal{Q}) - (\eta^{-1}\theta^{-1}\lambda_{min}(\mathcal{Q})\mathcal{J}^*(x))^{\beta} + (\eta^{-1}\theta^{-1}\lambda_{min}(\mathcal{Q})\mathcal{J}^*(x))^{\beta}\\ &\le -(\eta^{-1}\theta^{-1}\lambda_{min}(\mathcal{Q}))^{\beta}(\mathcal{J}^*(x))^{\beta} + \theta^{-1}\lambda_{min}(\mathcal{Q}) + (1-\beta)\beta^{\frac{\beta}{1-\beta}},\end{aligned} \tag{19}$$

for any $x \in \Omega$, we obtain

$$\dot{\mathcal{V}}_1(x) \le -\alpha_1(\mathcal{V}_1(x))^{\beta} + \omega_1, \tag{20}$$

where $\alpha_1 = (\eta^{-1}\theta^{-1}\lambda_{min}(\mathcal{Q}))^{\beta}$, $\omega_1 = \theta^{-1}\lambda_{min}(\mathcal{Q}) + (1-\beta)\beta^{\frac{\beta}{1-\beta}}$, and for any $0 < \varsigma < 1$,

the settling time is given by

$$t_f = \frac{1}{\varsigma\alpha_1(1-\beta)}\left[(\mathcal{V}(x_0))^{1-\beta} - \left(\frac{\omega_1}{\alpha(1-\varsigma)}\right)^{\frac{1-\beta}{\beta}}\right]. \tag{21}$$

From (20) hold, that is, the system (1) achieves the SGPFS for all $t > t_f$. The proof is completed.

3.2. Implementation of CNN

In order to avoid directly solving the HJB equation, similar to the works of literature [28], the cost function is approximated by a neural network. For the affine nonlinear system, we only need to construct a neural network with critic structure to achieve the design purpose of controller. For any $x \in \Omega$, we construct the CNN structure as follows

$$\mathcal{J}^*(x) = \mathcal{W}^{\mathrm{T}}\gamma(x) + \varepsilon(x), \tag{22}$$

where $\mathcal{W} \in \mathbb{R}^l$ and $\gamma(x) \in \mathbb{R}^l$ are the ideal weight and the activation function, respectively. l denotes the number of hidden neurons, and $\varepsilon(x)$ is the approximation error.

Taking the partial derivative of $\mathcal{J}^*(x)$, one has

$$\nabla_x\mathcal{J}^*(x) = \nabla_x\gamma^{\mathrm{T}}(x)\mathcal{W} + \nabla_x\varepsilon(x). \tag{23}$$

Based on (7) and (23), one can get

$$u^* = -\frac{1}{2}\mathcal{R}^{-1}g^{\mathrm{T}}(x)(\nabla_x\gamma^{\mathrm{T}}(x)\mathcal{W} + \nabla_x\varepsilon(x)). \tag{24}$$

Because the ideal CNN weight $\mathcal{W}$ is unknown, the control strategy is difficult to obtain. Therefore, we need to approximate the ideal weight:

$$\widehat{\mathcal{J}}(x) = \widehat{\mathcal{W}}^{\mathrm{T}}\gamma(x), \tag{25}$$

where $\widehat{\mathcal{W}}$ is the estimated value of $\mathcal{W}$.

The estimated value of optimal controller can be written as

$$\hat{u} = -\frac{1}{2}\mathcal{R}^{-1}g^{\mathrm{T}}(x)\nabla_x\gamma^{\mathrm{T}}(x)\widehat{\mathcal{W}}, \tag{26}$$

then, we can get the estimate of Hamiltonian as follows

$$\widehat{\mathcal{H}}(x,\hat{u},\nabla_x\widehat{\mathcal{J}}(x)) = \mathcal{A}(x,\hat{u}) + \Gamma^{\mathrm{T}}(x)\mathcal{R}\Gamma(x) + \nabla_x\widehat{\mathcal{J}}^{\mathrm{T}}(x)(f(x)+g(x)\hat{u}), \tag{27}$$

for $\mathcal{H}(x,u^*,\nabla_x\mathcal{J}^*(x)) = 0$, the approximation error of Hamiltonian can be given as

$$\xi = \widehat{\mathcal{H}}(x,\hat{u},\nabla_x\widehat{\mathcal{J}}(x)) - \mathcal{H}(x,u^*,\nabla_x\mathcal{J}^*(x)) = \widehat{\mathcal{H}}(x,\hat{u},\nabla_x\widehat{\mathcal{J}}(x)) - 0 = \widehat{\mathcal{H}}(x,\hat{u},\nabla_x\widehat{\mathcal{J}}(x)).$$

To keep ξ small enough, we need to train the weight $\widehat{\mathcal{W}}$ to converge to $\mathcal{W}$ by gradient descent method, and define the target function as $\zeta = \frac{1}{2}\xi^{\mathrm{T}}\xi$, the adaptive updating law of the weight can be designed as

$$\dot{\widehat{\mathcal{W}}} = -\kappa\frac{1}{(1+\sigma^{\mathrm{T}}\sigma)^2}\frac{\partial\zeta}{\partial\widehat{\mathcal{W}}}, \tag{28}$$

where κ is learning rate of the network, $1/(1+\sigma^{\mathrm{T}}\sigma)^2$ is the normalization factor and $\sigma = \nabla_x\gamma(x)(f(x)+g(x)\hat{u})$ .Together with (27) and (28), one can easily obtain

$$\dot{\widehat{\mathcal{W}}} = -\kappa\frac{1}{(1+\sigma^{\mathrm{T}}\sigma)^2}\widehat{\mathcal{H}}(x,\hat{u},\nabla_x\widehat{\mathcal{J}}(x))\sigma. \tag{29}$$

Generally, the persistent excitation (PE) [29] condition is considered in the weight adaptive update rate of CNN. From the initial time, in order to maintain the system active, a small noise signal is added to the control input, which ensures the convergence of the CNN weight. It is worth noting that the PE condition is relaxed by using the concurrent learning method. Now, we definite a stored data set $\{x(t)|_{t=t_s},\ s=1,2,...,N_2\}$, where $t_s \in [0,t_f]$, $N_2 \in \mathbb{N}$. The error of Hamiltonian at t_s can be written as

$$\xi(t_s) = \widehat{\mathcal{H}}(x(t_s),\hat{u}(t_s),\nabla_x\widehat{\mathcal{J}}(x(t_s))), \tag{30}$$

from (30) , the target function which includes the current data and the historical stored data is rewritten as

$$\zeta = \frac{1}{2}\left(\xi^{\mathrm{T}}\xi + \sum_{s=1}^{N_s}\xi^{\mathrm{T}}(t_s)\xi(t_s)\right). \tag{31}$$

The adaptive updating law of the weight can be redesigned as

$$\dot{\widehat{\mathcal{W}}} = -\frac{\widehat{\mathcal{H}}(x,\hat{u},\nabla_x\widehat{\mathcal{J}}(x))\sigma}{(1+\sigma^{\mathrm{T}}\sigma)^2}\kappa - \sum_{s=1}^{N_2}\frac{\widehat{\mathcal{H}}(x(t_s),\hat{u}(t_s),\nabla_x\widehat{\mathcal{J}}(x(t_s)))\sigma(t_s)}{(1+\sigma^{\mathrm{T}}(t_s)\sigma(t_s))^2}\kappa, \tag{32}$$

where $\sigma(t_s) = \nabla_x\gamma(x(t_s))(f(x(t_s)) + g(x(t_s))\hat{u}(t_s))$ .The weight estimation error of the network is defined as $\widetilde{\mathcal{W}} = \mathcal{W} - \widehat{\mathcal{W}}$ and noticing that $\widehat{\mathcal{W}}^{\mathrm{T}}\sigma = \sigma^{\mathrm{T}}\widehat{\mathcal{W}} \in \mathbb{R}$, we have

$$\begin{aligned}\dot{\widetilde{\mathcal{W}}} = {} & \kappa\frac{\sigma}{(1+\sigma^{\mathrm{T}}\sigma)^2}(-\sigma^{\mathrm{T}}\widetilde{\mathcal{W}} + \mathcal{A}(x,\hat{u}) + \Gamma^{\mathrm{T}}(x)\mathcal{R}\Gamma(x) + \mathcal{W}^{\mathrm{T}}\sigma) \\ & +\kappa\sum_{s=1}^{N_2}\frac{\sigma(t_s)}{(1+\sigma^{\mathrm{T}}(t_s)\sigma(t_s))^2}(-\sigma(t_s)^{\mathrm{T}}\widetilde{\mathcal{W}} + \mathcal{A}(x(t_s),\hat{u}(t_s)) + \Gamma^{\mathrm{T}}(x(t_s))\mathcal{R}\Gamma(x(t_s)) + \mathcal{W}^{\mathrm{T}}\sigma(t_s)).\end{aligned} \tag{33}$$

Remark 1: *For ensure the PE condition, the traditional CNN weight update rule is to add a small noise signal to the control input. However, the noise signal is not easy to design in practice. It is worth noting that the PE condition is replaced by a concurrent learning method using stored data set with adequate linearly independent elements, which can ensure the boundedness of* $\widehat{\mathcal{W}}$.

3.3. Stability Analysis

Assumption 3: *Assumed that the optimal control input* u^* *and its approximate control input* $\hat{u}$ *satisfy* $\|u^*-\hat{u}\|\le d_u$, *where* $d_u>0$ *is a constant.*

Assumption 4: $\nabla_x\gamma(x),\nabla_x\varepsilon(x)$ *are bounded, i.e.* , $\|\nabla_x\gamma(x)\|\le d_\gamma$, $\|\nabla_x\varepsilon(x)\|\le d_\varepsilon$, *where* $d_\gamma>0$ *and* $d_\varepsilon>0$ *are constants.*

Theorem 2: *Supposed that Assumptions 1-4 hold. Consider the system (1) with the controller (26), the critic network (25) and the weight update rule (33). Then, the SGPFS of the system is guaranteed provided that we make the following inequality hold:*

$$\lambda_{min}\left(\psi\psi^{\mathrm{T}}+\sum_{s=1}^{N_2}\psi(t_s)\psi^{\mathrm{T}}(t_s)\right)-d_g^2d_\gamma^2\|\mathcal{R}^{-\frac{1}{2}}\|^2>0 \tag{34}$$

with $\psi=\sigma/(1+\sigma^{\mathrm{T}}\sigma)$ *and* $\psi(t_s)=\sigma(t_s)/(1+\sigma^{\mathrm{T}}(t_s)\sigma(t_s))$.

Proof. Consider the Lyapunov function candidate

$$\mathcal{V}(x,\widetilde{\mathcal{W}})=\mathcal{V}_1(x)+\mathcal{V}_2(\widetilde{\mathcal{W}}), \tag{35}$$

where $\mathcal{V}_1(x)=\mathcal{J}^*(x)$, $\mathcal{V}_2(\widetilde{\mathcal{W}})=\frac{1}{2\kappa}\widetilde{\mathcal{W}}^{\mathrm{T}}\widetilde{\mathcal{W}}$.

Differentiating $\mathcal{V}_1(x)$ along the solution of $\dot{x}=f(x)+g(x)(\hat{u}+v(x))$, we have

$$\begin{aligned}\dot{\mathcal{V}}_1(x)&=(\nabla_x\mathcal{J}^*(x))^{\mathrm{T}}(f(x)+g(x)(\hat{u}+v(x)))\\&=(\nabla_x\mathcal{J}^*(x))^{\mathrm{T}}f(x)+(\nabla_x\mathcal{J}^*(x))^{\mathrm{T}}g(x)\hat{u}+(\nabla_x\mathcal{J}^*(x))^{\mathrm{T}}g(x)v(x),\end{aligned} \tag{36}$$

and according to (16) and (17), we get

$$\begin{aligned}\dot{\mathcal{V}}_1(x)&=-x^{\mathrm{T}}\mathcal{Q}x+(u^*)^{\mathrm{T}}\mathcal{R}u^*-\Gamma^{\mathrm{T}}(x)\mathcal{R}\Gamma(x)-2(u^*)^{\mathrm{T}}\mathcal{R}\hat{u}-2(u^*)^{\mathrm{T}}\mathcal{R}v(x)\\&=-x^{\mathrm{T}}\mathcal{Q}x+(u^*)^{\mathrm{T}}\mathcal{R}u^*-\hat{u}^{\mathrm{T}}\mathcal{R}\hat{u}-((u^*)^{\mathrm{T}}\mathcal{R}u^*+2(u^*)^{\mathrm{T}}\mathcal{R}v(x)+v^{\mathrm{T}}(x)\mathcal{R}v(x))\\&\quad+((u^*)^{\mathrm{T}}\mathcal{R}u^*-2(u^*)^{\mathrm{T}}\mathcal{R}\hat{u}+\hat{u}^{\mathrm{T}}\mathcal{R}\hat{u})-(\Gamma^{\mathrm{T}}(x)\mathcal{R}\Gamma(x)-v^{\mathrm{T}}(x)\mathcal{R}v(x))\\&\le-x^{\mathrm{T}}\mathcal{Q}x+(u^*)^{\mathrm{T}}\mathcal{R}u^*+(u^*-\hat{u})^{\mathrm{T}}\mathcal{R}(u^*-\hat{u}).\end{aligned} \tag{37}$$

Based on (24) and (26), using Assumption 4, it can be checked that

$$(u^*)^{\mathrm{T}}\mathcal{R}u^*\le\frac{1}{2}d_g^2d_\gamma^2\|\mathcal{R}^{-\frac{1}{2}}\|^2\|\mathcal{W}\|^2+\frac{1}{2}d_g^2d_\varepsilon^2\|\mathcal{R}^{-\frac{1}{2}}\|^2, \tag{38}$$

$$(u^*-\hat{u})^{\mathrm{T}}\mathcal{R}(u^*-\hat{u})\le\frac{1}{2}d_g^2d_\gamma^2\|\mathcal{R}^{-\frac{1}{2}}\|^2\|\widetilde{\mathcal{W}}\|^2+\frac{1}{2}d_g^2d_\varepsilon^2\|\mathcal{R}^{-\frac{1}{2}}\|^2, \tag{39}$$

we can find that (37) yields

$$\dot{\mathcal{V}}_1(x)\le-x^{\mathrm{T}}\mathcal{Q}x+\frac{1}{2}d_g^2d_\gamma^2\|\mathcal{R}^{-\frac{1}{2}}\|^2\|\widetilde{\mathcal{W}}\|^2+\Lambda_1, \tag{40}$$

where $\Lambda_1=\frac{1}{2}d_g^2d_\gamma^2\|\mathcal{R}^{-\frac{1}{2}}\|^2\|\mathcal{W}\|^2+d_g^2d_\varepsilon^2\|\mathcal{R}^{-\frac{1}{2}}\|^2$ .Further, the differential of $\mathcal{V}_2(\widetilde{\mathcal{W}})$ with respect to $\widetilde{\mathcal{W}}$ satisfies

$$\begin{aligned}\dot{\mathcal{V}}_2(\widetilde{\mathcal{W}})&=\frac{1}{\kappa}\widetilde{\mathcal{W}}^{\mathrm{T}}\dot{\widetilde{\mathcal{W}}}=\frac{\widetilde{\mathcal{W}}^{\mathrm{T}}\sigma}{(1+\sigma^{\mathrm{T}}\sigma)^2}(-\sigma^{\mathrm{T}}\widetilde{\mathcal{W}}+\mathcal{A}(x,\hat{u})+\Gamma^{\mathrm{T}}(x)\mathcal{R}\Gamma(x)+\mathcal{W}^{\mathrm{T}}\sigma)\\&\quad+\sum_{s=1}^{N_2}\frac{\widetilde{\mathcal{W}}^{\mathrm{T}}\sigma(t_s)}{(1+\sigma^{\mathrm{T}}(t_s)\sigma(t_s))^2}(-\sigma^{\mathrm{T}}(t_s)\widetilde{\mathcal{W}}+\mathcal{A}(x(t_s),\hat{u}(t_s))+\Gamma^{\mathrm{T}}(x(t_s))\mathcal{R}\Gamma(x(t_s))+\mathcal{W}^{\mathrm{T}}\sigma(t_s)).\end{aligned} \tag{41}$$

From Assumption 3, we have

$$\Lambda_2 \geq \mathcal{A}(x,\hat{u}) + \Gamma^{\mathrm{T}}(x)\mathcal{R}\Gamma(x) + \mathcal{W}^{\mathrm{T}}\sigma, \tag{42}$$

$$\Lambda_2(t_s) \geq \mathcal{A}(x(t_s),\hat{u}(t_s)) + \Gamma^{\mathrm{T}}(x(t_s))\mathcal{R}\Gamma(x(t_s)) + \mathcal{W}^{\mathrm{T}}\sigma(t_s), \tag{43}$$

where $\Lambda_2 = d_u^2 \| \mathcal{R}^{\frac{1}{2}} \|^2 - \nabla_x \varepsilon^{\mathrm{T}}(x)(f(x)+g(x)\hat{u})$, $\Lambda_2(t_s) = d_u^2 \| \mathcal{R}^{\frac{1}{2}} \|^2 - \nabla_x \varepsilon^{\mathrm{T}}(x(t_s))(f(x(t_s))+g(x(t_s))\hat{u}(t_s))$ .Combining (42) and (43), noticing that $1+\sigma^{\mathrm{T}}\sigma \geq 1$ and $1+\sigma^{\mathrm{T}}(t_s)\sigma(t_s) \geq 1$, and based on Young's inequality $\rho^{\mathrm{T}}\varrho \leq \frac{1}{2}\rho^{\mathrm{T}}\rho + \frac{1}{2}\varrho^{\mathrm{T}}\varrho$, one can obtain

$$\begin{aligned}
\dot{\mathcal{V}}_2(\widetilde{\mathcal{W}}) &\leq \frac{\widetilde{\mathcal{W}}^{\mathrm{T}}\sigma}{(1+\sigma^{\mathrm{T}}\sigma)^2}(-\sigma^{\mathrm{T}}\widetilde{\mathcal{W}} + \Lambda_2) - \sum_{s=1}^{N_2} \frac{\widetilde{\mathcal{W}}^{\mathrm{T}}\sigma(t_s)}{(1+\sigma^{\mathrm{T}}(t_s)\sigma(t_s))^2}(\sigma^{\mathrm{T}}(t_s)\widetilde{\mathcal{W}} - \Lambda_2(t_s)) \\
&\leq \frac{1}{(1+\sigma^{\mathrm{T}}\sigma)^2}\left(\frac{1}{2}\Lambda_2^2 - \frac{1}{2}\widetilde{\mathcal{W}}^{\mathrm{T}}\sigma\sigma^{\mathrm{T}}\widetilde{\mathcal{W}}\right) + \frac{1}{2}\sum_{s=1}^{N_2}\frac{1}{(1+\sigma^{\mathrm{T}}(t_s)\sigma(t_s))^2}(\Lambda_2^2(t_s) - \widetilde{\mathcal{W}}^{\mathrm{T}}\sigma(t_s)\sigma^{\mathrm{T}}(t_s)\widetilde{\mathcal{W}}) \\
&\leq -\frac{1}{2}\widetilde{\mathcal{W}}^{\mathrm{T}}\left(\psi\psi^{\mathrm{T}} + \sum_{s=1}^{N_2}\psi(t_s)\psi^{\mathrm{T}}(t_s)\right)\widetilde{\mathcal{W}} + \frac{1}{2}\Lambda_2^2 + \frac{1}{2}\sum_{s=1}^{N_2}\Lambda_2^2(t_s),
\end{aligned}$$

supposed that $\Lambda_2, \Lambda_2(t_s) \leq d_{\Lambda_2}$, where $d_{\Lambda_2} > 0$ is a constant, one has

$$\dot{\mathcal{V}}_2(\widetilde{\mathcal{W}}) \leq -\frac{1}{2}\lambda_{min}\left(\psi\psi^{\mathrm{T}} + \sum_{s=1}^{N_2}\psi(t_s)\psi^{\mathrm{T}}(t_s)\right)\|\widetilde{\mathcal{W}}\|^2 + \frac{1}{2}(N_2+1)d_{\Lambda_2}^2. \tag{44}$$

Together with (40) and (44), we derive

$$\begin{aligned}
\dot{\mathcal{V}}(x,\widetilde{\mathcal{W}}) &\leq -x^{\mathrm{T}}\mathcal{Q}x - \frac{1}{2}\left(\lambda_{min}(\psi\psi^{\mathrm{T}} + \sum_{s=1}^{N_2}\psi(t_s)\psi^{\mathrm{T}}(t_s)\right) - d_g^2 d_\gamma^2 \| \mathcal{R}^{-\frac{1}{2}} \|^2)\|\widetilde{\mathcal{W}}\|^2 + \Lambda_1 + \frac{1}{2}(N_2+1)d_{\Lambda_2}^2 \\
&= -x^{\mathrm{T}}\mathcal{Q}x - (\kappa\lambda_{min}(\psi\psi^{\mathrm{T}} + \sum_{s=1}^{N_2}\psi(t_s)\psi^{\mathrm{T}}(t_s)) - \kappa d_g^2 d_\gamma^2 \| \mathcal{R}^{-\frac{1}{2}} \|^2)\left(\frac{1}{2\kappa}\widetilde{\mathcal{W}}^{\mathrm{T}}\widetilde{\mathcal{W}}\right) + \Lambda_1 + \frac{1}{2}(N_2+1)d_{\Lambda_2}^2.
\end{aligned}$$

Based on Lemma 1 and Assumption 3, we get

$$\begin{aligned}
\dot{\mathcal{V}}(x,\widetilde{\mathcal{W}}) &\leq -\theta^{-1}\lambda_{min}(\mathcal{Q})(\theta\| x\|^2 + 1) - \Theta_1\left(\frac{1}{2\kappa}\widetilde{\mathcal{W}}^{\mathrm{T}}\widetilde{\mathcal{W}}\right) + \Theta_2 \\
&\leq -\eta^{-1}\theta^{-1}\lambda_{min}(\mathcal{Q})\mathcal{J}^*(x) - (\eta^{-1}\theta^{-1}\lambda_{min}(\mathcal{Q})\mathcal{J}^*(x))^\beta - \Theta_1\left(\frac{1}{2\kappa}\widetilde{\mathcal{W}}^{\mathrm{T}}\widetilde{\mathcal{W}}\right) \\
&\quad -\left(\Theta_1\left(\frac{1}{2\kappa}\widetilde{\mathcal{W}}^{\mathrm{T}}\widetilde{\mathcal{W}}\right)\right)^\beta + \left(\Theta_1\left(\frac{1}{2\kappa}\widetilde{\mathcal{W}}^{\mathrm{T}}\widetilde{\mathcal{W}}\right)\right)^\beta + (\eta^{-1}\theta^{-1}\lambda_{min}(\mathcal{Q})\mathcal{J}^*(x))^\beta + \Theta_2 \\
&\leq -(\eta^{-1}\theta^{-1}\lambda_{min}(\mathcal{Q})\mathcal{J}^*(x))^\beta + \Theta_2 + 2(1-\beta)\beta^{\frac{\beta}{1-\beta}} - \left(\Theta_1\left(\frac{1}{2\kappa}\widetilde{\mathcal{W}}^{\mathrm{T}}\widetilde{\mathcal{W}}\right)\right)^\beta \\
&\leq -\alpha_1(\mathcal{J}^*(x))^\beta - \Theta_1^\beta\left(\frac{1}{2\kappa}\widetilde{\mathcal{W}}^{\mathrm{T}}\widetilde{\mathcal{W}}\right)^\beta + \Theta_2 + 2(1-\beta)\beta^{\frac{\beta}{1-\beta}},
\end{aligned}$$

where $\Theta_1 = \kappa\lambda_{min}(\psi\psi^{\mathrm{T}} + \sum_{s=1}^{N_2}\psi(t_s)\psi^{\mathrm{T}}(t_s)) - \kappa d_g^2 d_\gamma^2 \| \mathcal{R}^{-\frac{1}{2}} \|^2$, $\Theta_2 = \Lambda_1 + \frac{1}{2}(N_2+1)d_{\Lambda_2}^2 + \theta^{-1}\lambda_{min}(\mathcal{Q})$.

Using Lemma 2, one can obtain

$$\dot{\mathcal{V}}(x,\widetilde{\mathcal{W}}) \leq -\alpha_2(\mathcal{V}(x,\widetilde{\mathcal{W}}))^\beta + \omega_2, \tag{45}$$

where $\alpha_2 = \min\{\alpha_1, \Theta_1^\beta\}$, $\omega_2 = \Theta_2 + 2(1-\beta)\beta^{\frac{\beta}{1-\beta}}$.

From Lemma 3, for any $0 < \varsigma < 1$, one has

$$t_f = \frac{1}{\varsigma\alpha_2(1-\beta)}\left((\mathcal{V}(x_0,\widetilde{\mathcal{W}}(0)))^{1-\beta} - \left(\frac{\omega_2}{\alpha_2(1-\varsigma)} \right)^{\frac{1-\beta}{\beta}} \right). \tag{46}$$

Hence, for $\forall t > t_f$, the trajectory of the closed-loop system is SGPFS. The proof is completed.

4. Numerical simulations

In this section, a practical example is given to verify the effectiveness of the proposed control method.

Example: *The rotational-translational actuator system*

Based on the inverted pendulum model, a translational oscillator model with rotary actuator is composed of a spring, a trolley and a ball [30]. The trolley is connected with the spring as a displacement oscillator, and the other end of the spring is fixed on the wall. The ball swings at the end of the swing rod of the inverted pendulum device. There is a coupling relationship between the translational motion of the trolley and the swing of the ball. Therefore, when the ball swings under the action of torque, the trolley also moves back and forth on the horizontal plane. Then, the dynamics of rotational-translational actuator system is denoted as

$$\dot{x} = \begin{bmatrix} x_2 \\ \dfrac{hx_4^2\sin(x_3) - x_1}{1-h^2\cos^2(x_3)} \\ x_4 \\ \dfrac{hx_1\cos(x_3) - h^2x_4^2\sin(x_3)\cos(x_3)}{1-h^2\cos^2(x_3)} \end{bmatrix} + \begin{bmatrix} 0 \\ \dfrac{-h\cos(x_3)}{1-h^2\cos^2(x_3)} \\ 0 \\ \dfrac{1}{1-h^2\cos^2(x_3)} \end{bmatrix}(u+v(x)),$$

where $x = [x_1,x_2,x_3,x_4]^{\mathrm{T}} \in \mathbb{R}^4$, $u \in \mathbb{R}$ and the uncertain disturbance term $v(x) = \hbar\sin(0.2x_3)$ and $\hbar$ randomly selected in $[0,1]$. In the system state vector, x_1 and x_2 are the standardized moving position and speed of the trolley respectively, x_3 and x_4 are the angular position and speed of the ball respectively. h represents the auxiliary parameter of the coupling relationship between the translational oscillation of the trolley and the rotational motion of the ball, and

$$h \triangleq \frac{M_bL}{\sqrt{(M_b+M_c)(J+M_bL^2)}} = 0.5(i.e., M_b = M_c = 1kg, L = 0.2m, J = 0.04kg\cdot m^2),$$

where M_b, M_c represent the mass of the ball and the trolley, respectively. L represents the length of the swing rod and J represents the moment of inertia of the ball about its center of mass. Let $\Gamma(x) = \| x\|$, $\mathcal{Q} = diag\{0.1,0.1,0.1,0.1\}$ and $\mathcal{R} = 1$, we have $\mathcal{J}(x) = \int_0^\infty (1.1\|x(t)\|^2 + \|u(t)\|^2)\mathrm{d}t.$

In this example, $N_2 = 5$, and we design the appropriate basis function vector $\gamma(x) = [x_1^2, x_1x_2, x_1x_3, x_1x_4, x_2^2, x_2x_3, x_2x_4, x_3^2, x_3x_4, x_4^2]^{\mathrm{T}}$ .The initial state is designed as $x_0 = [0.4,0,0.1,0]^{\mathrm{T}}$, the learning rate is selected as $\kappa = 0.9$, and the initial weights $\widehat{\mathcal{W}}^{initial}$ are randomly chosen in $[0,\ 1.8]$.

It is shown in Figure 4 that the adaptive weights of CNN converge to stable value. Based on (30), we get the change of control input in Figure 2. Further, in Figure 1, we can see that the system state converges to the equilibrium point in a finite time under the controller. And the evolution of the cost function is shown in Figure 3, similarly, we can see that the cost function is always less than or equal to the designed function.

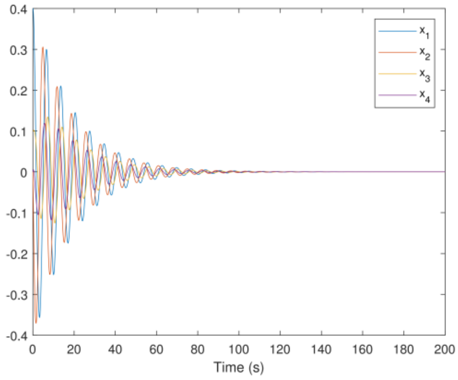

Figure 1: The system states

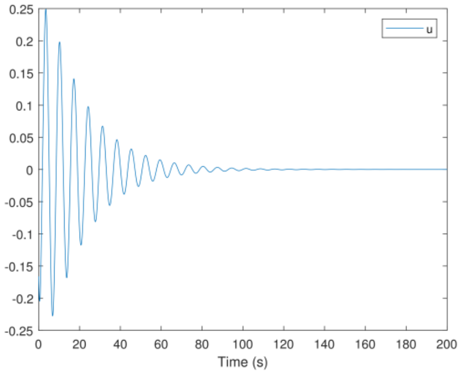

Figure 2: The control input

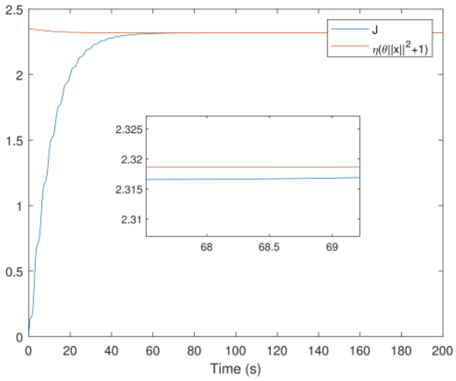

Figure 3: The cost function

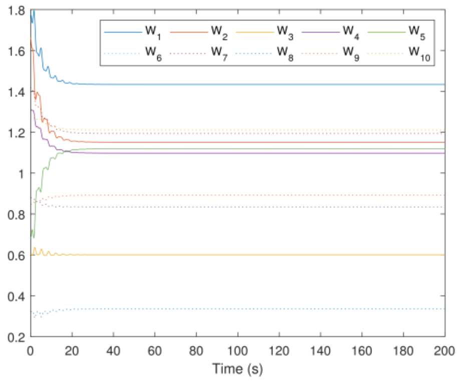

Figure 4: The CNN weights

5. Conclusions

In this paper, FT stabilization of a class of nonlinear systems with matched uncertainty via ADP-based is studied. Firstly, the control problem of the original uncertain system is transformed into the robust optimal control problem of the nominal system, and a modified cost function is defined to deal with the uncertainty. The unknown cost is approximated by a ADP method. Through the FT stability theory, it is proved that the optimal control strategy of the nominal system can make the original system signal converge to a small field of the origin in a finite time. The simulation results shown that the proposed optimal control strategies are corrected. Unfortunately, the control method in this paper can only make the system semi-globally stable in finite time. Therefore, the global FT stability of nonlinear systems based on ADP-based method will be considered in future research.

Acknowledgements

The authors would like to thank the editor and the anonymous reviewers for their valuable comments and constructive suggestions. This paper was supported by the self-initiated scientific research projects of Sichuan Minzu College (Grant No.XYZB2510ZB).

References

[1] Feng L, Robert D B, Jing S. Robust control of nonlinear systems: compensating for uncertainty[J]. International Journal of Control, 1992, 56(6): 1453-1459.

[2] Gahinet P, Nemirovskii A, Laub A J, et al. The LMI control toolbox[C]. Proceedings of 1994 33rd IEEE Conference on Decision and Control, 1994, 3: 2038-2041.

[3] Werbos P J. Approximate dynamic programming for real-time control and neural modeling[J]. Handbook of Intelligent Control Neural Fuzzy & Adaptive Approaches, 1992.

[4] Lewis F L, Vrabie D. Reinforcement learning and adaptive dynamic programming for feedback control [J]. IEEE Circuits and Systems Magazine, 2009, 9(3): 32-50.

[5] Shen M, Wang X, Zhu S, et al. Data-driven event-triggered adaptive dynamic programming control for nonlinear systems with input saturation[J]. IEEE Transactions on Cybernetics, 2024, 54(2): 1178-1188.

[6] Xue S, Zhao N, Zhang W, et al. A hybrid adaptive dynamic programming for optimal tracking control of USVs[J]. IEEE Transactions on Neural Networks and Learning Systems, 2025, 36(6): 9961-9969.

[7] Sun T, Sun X. An adaptive dynamic programming scheme for nonlinear optimal control with unknown dynamics and its application to turbofan engines[J]. IEEE Transactions on Industrial Informatics, 2021, 17(1): 367-376.

[8] Zhao Fu, Gao W, Jiang Z, et al. Event-triggered adaptive optimal control with output feedback: An adaptive dynamic programming approach[J]. IEEE Transactions on Neural Networks and Learning Systems, 2021, 32(11): 5208-5221.

[9] Zhao J, Na J, Gao G. Robust tracking control of uncertain nonlinear systems with adaptive dynamic programming[J]. Neurocomputing, 2022, 471: 21-30.

[10] Li C, Ding J, Lewis F L, et al. A novel adaptive dynamic programming based on tracking error for nonlinear discrete-time systems[J]. Automatica, 2021, 129: 109687.

[11] Ryan E P. Finite-time stabilization of uncertain nonlinear planar systems[J]. Dynamics & Control, 1991, 1(1): 83-94.

[12] Bhat S P, Bernstein D S. Continuous finite-time stabilization of the translational and rotational double integrators[J]. IEEE Transactions on Automatic Control, 1998, 43(5): 678-682.

[13] Li S, Wu Q. Finite-time control algorithm based on modal state observer for flexible satellite attitude tracking[J]. IEEE Access, 2020, 8: 184722-184730.

[14] Pang, Hongbo and Liu, Shuo. Robust finite time passivity and stabilization of uncertain switched nonlinear system[J]. IEEE Access,2021, 9:36173-36180.

[15] Liu K, Tang W, Sheng Y, et al. Event-triggered finite-time stabilization of delayed ts fuzzy systems on time scales[J]. IEEE Transactions on Cybernetics, 2025, 55(8): 3737-3746.

[16] Wang F, Jin N, Liu D, et al. Adaptive dynamic programming for finite-horizon optimal control of discrete-time nonlinear systems withε-error bound[J]. IEEE Transactions on Neural Networks, 2011, 22(1): 24-36.

[17] Heydari A, Balakrishnan S N. Finite-horizon control-constrained nonlinear optimal control using single network adaptive critics[J]. IEEE Trans Neural Netw Learn Syst, 2013, 24(1): 145-157.

[18] Xue S, Luo B, Liu D. Event-triggered adaptive dynamic programming for unmatched uncertain nonlinear continuous-time systems[J]. IEEE Transactions on Neural Networks and Learning Systems, 2021, 32(7): 2939-2951.

[19] Zhang K, Zhang Z, Xie X, et al. An unknown multiplayer nonzero-sum game: Prescribed-time dynamic event-triggered control via adaptive dynamic programming[J]. IEEE Transactions on Automation Science and Engineering, 2025, 22: 8317-8328.

[20] Yang X, Meng J, Zhang S, et al. Decentralized event-sampled control of multi-unit power systems via adaptive dynamic programming[J]. IEEE Transactions on Automation Science and Engineering, 2026, 23: 1617-1630.

[21] Jun Z, Jing N, Guanbin G. Adaptive dynamic programming based robust control of nonlinear systems with unmatched uncertainties[J]. Neurocomputing, 2020, 395: 56-65.

[22] Liu D, Wei Q, Wang D, et al. Adaptive dynamic programming with applications in optimal control [J]. Advances in Industrial Control, 2017, 9: 345-360.

[23] Wang D, Liu D, Li H, et al. Neural-network-based robust optimal control design for a class of uncertain nonlinear systems via adaptive dynamic programming[J]. Information Sciences, 2014, 282: 167-179.

[24] Zheng Z, Xia Y, Fu M. Attitude stabilization of rigid spacecraft with finite-time convergence[J]. International Journal of Robust and Nonlinear Control, 2011, 21(6): 686-702.

[25] Huang X, Wei L, Bo Y. Global finite-time stabilization of a class of uncertain nonlinear systems[J]. Automatica, 2005, 41(5): 881-888.

[26] Zuo Z, Tie Lin. Distributed robust finite-time nonlinear consensus protocols for multi-agent systems [J]. International Journal of Systems Science, 2014, 47(6): 1-10.

[27] Sun Y, Chen B, Lin C, et al. Finite-time adaptive control for a class of nonlinear systems with nonstrict feedback structure[J]. IEEE Transactions on Cybernetics, 2017, 1-9.

[28] Xue S, Luo B, Liu D, et al. Constrained event-triggered H_∞ control based on adaptive dynamic programming with concurrent learning[J]. IEEE Transactions on Systems, Man, and Cybernetics: Systems, 2022, 52(1): 357-369.

[29] Gu H, Mei J. Optimal sampling control of nonlinear systems based on adaptive dynamic programming[C]. 2021 8th International Conference on Information, Cybernetics, and Computational Social Systems (ICCSS), 2021, 429-434.

[30] Xue S, Luo B, Liu D. Integral reinforcement learning based event-triggered control with input saturation [J]. Neural Networks, 2020, 131: 144-153.