ISSN 2616-5805 Vol. 6, Issue 2: 33-40, DOI: 10.25236/AJMS.2025.060205

Steady-State Properties of Forest Growth Model under the Influence of Correlation Noise

Fu Yan[1,a], Pan Xilin[1,b], Xie Shu[1,c], Zheng Yihan[1,d], Peng Yuxin[1,e], Yang Yi[1,f], Wang Guowei[2,g,*]

[1]*School of Mathematics and Computer Science, Yuzhang Normal University, Nanchang, China*
[2]*School of Education, Nanchang Institute of Science and Technology, Nanchang, China*
[a]*fuyan@yuznu.edu.cn*, [b]*panxilin@yuznu.edu.cn*, [c]*xieshu@yuznu.edu.cn*, [d]*zhengyihan@yuznu.edu.cn*, [e]*pengyuxinuyan@yuznu.edu.cn*, [f]*yangyi@yuznu.edu.cn*, [g]*wanggw@ncpu.edu.cn*
**Corresponding author*

Abstract: *A non-deterministic Langevin equation is obtained by introducing multiplicative and additive Gaussian white noise in the Logistic model of tree growth, and an approximate Fokker-Planck equation is derived through calculations by using the Liouville equation, Novikov's theorem, and nonlinear approximation. The equation is solved under steady-state conditions, and the impact of noise-related parameters on the steady-state probability distribution function is systematically discussed. Obtained results show that changing the multiplicative white noise intensity D and additive white noise intensity Q can lead to the change of peak height and peak position of steady-state probability distribution curve, and have a drift effect on the probability density distribution. However, the change of the value of the steady-state probability distribution curve and the direction of the peak position are different in the process of increasing D and Q. The height of the peak becomes higher as multiplicative noise intensity D increases, but the width of the peak becomes narrower and the position of the peak shifts to the left. When additive noise intensity Q increases, the peak height decreases, but the width of the peak increases and the position of the peak shifts to the right. In addition, when noise correlation strength* $\lambda>0$*, it is found that there are no two peaks in the image, which does not conform to the regularity of tree growth, so we only discuss the case of noise correlation strength* $\lambda<0$*. With the increase of noise correlation strength* λ*, the peak height of steady-state probability distribution function shows a decreasing trend, accompanied by the phenomenon of increasing peak width and right shift of peak position.*

Keywords: *Trees Grow Model; Logistic Model; White Noise; Steady-State Probability Distribution*

1. Introduction

Forests are a vast natural resource for humanity, serving multiple functions and contributing significant economic, social, and ecological benefits to human society. However, due to human activities and natural factors, tree growth is subject to numerous constraints [1]. This makes it difficult for people to accurately grasp the laws of forest growth. Scientists in various fields have attempted to use their professional knowledge to change this situation. Thus, many articles have emerged documenting outstanding research findings, among which the topic of noise and its correlation affecting nonlinear systems has sparked extensive discussions [2]. The growth and change process of forest trees is actually a complex nonlinear dynamic process.

Therefore, many researchers have considered the impact of noise and its correlation on the growth process of forest trees. Chinese scholars such as Deng Hongbing *et al*. studied the growth model of single red pine trees, confirming that the growth of forest trees, represented by the height of individual red pine trees, is a complex nonlinear process [3]. Botany *et al*. explored the adverse effects of noise pollution on plants, noting that leaves exposed to prolonged noise exposure increase the secretion of two compounds, which reflect stress in the plant [4]. Additionally, noise reduces hormone levels related to healthy growth within plants; Zhou Xiaoting *et al*. investigated the noise reduction characteristics and influencing factors of 10 garden tree species before and after leaf fall, indirectly reflecting the impact of noisy environments on plant growth and morphology [5]. Clint Francis *et al*. examined the persistent effects of noise pollution on plant diversity in ecosystems, finding that human-generated noise pollution has a lasting impact on plant diversity in ecosystems, even after noise is removed [6]. Fulinski considered the correlation state between noises and found that in certain cases, there is a certain form of correlation between noises, and

ISSN 2616-5805 Vol. 6, Issue 2: 33-40, DOI: 10.25236/AJMS.2025.060205

this correlation state has a significant impact on the state of nonlinear systems [7]. After these research findings emerged, Wang Guowei *et al*. investigated the statistical properties of bi-stable systems and collective populations under noise effects, using the Langevin equation to explore the statistical properties of bi-stable systems, collective populations, and forest growth Logistic models driven by noise [8]. Xu Dahai *et al*. investigated the impact of color-related noise on the Logistic growth model of trees, revealing the mechanisms by which noise affects the growth process of trees [9]. These studies have confirmed that noise and its associated factors significantly influence the complex nonlinear dynamics of tree growth. The effects are not only reflected in the trees' reproductive capacity, growth rate, and density but also in photosynthesis and respiration within the tree, as well as indirectly through changes in animal behavior and adjustments in plant community structure. Therefore, under the simultaneous influence of multiple factors, whether these conditions promote or inhibit tree growth requires specific analysis [10].

This paper is based on the nonlinear growth of trees. By using the tree growth Logistic model, a more realistic one-dimensional Langmuir equation is derived. External and internal factors affecting tree growth are introduced into the model as multiplicative and additive noise, respectively. The impact of these two types of noise on tree growth is considered under different color correlation strengths and color correlation times. Using linear approximation methods and the steepest descent method, we derive the steady-state probability distribution function of the system. By observing changes in two noise intensities and the color correlation strength and duration between noises, we explore under what external conditions the growth needs of trees can be better met. We aim to study the growth of trees in such a complex environment to provide more realistic theories for ecological afforestation. Based on this theory, reasonable and effective planting plans can be established for ecological afforestation.

2. Models and methods

Logistic Equation is widely used in ecology, economics, demography and other fields to describe the change of population size. At the same time, Logistic equation is also the most commonly used model to simulate population dynamics in ecology.

2.1 Tree growth Logistic model

The growth of trees satisfies the equation as follows:

$$\frac{1}{x}\frac{dx}{dt}=r-\frac{r}{A}x. \tag{1}$$

Where r is the maximum growth rate of trees, A is the maximum growth parameter of trees, x is the size of trees, r/A and is the crowding effect coefficient. This is an ideal equation.

Through further literature review and group discussions, we found that we had overlooked the influence of genetic factors on tree growth and the limited nutritional space for trees within a forest stand (the existence of competition). The external environment, such as light, temperature, soil, and rainfall, is complex and variable. As a result, the maximum growth rate will inevitably fluctuate with the environment. The growth rate of trees will vary around the initial differential equation, then the initial differential equation will be revised. The revised equation is as follows:

$$\frac{dx}{dt}=rx-\frac{r}{A}x^2+\left(x-\frac{1}{A}x^2\right)\xi(t)+\eta(t), \tag{2}$$

Where $\xi(t)$ and $\eta(t)$ are multiplicative and additive Gaussian white noise respectively, and they have the following statistical properties:

$$\langle\xi(t)\rangle=\langle\eta(t)\rangle=0, \tag{3}$$

$$\left\langle\xi(t)\xi\left(t'\right)\right\rangle=2D\delta\left(t-t'\right), \tag{4}$$

$$\left\langle\eta(t)\eta\left(t'\right)\right\rangle=2Q\delta\left(t-t'\right), \tag{5}$$

ISSN 2616-5805 Vol. 6, Issue 2: 33-40, DOI: 10.25236/AJMS.2025.060205

$$\left\langle \xi(t)\eta\left(t'\right)\right\rangle = \left\langle \eta(t)\xi\left(t'\right)\right\rangle = \frac{\lambda\sqrt{DQ}}{\tau} exp\left(-\frac{\left|t-t'\right|}{\tau}\right). \tag{6}$$

Where D and Q are the intensity of multiplicative and additive Gaussian white noise respectively; λ is the correlation strength between the two noises, when $-1<\lambda<0$, the two noises are negatively correlated, when $0<\lambda<1$, the two noises are positively correlated; τ is the correlation time between cross-correlation white noise; t and t' are two different moments respectively.

The potential function of x in equation (2) is as follows:

$$V(x) = -\frac{r}{2}x^2 + \frac{r}{3A}x^3. \tag{7}$$

It has an unstable state $x_u = 0$ and a steady-state $x_s = A$.

2.2 Steady-state probability distribution function

In order to obtain the evolution equation of the probability distribution function satisfying equation (2), the substitution is made as the following:

$$f(x) = rx - \frac{r}{A}x^2, \tag{8}$$

$$g_1(x) = x - \frac{x^2}{A}, \tag{9}$$

$$g_2 = 1. \tag{10}$$

Then, the above Langevin equation (2) can be transformed into as the following:

$$\frac{dx}{dt} = f(x) + g_1(x)\xi(t) + g_2(x)\eta(t). \tag{11}$$

According to Liouville equation, we get:

$$\frac{\partial \rho(x,t)}{\partial t} = -\frac{\partial}{\partial x}\left[f(x) + g_1(x)\xi(t) + g_2(x)\eta(t)\right]\times \rho(x,t). \tag{12}$$

To average the above formula together, and:

$$P(x,t) = \langle \delta(x(t)-x)\rangle, \tag{13}$$

The following can be obtained:

$$\frac{\partial P(x,t)}{\partial t} = -\frac{\partial}{\partial x} f(x)P(x,t) - \frac{\partial}{\partial x} g_1(x)\langle \xi(t)\delta(x(\mathrm{t})-x)\rangle - \frac{\partial}{\partial x} g_2(x)\langle \eta(t)\delta(x(t)-x)\rangle. \tag{14}$$

According to Novikov's theorem and unified white noise approximation, the approximate Fokker-Planck equation satisfied by the probability distribution density function can be obtained through calculation:

$$\frac{\partial P(\mathrm{x},t)}{\partial t} = -\frac{\partial}{\partial x}A(x)P(x,t) + \frac{\partial^2}{\partial x^2}B(x)P(x,t), \tag{15}$$

Where,

$$A(x) = r\left(x - \frac{x^2}{A}\right) + D\left(x - \frac{x^2}{A}\right)\left(1 - \frac{2x}{A}\right) + \lambda\sqrt{DQ}\left(1 - \frac{2x}{A}\right), \tag{16}$$

ISSN 2616-5805 Vol. 6, Issue 2: 33-40, DOI: 10.25236/AJMS.2025.060205

$$B(x)=D\left(x-\frac{x^2}{A}\right)^2+2\lambda\sqrt{DQ}\left(x-\frac{x^2}{A}\right)+Q. \tag{17}$$

Equation (15) is solved in the steady-state, and its steady-state probability distribution function can be obtained by using the expressions of $A(x)$ and $B(x)$.

$$P_{st}(x)=\frac{N}{B(x)}exp\left(\int\frac{f(x)}{B(x)}dx\right). \tag{18}$$

Where, $U(x)=-\int\frac{f(x)}{B(x)}dx$ is the generalized potential, N is the steady-state normalized constant probability distribution, the value of N is determined by $\int_{-\infty}^{+\infty}P_{st}(x)dx=1$. By substituting the expression of $f(x)$ and $B(x)$ into formula (18), the influence of different parameters on the probability distribution function can be analyzed.

3. Discussion and Results

Figure 1 shows the effect of different multiplicative white noise intensities D on the steady-state probability distribution curves.As can be seen from Figure 1, when the intensity of fixed multiplicative white noise is fixed, the curve variation of steady-state probability distribution function is divided into four different stages. The steady-state probability distribution function decreases first, then goes through a period of stability, then increases rapidly to reach a peak, and finally decreases quickly and tends to equilibrium. This "fast-slow-fast-slow" pattern means that seeds need to cross a critical density (such as the minimum resources required for germination).

Above results indicate that the intensity of multiplicative white noise may reduce seed survival rates. Surviving seeds will first store resources, entering the first stable phase. After storing sufficient resources, the seeds will begin to germinate and enter a rapid growth stage, with the sapling growth rate reaching its maximum. Subsequently, the growth rate will gradually slow down, transitioning into the hardening period, thus entering another lower stable phase, which aligns with the growth trend of trees.

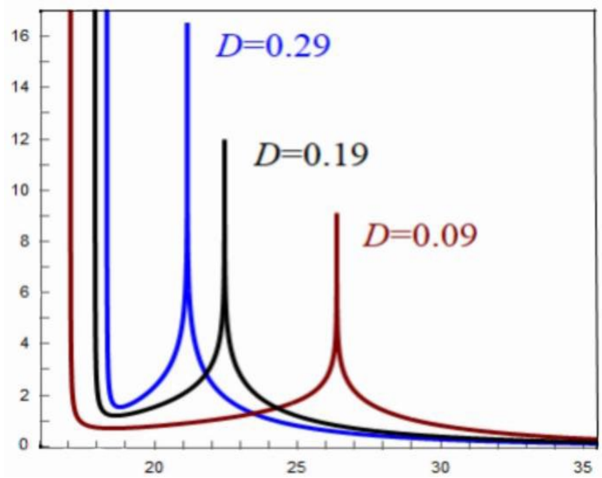

Figure 1 Effect of the multiplicative white noise intensity D on the steady-state probability distribution curve (r=1, A=20, Q=1.53, λ=-0.94)

As D increases, the steady-state probability distribution function grows at a faster rate to a larger value and approaches the first stationary phase. Moreover, the larger D is, the earlier this stationary phase ends, followed by a rapid rise and reaching its peak. The height of the peak increases with D, but the width narrows and the position shifts leftward. After the peak, the larger D results in a steeper decline, ultimately stabilizing into another lower stationary phase.

Figure 1 shows that in the first stationary period, the trees experience a relatively slow growth process, and the greater the multiplicative white noise intensity, the earlier the tree growth ends the slow growth period. This means that the rapid growth period of trees is shortened and the growth rate of trees changes faster.

ISSN 2616-5805 Vol. 6, Issue 2: 33-40, DOI: 10.25236/AJMS.2025.060205

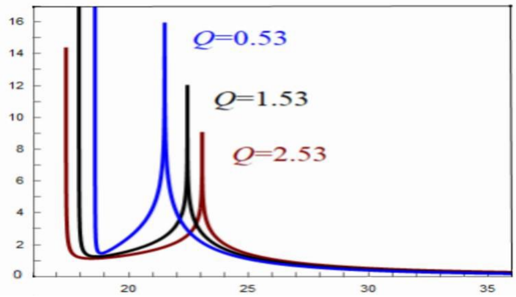

Figure 2 Effect of the additive white noise intensity Q on the steady-state probability distribution curve (r=1, A=20, D=0.19, λ=-0.94)

Figure 2 shows the effect of additive white noise Q on the steady-state probability distribution curve at different intensities. As can be seen from Figure 2, when the intensity Q of additive noise is fixed, the variation trend of steady-state probability distribution function curve is similar to that of multiplicative noise, that is, the steady-state probability distribution function first decreases, then experiences a stable period, then increases rapidly to reach a peak, and finally decreases quickly and tends to balance. This means that the seeds of trees need to cross a certain critical density, thus the intensity of additive white noise may lead to a decrease in seed survival rates. Those seeds that survive will first store resources, at which point the seeds are in their first stable phase. After accumulating sufficient resources, the seeds will begin to germinate and enter a rapid growth stage, during which the growth rate of saplings reaches its peak. Subsequently, the growth rate of saplings gradually slows down, entering the hardening period, ultimately reaching another relatively low stable state, consistent with the growth trend of trees.

As can be seen from Figure 2, when the intensity of multiplicative noise Q gradually increases, it shows the opposite situation to that of the increase of multiplicative noise intensity D. In the initial stage, the larger the value of Q, the smaller the initial value of the steady-state probability distribution curve, and it increases to the first steady-state period at a slower rate, then increases to a peak at a slower rate. The height of the peak decreases as Q increases, but the width of the peak increases with the increase of Q, and the position of the peak shifts to the right, and finally gradually decreases and tends to stabilize.

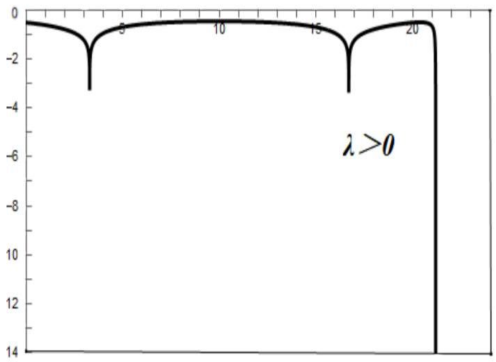

Figure 3 Effect of the positive correlation noise intensity λ on the steady-state probability distribution curve(r=1, A=20, Q=1.53, D=0.19)

Compared Figures 1 and 2, one can find the differences between the steady-state probability distribution curves when they reach their peaks. The height of the peak increases with the increase of D in Figure 1, but the width of the peak becomes narrower and the position of the peak shifts to the left. However, in Figure 2, the height of the peak of the steady-state probability distribution function decreases as Q increases, but the width of the peak becomes wider and the position of the peak shifts to the right.

Previous studies have shown that multiplicative noise and additive noise have significantly different effects on tree growth. Although the two sources are different (multiplicative noise comes from the internal fluctuations of the system, and additive noise reflects the disturbance of the external environment), the change of the external environment will disturb the internal system through the coupling action, leading to the correlation between the two types of noise. For example, precipitation change(similar to additive noise), which mesas that when regional precipitation increases significantly, although it promotes forest growth in the short term, it may reduce soil permeability in the long term, causing the problem of root hypoxia (internal metabolism limitation, similar to multiplicative noise effect). However, in the actual observation, it was found that after the annual rainfall increased by 30%, the initial growth rate of some tree species increased by 12%, but 5 years later, due to the lack of oxygen in the root system, the biomass accumulation decreased by 8%. The interaction between exogenous precipitation disturbance and endogenous metabolic limitation confirms the potential role of noise correlation mechanism in ecological processes.

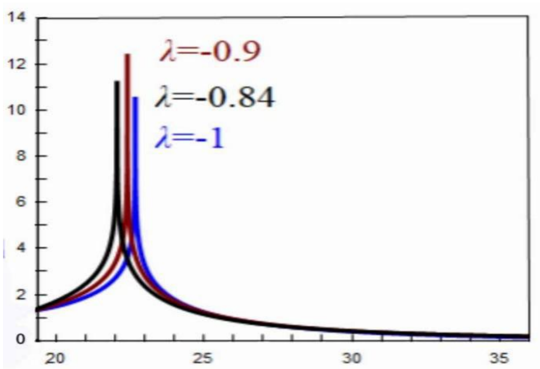

Figure 4 Effect of the negative correlation noise intensity λ on the steady-state probability distribution curve(r=1, A=20, Q=1.53, D=0.19)

As can be seen from Figure 3, the last stage of tree growth is the hardening stage, which is another lower stable state of tree growth. When the value of noise correlation strength is $\lambda>0$, it can be found from Figure 3 that there is no second steady state in its image, which does not conform to the stage of tree growth. Therefore, our study does not consider this situation.

Figure 4 presents the characteristics of steady-state probability distribution with negative association between multiplicative white noise and additive white noise. In general, under the condition of fixed noise correlation strength, its distribution curve trend is similar to that in Figure 1 and Figure 2, but there are also two significant differences. When the correlation strength approaches 0, Figure 3 shows that the duration of the first stable period is significantly extended, and the peak of the probability distribution shows an increasing trend, and its sharpness and value are significantly improved. These results show that the system has higher state aggregation for weak correlation strength in the initial stage.

Further analysis of the influence on the steady-state probability distribution function shows that before the value of the steady-state probability distribution function reaches its peak, the curve with the minimum absolute value of the correlation strength(dark gray dotted line) is always located at the high end of the distribution, while the curve with the large correlation strength (gray dotted line) is at the low end. Once the peak of the steady-state probability density distribution function is formed, this trend will be reversed. The curve with the minimum absolute value of correlation strength(dark gray dotted line) is located at the low end of the distribution, while the curve with the larger correlation strength (gray dotted line) is located at the high end.

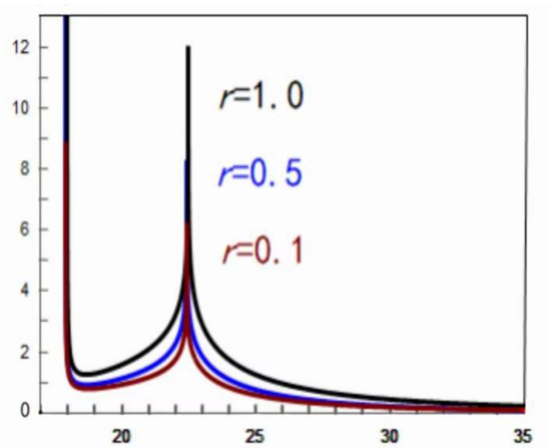

Figure 5 Effect of the maximum growth rate r on the steady-state probability distribution curve(λ=-0.94, A=20, Q=1.53, D=0.19)

ISSN 2616-5805 Vol. 6, Issue 2: 33-40, DOI: 10.25236/AJMS.2025.060205

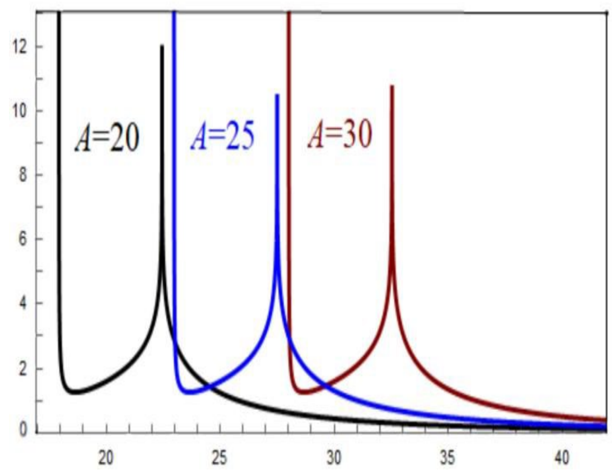

Figure 6 Effect of tree growth maximum parameter A on the steady-state probability profile(r=1, λ=0.94, Q=1.53, D=0.19)

In addition, when the value of the correlation strength between noise increases, the height of the peak shows a decreasing trend, accompanied by the phenomenon that the peak width of the steady-state probability distribution function increases and the peak position moves to the right. From an ecological perspective, these results suggest that lower noise correlation strength is more likely to occur at a given tree size before the maximum growth of the tree. With the increase of correlation strength, the peak height of steady state probability distribution function becomes smaller, indicating that the probability of tree size at the maximum growth is reduced. However, the peak position of the steady-state probability distribution function moved to the right, indicating that increasing the correlation strength between noise could promote the increase of the maximum value in the growth process of trees. These trends show that the introduction of noise has a certain effect on tree growth, and this counterintuitive result confirms that the noise correlation mechanism has a dual regulatory effect on ecosystem dynamics, which may inhibit the stability of deterministic trajectory and produce new steady state characteristics through nonlinear coupling.

Figure 5 shows the effect of the maximum growth rate of different trees on the steady-state probability distribution curve, and Figure 6 shows the effect of the maximum parameter A for different tree growth on the steady-state probability distribution curve. As can be seen from Figure 5 and Figure 6, the curves of the two plots are the same, with four stages under the influence of multiplicative white noise and additive white noise. The steady-state probability distribution function decreases, then experiences a plateau, then increases rapidly, reaches a peak, and finally decreases quickly and tends to equilibrium. It can be seen from Figure 5 that with the greater the maximum growth rate, the probability of the maximum growth rate is the highest at this point, thus shortening the time required for the tree growth to enter the hardening period. Figure 6 shows that with the increase of the maximum parameter A, the time of seed reserve resources increases, and the germination period is delayed.

4. Conclusions

By introducing multiplicative and additive Gaussian white noise into the growth Logistic model of trees, using Liouville equation, Novikov's theorem and nonlinear approximation, the approximate Fokker-Planck equation of the system is obtained through calculation. The steady-state probability distribution function is solved under the steady state condition, and the influence of relevant parameters on the steady-state probability distribution function is analyzed.

Obtained results show that changing the multiplicative white noise intensity D and the additive white noise intensity Q can lead to the change of the peak height of the steady-state probability distribution curve and the movement of the peak position, showing a drift on the probability density distribution. However, in the process of increasing D and Q, the height of the steady-state probability distribution curve and the movement direction of the peak position are different. When the multiplicative white noise intensity D increases, the height of the peak becomes higher, but the width of the peak rows the width and the position of the peak shifts to the left. When the additive white noise intensity Q increases, the height of the peak decreases, but the width becomes larger, and the position of the peak shifts to the right. In addition, the peak height of steady-state probability distribution function shows a decreasing trend with the increase of noise correlation strength, accompanied by the phenomenon of peak width increasing and peak position moving to the right.

In conclusion, tree growth can be promoted by controlling internal and external factors, such as considering the influence of tree seed selection, environment and soil. In the actual production, the above analysis can theoretically guide us to choose the forest species, climate environment and planting conditions, as well as how to ensure that the forest growth has a faster and more lasting growth period, and reach the maximum growth value faster.

Acknowledgements

The authors gratefully acknowledge the support of the College Students Innovation Training Program project Yuzhang Normal University(yzcxcy2024066), Natural Science Foundation of Jiangxi Provincial(No. 20232BAB201048) and Science and Technology Plan Project of Nanchang Institute of Science & Technology(NGKJ-24-01).

References

[1] Gardiner C W. Stochastic Methods: A Handbook for the Natural and Social Sciences[M]. 4th ed. Berlin: Springer, 2009: 145-182.
[2] Horsthemke W, Lefever R. Noise-Induced Transitions: Theory and Applications in Physics, Chemistry, and Biology[M]. Berlin: Springer-Verlag, 1984: 67-112.
[3] Van Kampen N G. Stochastic Processes in Physics and Chemistry[M]. 3rd ed. Amsterdam: Elsevier, 2007: 230-255.
[4] Turelli M. Stochastic community theory: A partially guided tour[J]. Theoretical Population Biology, 2021, 142: 1-12.
[5] Bressloff P C. Stochastic Processes in Cell Biology[M]. Cham: Springer, 2022: 333-378.
[6] Mangioni S E, Deza R R, Wio H S. Effects of correlated noises on dynamical systems[J]. Physical Review E, 1997, 56(3): 2497-2503.
[7] Ditlevsen P D, Johnsen S J. Tipping points: Early warning and wishful thinking[J]. Geophysical Research Letters, 2010, 37(19): L19703.
[8] Hänggi P, Talkner P, Borkovec M. Reaction-rate theory: Fifty years after Kramers[J]. Reviews of Modern Physics, 1990, 62(2): 251-341.
[9] Shnerb N M, Louzoun Y, Bettelheim E, et al. The importance of being discrete: Life always wins on the surface[J]. Proceedings of the National Academy of Sciences, 2000, 97(19): 10322-10324.
[10] Bena I M. Dichotomous Markov noise: Exact results for out-of-equilibrium systems[J]. International Journal of Modern Physics B, 2006, 20(20): 2825-2888.