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Research and Application of System Reliability Allocation based on Markov Chain

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Abstract: *In order to maximize the reliability of a weapon system, a continuous time Markov method is used to allocate the system reliability redundancy. The choice of reliability redundancy strategy is considered as a decision variable, which strives to maximize the reliability of the system under the constraints of cost, weight and volume. In many model applications studied in the past, each system generally adopts a single active redundancy strategy or cold standby redundancy strategy. In the model cited in this paper, each subsystem can adopt the selective redundancy strategy of active or cold standby components, increasing the flexibility of the redundancy strategy, and establishing the precise reliability function for the system using the selective redundancy strategy through the continuous time Markov chain. Because reliability redundancy assignment is a nonlinear mixed integer programming problem, a pseudo-parallel genetic algorithm is designed to solve the model. Finally, the results show that the reliability of the weapon system is significantly improved.*

Keywords: *System reliability allocation; Weapon system; Reliability function; Genetic algorithm*

1. Inroduction

To maximize system reliability, the traditional reliability redundancy assignment problem determines the component reliability and redundancy level of each subsystem. Reliability allocation is an important reliability design method that has a wide range of applications in engineering fields where very high system reliability is strongly required, such as the stop-stack system of nuclear power plants [1], the resource allocation system of national defense [2], and power supply system [3]. Therefore, it is necessary to conduct a study on reliability redundancy allocation for systems with high reliability requirements.

In the selection of redundancy strategies, active redundancy [4-5] or cold-standby redundancy [6-7] were generally used in the past to improve the reliability of the system. In active redundancy, all components in the system operate simultaneously and all components are subjected to the influence from the operating environment (temperature, pressure, etc.), however, it is not necessary for all components to operate simultaneously to ensure the proper functioning of the system, which is the biggest drawback of the active redundancy strategy. In standby redundancy, most of the past studies have focused on cold standby redundancy, which, unlike active redundancy, requires the operation of only one component from the moment the system task starts, with the other secondary components as redundant components, and these standby components are not affected from the working environment when they are not working. This process then requires the use of a fault detection/switching system to continuously monitor the entire subsystem, and when a component failure is detected, the fault detector switches the redundant component at the same time.

In order to maximize the reliability of a weapon system, a continuous-time Markov chain based on continuous time is used in this paper to establish an exact reliability function for a weapon system with selected redundancy [8]. A radar receiving subsystem is redesigned for reliability redundancy assignment. Since reliability redundancy allocation is a nonlinear mixed integer programming problem, an improved pseudo-parallel genetic algorithm is finally designed to solve the mathematical model.

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2. Reliability redundancy allocation that integrates cost and quality

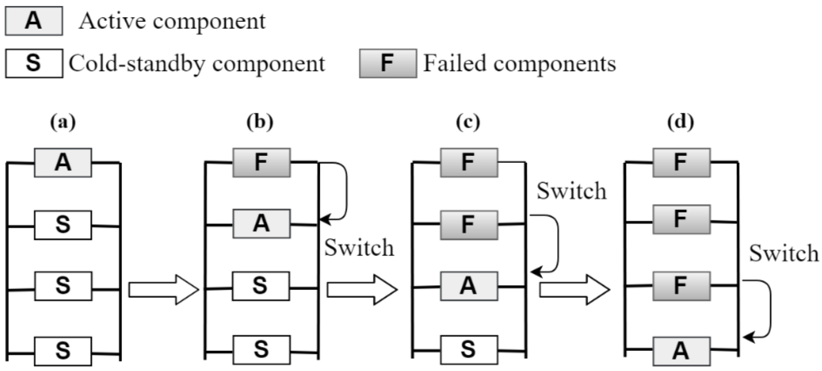

Figure 1: Cold-standy redundance strategy

2.1 Objective Functions and Constraints

The practical application of this paper is derived from [9], where a weapon system is assumed to consist of *m* subsystems connected in series and parallel, as shown in Figure 1, which illustrates the structure and normal operation process of subsystem *i* using cold-standby redundancy strategy. The subsystem consists of one active component and three cold-standby components, respectively. In this case, when the first active component fails, the switchover system replaces the first redundant component, and so on, to ensure the normal operation of the system. In the system reliability allocation, in order to improve the reliability of the whole system work, redundant parts are equipped on each component and put into the system work automatically. The more redundant parts, the higher the system reliability, but the cost and quality of the whole system are increased accordingly, and the mobility and working accuracy of the system are reduced. In order to maximize the reliability of the whole system in the working time t_s=1000 h under the above constraints, it is necessary to choose the number of redundant parts of each unit reasonably. Let the number of components on the ith subsystem be n_i, and the reliability of subsystem *i* be $R_i(n_i,t_s)$, $R_i(n_i,t_s)$ = Max($R_{i,A}$,$R_{i,S}$), where $R_{i,A}$ is the reliability value of subsystem *i* with active redundancy, and $R_{i,S}$ is the reliability value of subsystem *i* with cold-standby redundancy. The reliability of the whole system at the moment of t_s is

$$R(N,t_s) = \prod_{i=1}^{m} R_i(n_i,t_s) \tag{1}$$

Its cost and quality constraints are:

$$\sum_{i=1}^{m} c_i n_i \le C \ , \ \sum_{i=1}^{m} w_i n_i \le W \ , \ n_i, i \in Z^+ \ , \ 1 \le i \le m \tag{2}$$

The cost and mass of each redundant component of the ith subsystem of the above system are c_i, w_i, respectively, and the total cost and mass of the system spare parts do not exceed C, W.

2.2 Reliability of subsystems with active redundancy strategy

In the system with the active redundancy strategy, all components start working at the same time. As in past studies, we assume that the lifetimes of the components in the subsystem obey an λ_i and λ_{id} exponential distribution of *i*, so that the reliability of the individual components of subsystem *i* at the moment of time t_s is:

$$ri(ts) = \exp(-\lambda its) \tag{3}$$

Therefore, the reliability function of subsystem I with active redundancy at TS time is:

$$R_{i,A}(t_s,n_i) = 1 - \left(1 - r_i(t_s)\right)^{n_i} \tag{4}$$

2.3 Reliability of Subsystems with Cold Standby Redundancy Strategy

We assume that the lifetimes of the components in the subsystem obey an λ_i and λ_{id} exponential distribution of *i,* The general reliability function for the system with cold standby redundancy strategy when the fault detector/switch continuously monitors the failure of the activated component is [7]:

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$$\widetilde{R_i(t_s)} = e^{-\lambda_i t_s} + \rho_i(t_s)\sum_{x=1}^{n_i-1} \frac{e^{-\lambda_i t_s}(\lambda_i t_s)^x}{x!} \quad (5)$$

Since the switch continuously monitors the failure of the activating component as time passes, the reliability of the switch is a non-increasing function with increasing time as the component, so the reliability of the imperfect switch $\rho_i(t_s) < 1.0$ for the subsystem i in the subsystem i. The reliability value of Eq. (5) is a lower bound approximation of the subsystem i with the formula. We assume $\rho_i(t_s{=}1000) = 0.99$,$\lambda_{id} = 1.005\times10^{-5}$.

In this paper, a structured continuous-time Markov model proposed by Kim [8] is used to calculate the reliability values of a weapon system using cold-standby redundancy. The normal and failure of the switch are described as two different scenarios:

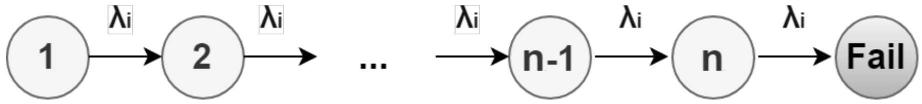

Figure 2: State transition: scenarios 1

Scenario 1: When the fault detector is operating normally, the task of the faulty component is switched to a redundant component and the subsystem operates normally until the last component fails, as shown in Figure 2(a). The transfer rate matrix $\mathbf{D}_{i1}$ describing this scenario can be represented byEq. (6).

$$\left[\mathbf{D}_{i1}\middle|\mathbf{d}_{i1}\right] = \left[\begin{array}{ccccc|c} -\lambda_i & \lambda_i & & & & 0 \\ & -\lambda_i & \lambda_i & & & 0 \\ & & \ddots & \ddots & & 0 \\ & & & -\lambda_i & \lambda_i & 0 \\ & & & & -\lambda_i & \lambda_i \end{array}\right] \quad (6)$$

Here $\mathbf{D}_{i1}$ is a square matrix of size n_i order, indicating that subsystem i works properly by switching sequentially between the primary and redundant components. The last column $\mathbf{d}_{i1}$ indicates that the failure of subsystem i is caused due to the failure of the last component.

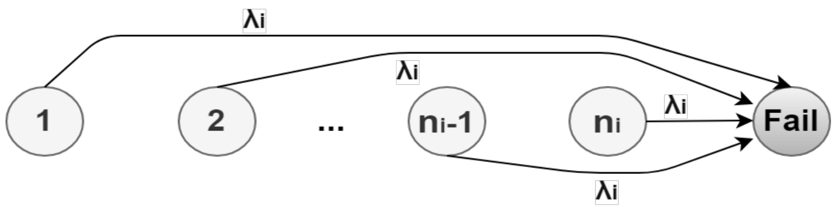

Figure 3: State transition: scenarios 2

Scenario 2: Due to the failure of the fault detector, the redundant component cannot be replaced when it fails, so the subsystem operates normally until the current main working component fails, as shown in Figure 2(b). The transfer rate matrix $\mathbf{D}_{i2}$ describing this scenario can be represented by Eq.(7).

$$\left[\mathbf{D}_{i2}\middle|\mathbf{d}_{i2}\right] = \left[\begin{array}{cccc|c} -\lambda_i & & & & \lambda_i \\ & -\lambda_i & & & \lambda_i \\ & & \ddots & & \vdots \\ & & & -\lambda_i & \lambda_i \end{array}\right] \quad (7)$$

Here $\mathbf{D}_{i2}$ is a square matrix of size n_i order, indicating the current operational state maintained by the main diagonal element. The last column $\mathbf{d}_{i2}$ describes the failure of subsystem i due to the current failure of the main working component.

The transfer rate matrix $\mathbf{S}_i$ represents the two states of the switching system in subsystem i (S1: working, S0: failed), which can be expressed by equation Eq.(8).

$$\mathbf{S}_i = \begin{matrix} S_1 \\ S_0 \end{matrix}\begin{bmatrix} -\lambda_{id} & \lambda_{id} \\ 0 & 0 \end{bmatrix} \quad (8)$$

In summary, the state transfer matrix $\mathbf{D}_i$ of the whole subsystem can be represented by Eq.(9), so the

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exact reliability formula for subsystem i with cold-standby redundancy strategy at the moment of t_s is Eq.(10)[8].

$$\mathbf{D_i} = \begin{bmatrix} D_{i1} - \lambda_{id} I_{ni} & \lambda_{id} I_{ni} \\ 0 & D_{i2} \end{bmatrix} \tag{9}$$

$$R_{i,S}(t_s, n_i) = \pi_i \exp(D_i t_s) 1, \; \pi_i = [1,0], \quad 1 = [1,1], \quad 0 = [0,0] \tag{10}$$

Here $\mathbf{D_i}$ is a square matrix of order $2n_i$ in size; 1 and i are row vectors consisting of $2n_i$ elements and i is also the initial distribution of all components in subsystem i; $\mathbf{I_{ni}}$ denotes a unit vector of order n_i; and 0 is a zero vector consisting of n_i elements.

3. Pseudo-Parallel Genetic Algorithm Design

Genetic algorithm (GA) is widely used in reliability design optimization as a heuristic optimal solution search technique. In this paper, a pseudo-parallel genetic algorithm (PPGA) for solving the reliability redundancy assignment problem will be proposed.

3.1 Encoding

In this paper, we use truth values to encode single chromosomes. A single chromosome consists of information about the type of each subsystem component and the number of components that employ active redundancy and cold standby redundancy.

3.2 Fitness Function

In genetic algorithms, the function that measures the fitness of an individual is called the fitness function. In this paper, the objective function is directly used as the fitness function according to the characteristics of the objective function. By generating a judgment of weight and cost constraint for each individual in advance to screen the individuals that satisfy the condition, the quality of the solution is guaranteed by replacing the conventional penalty function in this way.

3.3 Selection, Crossover and Mutation

In the selection phase, this paper used the elite conservation strategy and the roulette wheel method to select parents from the population for later propagation. In the elite conservation strategy, the best individuals in the population are selected and retained intact for the offspring. In the roulette wheel method, a parent is randomly selected based on a probability proportional to fitness, so the process ensures that all individuals have the possibility of being selected and remaining. Crossover, the core operator of the genetic algorithm, creates two new individuals by exchanging two mutually paired parental chromosomes with each other in some way for some of their genes. In this paper, we use two-point crossover, in which two crossover points are randomly set in the coding strings of two individuals paired with each other. The exchanged two individuals have part of their chromosomes between the two set crossover points. Variation is used as an auxiliary method for the genetic algorithm to generate new individuals, and variation plays a role in maintaining the diversity of the population and preventing the occurrence of premature maturity. In this paper, the maximum-minimum variation operator is used. The genes corresponding to the variables of the selected subsystem are regenerated between the lower and upper bounds of each variable.

3.4 Parallelization

In order to avoid the premature phenomenon that tends to occur in the process of genetic algorithm, the idea of parallel genetic algorithm is introduced to improve the above mentioned process of genetic algorithm selection, crossover and mutation to parallelize the genetic algorithm. Due to the limited conditions, this paper is executed serially on a single processor, so it is called pseudo-parallel genetic algorithm (PPGA). In this paper, a subpopulation information exchange model for island populations is used. The initial population is divided equally into four subpopulations, and each subpopulation evolves independently, and all subpopulations share the optimal part of individuals with each other each generation, and the model is shown in Figure 4.

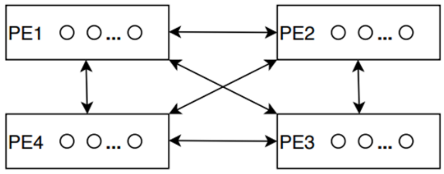

Figure 4: Island Mode

4. Example Application and Analysis

4.1 Example Applications

In order to verify the validity of the reliability optimization model, the selected redundancy strategy model of [8] and the algorithm designed in this paper were applied to a weapon system [9]. A radar receiving subsystem consists of four subsystems: receiver, excitation source, frequency source and power supply, and the reliability value, cost and quality of each subsystem component at the time of t_s=1000 are shown in Table 1. If the system is designed for redundancy, the total weight of redundant parts $W \leq 450$ kg and the total cost of redundant parts $C \leq 1.25$, how can the system be designed for maximum reliability? The program parameters are set as follows: chromosome length is 4, population size is 100, evolutionary generation is 100, crossover rate is 0.8, and variation rate is 0.05.

Table 1: List of reliability, price and weight of each component

subsystems	reliability	ci	wi
receiver 1	0.96	0.2	80
excitation source 2	0.98	0.2	60
frequency source 3	0.95	0.25	50
power supply 4	0.90	0.1	30

Table 2 shows the different results obtained by different models when t_s = 1000. Where n_{Ai} and n_{Si} denote the number of components in subsystem i with active redundancy strategy and cold-standby redundancy strategy, respectively. The maximum possible index (MPI) is used for the improvement effect of the new model relative to other studies, and its higher value indicates the more significant improvement effect, and the formula is as follows.

$$MPI(\%) = [R_s(\text{New Approach})-R_s(\text{other})]/[1- R_s(\text{other})] \tag{11}$$

4.2 Data analysis

Table 2: Comparison results of various models

parameter	[9]	PPGA	By Eq.(5)	By Eq.(10)
	A	A	A or S	A or S
R(ts=1000)	0.911544	0.938620	0.942670	0.943397
nAi	(1,2,1,4)	(2,2,1,2)	(1,1,1,1)	(1,1,1,1)
nSi	(0,0,0,0)	(0,0,0,0)	(1,1,0,1)	(1,1,0,1)
redundancy strategy	(A,A,A, A)	(A,A,A,A)	(S,S,A,S)	(S,S,A,S)
Cost	1.25	1.25	1.25	1.25
Weight	370	390	390	390
MPI(%)	36.01	7.78	1.27	-

∗redundancy strategy: A (Active redundancy strategy), S (cold-standby redundancy strategy)

The MPIs of the selected redundancy strategy based on the continuous time Markov chain compared with the design of the redundancy allocation for the reliability of the weapon system using the active redundancy strategy with two different algorithms are 36.01 and 7.78, respectively, and the numerical results show that the reliability value of the weapon system is significantly improved under the new model. Under the same selective redundancy strategy, the MPI is 1.27 compared to the lower bound equation Eq.(5), and the results obtained by the new Markov model are more accurate. The reliability value of 0.938620 obtained by the pseudo-parallel genetic algorithm designed in this paper under the same active redundancy strategy is higher than 0.911544 obtained in [9] using dynamic programming method, which illustrates the efficiency of the genetic algorithm designed in this paper. Figure 5 shows the optimal configuration and structure of the entire radar receiving subsystem.

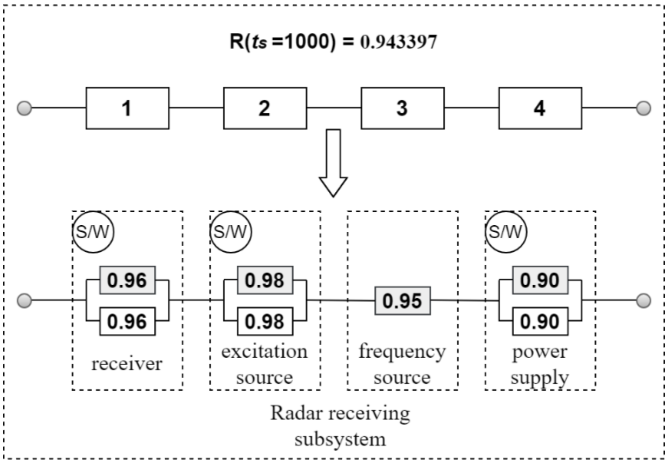

Figure 5: The best configurations for the radar reception subsystem

5. Conclusion

In this paper, based on the exact reliability function proposed by other scholars, a practical application is carried out to redesign the reliability redundancy allocation for a radar receiving subsystem, and a pseudo-parallel genetic algorithm is designed to solve the nonlinear mixed integer programming problem. The results show that the reliability of the weapon system is improved, which proves the effectiveness of the new reliability allocation application model and algorithm, and also provides a reference opinion for the reliability redundancy allocation design in some engineering fields that strongly require extremely high system reliability.

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