ISSN 2618-1584 Vol. 7, Issue 3: 74-79, DOI: 10.25236/FMSR.2025.070310

Steady-State Properties Analysis of SIS Infectious Disease Model with Correlated Noise

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Abstract: *A model closer to real life and applicable to a wider range of scenarios and conditions is obtained by adding internal and external disturbance factors based on the original SIS infectious disease model when the differences between different individuals and environments, such as health status, protection measures and uncertainty of model parameters, are considered. The equilibrium point of the SIS infectious disease model with correlated noise and the steady-state solution of the Fokker-Planck equation are solved through linearized approximate numerical methods. The steady-state probability distribution function of the SIS infectious disease model are plotted by using Geometer's Sketchpad, the impact of various parameters on the SIS infectious disease model and the fluctuations of the equilibrium point of the SIS infectious disease model are dynamically examined. The results show that the SIS infectious disease model can effectively improve the fitting accuracy of the real epidemic transmission by regulating the noise intensity parameter and color correlation characteristics (including correlation strength and duration), and provide theoretical support and quantitative basis for the formulation of public health intervention measures.*

Keywords: *SIS Model; Langevin Equation; Noise; Numerical Solution; Stability*

1. Introduction

In the context of rapid development of contemporary science and technology, the popularization and application of electronic computers have greatly promoted the deep integration of mathematical methods in biological research. This interdisciplinary integration trend has become an important development direction in modern biology, especially in the field of medical research. With the continuous improvement of public health system, the remarkable improvement of medical technology and the continuous progress of human civilization, infectious diseases such as smallpox, monkeypox and novel coronavirus, which caused global public health crises in history, have been effectively contained. However, in some economically underdeveloped countries and regions, local outbreaks of infectious diseases still occur from time to time.

In 2014, Jiang Daqing *et al*. carried out a long-term dynamics study on a random SIS infectious disease model included in the vaccination mechanism, systematically derived the sufficient conditions for the average extinction and persistence of the disease, and strictly proved that the basic reproduction number as a key threshold played a decisive role in the outcome of disease transmission when the intensity of environmental white noise was low. In 2016, Meng Xinzhu *et al*. constructed a stochastic SIS model containing nonlinear incidence function and double hypothesis conditions, determined the dynamic characteristics of the equilibrium point of the system through stability analysis, revealed the double threshold conditions for the extinction and persistence of infectious diseases, and innovatively demonstrated the promoting effect of high intensity random disturbance on disease elimination [1]. In 2021, the Lahrouz team studied the classical SIS model with a generalized transmission mechanism (covering non-monotonic infection rate), focusing on the impact mechanism of white noise interference on the dynamic change of disease transmission rate, which provided a new theoretical perspective for understanding the transmission law of infectious diseases under complex environmental disturbances. Foreign researchers have made rich achievements in the study of SIS model. They not only deeply discussed the dynamic characteristics and stability conditions of the model, but also put forward a variety

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of optimization prevention and control strategies, such as vaccination and isolation measures. These prevention and control measures have been widely implemented in practice, and the results are very significant.

This paper derives a more practical one-dimensional Langmuir equation based on the nonlinear transmission of infectious diseases under the mechanism of organism immunity. It introduces external and internal factors affecting the spread of infectious diseases into the model as white noise and colored noise, respectively, and explores the impact of these two types of noise on SIS infectious disease transmission under different color correlation strengths and color correlation times. Using linear approximation method, the steady-state probability distribution function of the system was derived and calculated. By observing the change of two noise intensities and the color correlation strength and color correlation time between the noises, it was investigated whether the SIS model could better simulate the disease transmission in the real world, which provided scientific support for the construction of effective prevention and control strategies.

2. Model construction

According to the transmission mechanism of diseases, the transmission that is similar to the spread of cold virus and can be infected multiple times but can be cured and cannot be immune is summarized as SIS virus transmission, and its transmission mechanism is shown in Figure 1.☐

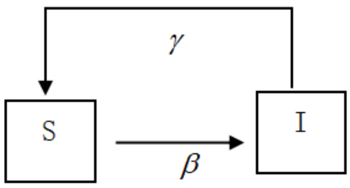

Figure 1 Mechanism of transmission of classic SIS class infectious diseases.

Among them, S represents the healthy state and I represents the infected state. After contact between S individuals and I individuals, they will be transformed into infected individuals with a certain probability β, and then they will be transformed into S individuals with a probability γ after being cured.

According to the actual situation, infectious diseases are transmitted through contact. The number of times a healthy person comes into contact with a patient in a unit time is called contact rate, which has a close relationship with the number of individuals N in the network, denoted as $C(N)$. We call the contact rate of the probability of infection β the effective contact rate, that is $\beta C(N)$, which can reflect the disease resistance of healthy individuals, environmental conditions and other practical factors. The proportion of susceptible people S is S/N, so the average infection rate of susceptible people is $\beta C(N)S/N$.

According to the above analysis, the number of susceptible persons converted into infected persons at time t is $\beta C(N)I(t)S/N$. If we assume that the contact rate is linearly related to the total number of individuals in the network, the proportional coefficient is k, i.e,

$$C(N) = kN, \tag{1}$$

Thus, the rate of infection is obtained as follows:

$$\beta C(N)I(t)S/N = \beta S(t)I(t), \tag{2}$$

The classical SIS model is obtained as follows [2]:

$$\begin{cases} \dfrac{dS}{dt} = -\beta SI + \gamma I, \\ \dfrac{dI}{dt} = \beta SI - \gamma I. \end{cases} \tag{3}$$

It is clear that the transmission and proliferation of infectious disease pathogens are also inevitably profoundly affected by external environmental factors, which involve many aspects [3]. In this paper, the

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external and internal factors affecting the spread of infectious diseases are introduced into the model in the form of white noise and color noise respectively [4]. After the above influencing factors are brought into the equation, the Langvin equation which is more consistent with the SIS infectious disease model is obtained as follows:

$$\frac{dx}{dt} = kx\left(n - \frac{1}{\sigma} - x\right) + \varphi(n-x) + \eta. \tag{4}$$

Based on the above equation, the following expression of deterministic potential function can be obtained by calculation as follows:

$$V(x) = \frac{1}{3}kx^3 - \frac{k}{2}\left(n - \frac{1}{\sigma}\right)x^2. \tag{5}$$

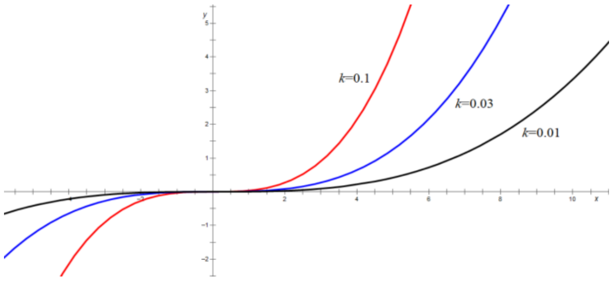

Figure 2 Potential function graph.

The sum in the above equation represents white noise and color noise with zero mean, respectively, and they have the following statistical relationship as follows:

$$\xi(t) = \eta(t) = 0, \tag{6}$$

$$\left\langle \xi(t)\xi(t')\right\rangle = 2D\sigma(t-t'), \tag{7}$$

$$\left\langle \eta(t)\eta(t')\right\rangle = 2Q\sigma(t-t'), \tag{8}$$

$$\left\langle \xi(t)\eta(t')\right\rangle = \left\langle \eta(t)\xi(t')\right\rangle = \frac{\lambda\sqrt{DQ}}{\tau} exp\left\{\frac{-|t-t'|}{\tau}\right\}. \tag{9}$$

In the above formula, the size of D and Q values represents the noise intensity, λ represents the color correlation strength between the two noises, and t represents the time, τ represents the color correlation time between the two noises[5].

3. Fokker-Planck equation and its steady state solution

Fokker-Planck equation and its steady-state solution are simplified according to the method of relevant literature, and written in the following form as follows:

$$\frac{dx}{dt} = f(x) + g_1(x)\xi(t) + g_2(x)\eta(t), \tag{10}$$

$$f(x) = kx\left(n - \frac{1}{\sigma} - x\right), \tag{11}$$

$$g_1(x) = n - x, g_2(x) = 1. \tag{12}$$

The corresponding Fokker-Planck equation is obtained by using Liu's equation[6]:

$$\partial P(x,t) = -\frac{\partial}{\partial(x)}A(x)P(x,t) + \frac{\partial^2}{\partial x^2}B(x)P(x,t). \tag{13}$$

The steady-state probability distribution function in the model is expressed by $A(x)$ and $B(x)$, which

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can be expressed as follows:

$$A(x)=f(x)+Dg_1(x)g_1{}'(x)+\frac{\lambda\sqrt{DQ}}{1-\tau f'(x_s)}\left(g_1(x)g_2{}'(x)\right), \tag{14}$$

$$B(x)=Dg_1^2(x)+\frac{2\lambda\sqrt{DQ}}{1-\tau f'(x_s)}g_1(x)g_2(x)+\theta g_2^2(x). \tag{15}$$

According to the reflection boundary condition, the steady state solution of the Fokker-Planck equation, *i.e.*, Eq.(13), can be obtained [7]:

$$P_{st}(x)=\frac{N}{\sqrt{B(x)}}exp\left\{\int\frac{f(x)}{B(x)}dx\right\}=\frac{NE}{\sqrt{B(x)}}|F|^{kG}. \tag{16}$$

Take the solution when $\lambda>1$ or $\lambda<-1$ is taken, where:

$$E=e^{\frac{-kx}{D}}\left(\left|\left(x^2-2nD\right)x+\left(Dn^2+Hn+Q\right)\right|\right)^{\frac{k\left(-n-\frac{H}{D}-\frac{1}{\sigma}\right)}{2D}}, \tag{17}$$

$$F=\left|\frac{2Dx-(2nD+H)-\sqrt{H^2-4DQ}}{2Dx-(2nD+H)+\sqrt{H^2-4DQ}}\right|, \tag{18}$$

$$G=\frac{\frac{Q}{D}-\frac{n}{\sigma}-\frac{Hn}{2D}-\frac{H^2}{2D^2}-\frac{H}{2D\sigma}}{\sqrt{H^2-4DQ}}. \tag{19}$$

4. Numerical analysis and discussion

In the theoretical analysis work carried out in the early stage, all kinds of factors that may affect the research results have been taken into consideration, and these influencing factors have been organically integrated with the relevant variables in the formula [8]. Figure 2 shows the relationship between the deterministic potential function and the correlation coefficient *k* between the contact rate and the total number of individuals in the network. It can be seen from Figure 2 that the value of the correlation coefficient significantly affects the distribution of the potential function.

The numerical simulation of the steady-state probability distribution function *Pst*(*x*) is carried out by keeping other variables unchanged and only changing the value of noise intensity *D*. The results are shown in Figure 3. It can be found that when the intensity *D* of white noise gradually increases from the analysis of Figure 3, especially the specific value changes from *D*=0.89 to *D*=0.9, and then further increases to *D*=0.91, the curve shape corresponding to the steady state probability distribution function changes. On the whole, the peak position of the curve has a certain movement, and the width and other morphological characteristics of the curve have also changed. This shows that the increase of white noise intensity has an obvious influence on the shape of the steady state probability distribution.

By comparing the function values of these points under different *D* values, the influence of white noise intensity change on the steady state probability in a specific state can be quantitatively seen. For example, at a specific horizontal coordinate value, as *D* increases from 0.89 to 0.91, the corresponding function value may decrease or increase, which reflects the change of steady-state probability in this state with the increase of white noise intensity. It can be found from the change trend of the curve in the figure that when the value of the independent variable on the horizontal axis gradually increases (that is to say that the curve extends to the right), the steady-state probability distribution function corresponding to different *D* values all tend to 0, but the speed of convergence is different. Curves with large *D* values (e.g., *D*=0.91) may approach 0 more quickly at relatively small independent variable values, which means that the probability of the system being in a larger state value decreases more quickly at higher white noise intensity.

In general, the influence of the white noise intensity parameter *D* on the steady state probability distribution function can be clearly defined by numerical analysis of the shape of the curve, specific points and asymptotic behavior in the figure.

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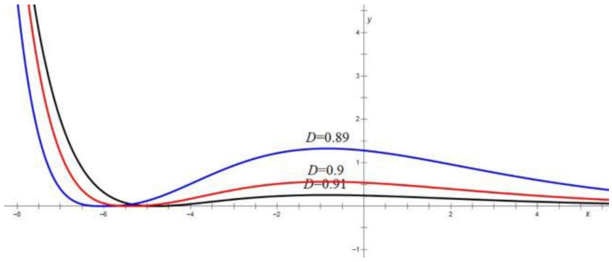

Figure 3 Influence of white noise intensity D on steady state probability distribution function.

Figure 4 shows the numerical analysis of the influence of color noise intensity Q on the steady-state probability distribution function. With the increase of color noise intensity *Q* (from *Q*=1.01 to *Q*=1.02 and then to *Q*=1.03), the peak of steady-state probability distribution function curve gradually increases. This indicates that with the increase of color noise intensity, the probability of the system being in the state corresponding to the peak of the probability distribution increases. The curve corresponding to a larger *Q* value is relatively wider, indicating that with the increase of color noise intensity, the distribution range of system state presents an expanding trend.

By comparing the values of steady-state probability distribution function under different *Q* values, it can be found that as *Q* increases from 1.01 to 1.03, the function values of these specific points may increase, reflecting that in these specific states, the steady-state probability of the system increases with the increase of color noise intensity. From the change trend of the curve in Figure 4, the steady state probability distribution function corresponding to different *Q* values all tend to 0 as the value of the independent variable on the horizontal axis increases to both sides (to the left and right). However, the curve with a large *Q* value (such as *Q*=1.03) tends to 0 at a slower rate than the curve with a small Q value (such as *Q*=1.01) when the independent variable is relatively small, which means that the probability of the system being far away from the peak state decreases relatively slowly under the condition of large color noise intensity.

In general, the change of color noise intensity *Q* significantly affects the peak, width and asymptotic behavior of steady-state probability distribution function.

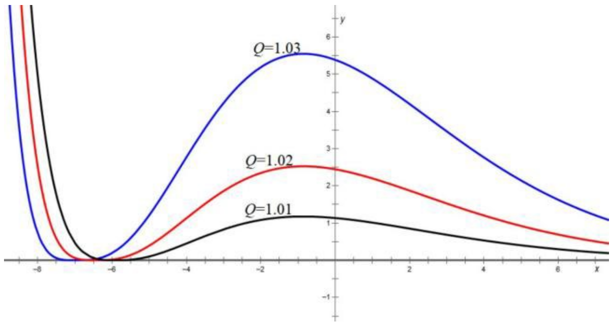

Figure 4 Influence of color noise correlation strength Q on steady state probability distribution

With the increase of the correlation strength λ (from λ=-50.10 to λ=-50.20 and then to λ=-50.60), the peak of the steady-state probability distribution function curve increases first and then decreases, as shown in Figure 5. For example, the peak value of the curve corresponding to λ=-50.60 is higher than that of the curve corresponding to λ=-50.20 and λ=-50.10, indicating that within a certain range, the change of correlation strength will change the probability of the system being in the most likely state, and it is not a simple monotonic relationship. The overall shape of the curve also changes, and the width of the curve seems to vary with λ, indicating that the correlation strength affects the distribution range of the system state, and the probability distribution of the system in different states changes with the change of λ.

As can be seen from Figure 5, with the change of λ from-50.10 to-50.60, the function values of these specific points change correspondingly, reflecting that in these specific states, the steady-state probability of the system changes with the change of correlation strength, which may increase at some points and decrease at some points. From the trend of the curve, as the value of the independent variable on the horizontal axis increases to both sides (extending to the left and right), the steady-state probability distribution function corresponding to different λ values all tend to 0. However, when the λ value is different, the speed of approaching 0 is different. The curve corresponding to λ=-50.60 approaches 0 relatively slowly when it is far away from the peak region, which means that the probability of the system being far away from the peak state decreases more slowly under a large correlation strength.

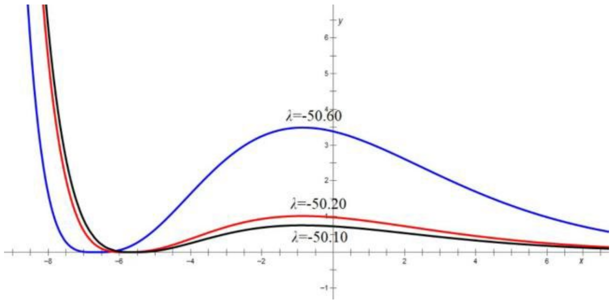

Figure 5 Influence of noise correlation strength λ on steady state probability distribution function.

In general, when the intensity of color noise Q changes, it will significantly affect the steady state solution, width and asymptotic trend of the steady state probability distribution function.

5. Conclusion

This paper considers the internal and external influencing factors starting from the SIS model, introduces them in the form of noise to obtain the Langvin equation, and then uses the nonlinear approximation method to obtain the steady-state probability distribution function. By adjusting the color correlation strength and correlation duration between the noise, this paper explores the influence of the noise on the transmission of SIS infectious disease under different circumstances. The results show that the intensity of white noise D and color noise Q, the observation of the change of two noise intensities and the color correlation strength and color correlation time between noises can better simulate the disease transmission in the real world by SIS model, which provides a good scientific basis for the formulation of effective prevention and control strategies.

Acknowledgements

The authors gratefully acknowledge the support of the Natural Science Foundation of Jiangxi Provincial(20232BAB201048), College Student Innovation Training Program Project of Yuzhang Normal University(YZCXCY2024048) and Science and Technology Plan Project(NGKJ-24-01), First Class Undergraduate Construction and Cultivation Project(JXCGJ-PY-2024-065) of Nanchang Institute of Science and Technology.

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