# A Parameters Adaptive Non-Singular Terminal Sliding Mode Control Method for Overhead Crane System

## Xin Wanga, Chuntong Liub, Zhenxin Hec,\*

Xi'an Research Institute of High Technology, Baqiao District, Tongxin Road, Xi'an City, Shaanxi 710025. China

<sup>a</sup> 3059237219@qq.com, <sup>b</sup> Liuchuntong72@.sina.com, <sup>c</sup> hezhenxin1986@126.com

Abstract: In this paper, a non-singular terminal sliding mode control method based on parameter adaptation is proposed for trajectory tracking control of overhead cranes with parameter uncertainties. The method can realize fast tracking of the planned overhead crane trajectory when the payload mass and the length of the suspension rope are unknown. At the same time, the hyperbolic tangent function is used to replace the switching term in the sliding mode control, which improves the robustness of the control method to external disturbances to suppress the charting during the crane operation. Using Lyapunov theorem, it is proved that the displacement trajectory can converge to the target trajectory in a finite time. The effectiveness of the proposed control scheme is verified by platform experiments.

Keywords: Underactuated Overhead Crane, Adaptive Control, Non-Singular Terminal Sliding Mode

#### 1. Introduction

As a heavy machinery frequently used in infrastructure, freight and industrial production, the main task of overhead crane is to safely and stably transfer the payload to the target position within a period of time, but due to its underactuated characteristic, it is difficult to be used in the actual production process[1-3]. At present, the overhead crane still adopts the manual operation mode, but the manual operation is limited by the operator's experience and attention, which is prone to safety accidents. Therefore, it is of great significance to realize the automation of the overhead crane handling process. At present, scholars at home and abroad have also proposed a variety of positioning control methods for overhead suspension systems, but they are all limited by known system parameters, which also limit the application of existing methods in practical systems.

In the past few years, various advanced control methods such as: sliding mode control (SMC) [4], neural network control[5] and observer-based feedback control[6] have been proposed by researchers to achieve precise position control. Among these control methods, sliding mode control has received considerable attention for its superior robustness and simplicity of design. The traditional sliding mode control method has been applied in the positioning control of the overhead crane due to its advantages of simple design and strong anti-interference ability, but it also has the problem of slow tracking convergence[7]. Terminal Sliding Mode Control (TSMC) is an improvement on traditional sliding mode, which can achieve finite-time stabilization of overhead crane systems[8], but there are discontinuous terms in the sliding mode control signal, which can lead to chattering and chattering. There are non-singular phenomena.

Based on the above discussion, this paper aims to study a control method that can quickly and accurately complete the positioning even when the model parameters are unknown, while avoiding the occurrence of non-singular phenomena and reducing system chattering.

## 2. Modeling of 2-D Overhead Crane System

In this paper, the two-dimensional(2D) overhead crane system is shown in Figure 1. X and  $\theta$  are the horizontal displacement of the trolley and the swing angle of the payload, respectively, which are regarded as the states and the outputs of the system. L is the length of the rope, M and m are the mass of the trolley and payload respectively, f is the rail friction and Fr is the actuator force.

<sup>\*</sup>Corresponding author

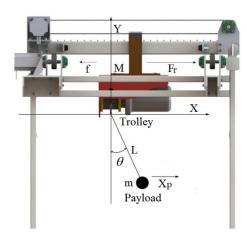


Fig. 1: 2-D diagrammatic of an overhead crane

By assuming that the payload has a point mass, and the rope has no mass and elasticity, with air resistance being ignored, the nonlinear crane dynamics with constant rope length can be depicted as follows [9]:

$$(M+m)\ddot{x} + mL\ddot{\theta}\cos\theta - mL\dot{\theta}^2\sin\theta = F \tag{1}$$

$$mL^2\ddot{\theta} + mL\ddot{x}\cos\theta + mgL\sin\theta = 0 \tag{2}$$

Where g is gravity constant,  $F = F_r - f$  is the control input to be designed. The form of f is given by [10]:

$$f(\dot{d}) = f_{r0} \tanh(\frac{\dot{d}}{\xi_r}) - k_r |\dot{d}| \dot{d}$$
(3)

Where  $f_{r0}$ ,  $k_r \in \mathbb{R}$  are friction-related parameters and  $\xi_r \in \mathbb{R}$  is a static friction coefficient, which can be obtained from offline experimental data analysis. During the overall process, the payload is always beneath the trolley, namely  $-\pi/2 < \theta(t) < \pi/2$ .

#### 3. Adaptive Tracking Controller Design

As mentioned above, most of the existing overhead crane positioning control methods require accurate model parameter information. However, in many cases, it is difficult to measure the exact value of the actual payload mass and rope length of an overhead crane system. For this reason, this paper proposes an adaptive non-singular terminal sliding mode control method, which can accomplish the above tasks when the model parameters are unknown, while improving the convergence of the system, and does not require linearization of the model.

#### 3.1 S Curve Trajectory

According to operation experience, it is known that to achieve a superior performance, the desired trajectory should be a S curve. The desired trajectory  $x_{dx}(t) \in \mathbb{R}^1$  takes a limit of a positive constant  $p_{dx} \in \mathbb{R}^+$ :

$$\lim_{t \to \infty} x_{dx}(t) = p_{dx} \tag{4}$$

Where  $p_{dx}$  denotes the desired position.

In this paper, we choose the S curve trajectory in [11] as the positioning reference trajectory, and its expression is as follows:

$$x_{dx}(t) = \frac{p_{dx}}{2} + \frac{k_1^2}{4k_2} ln \left[ \frac{\cosh(2k_2 t/k_1 - \varepsilon)}{\cosh(2k_2 t/k_1 - \varepsilon - 2k_2 n_{dx}/k_1^2)} \right]$$
 (5)

#### 3.2 Adaptive Tracking Controller Design

Recently, a series of adaptive control methods have been reported to address overhead crane. In this paper, an adaptive non-singular terminal sliding mode control method is proposed, which is described

as follows.

First of all, it is clear that the control goal of the system is to make the trolley quickly and accurately track the planned trajectory to the target position, namely:

$$x(t) \to x_{dx}(t) \tag{6}$$

According to equations (6), the positioning error of the payload is defined as follows e(t):

$$e(t) = x_1 - x_{dx}(t) \tag{7}$$

The sliding surface of the non-singular terminal sliding mode is designed as follows:

$$S = e + \frac{1}{\beta} \dot{e}^{\frac{p}{q}} \tag{8}$$

Where,  $\beta > 0$ , p > q, p and q are positive odd.

Then we designed Lyapunov function as follow:

$$V_{S} = \frac{1}{2}g(\theta)S^{2} + \frac{1}{2r_{1}}(\phi_{1} - \widehat{\phi_{1}})^{2} + \frac{1}{2r_{2}}(\phi_{2} - \widehat{\phi_{2}})^{2}$$
(9)

Which  $g(\theta) = M + m \cdot \sin^2 \theta$ ,  $\widehat{\phi}_1$ ,  $\widehat{\phi}_2$  are the estimates of m, m \* L, respectively.

According to the above equations, we design an adaptive tracking controller as follows:

$$F = \frac{q\beta}{pS} \dot{e}^{1-\frac{p}{q}} \left[ -MS\dot{e} - \widehat{\phi}_{1}(\dot{\theta}S^{2}\sin\theta\cos\theta + S\dot{e}\sin^{2}\theta + \frac{pS}{q\beta}\dot{e}^{\frac{p}{q}-1}g\sin\theta\cos\theta) - \widehat{\phi}_{2}\frac{pS}{q\beta}\dot{e}^{\frac{p}{q}-1}\dot{\theta}^{2}\sin\theta + f \right] - \eta \cdot \tanh(S)$$
 (10)

Where  $\eta \in \mathbb{R}^+$  are positive gain, tanh(\*) is the hyperbolic tangent function, then  $\widehat{\phi}_1$ ,  $\widehat{\phi}_2$  are generated by the following update law:

$$\begin{cases}
\dot{\overline{\phi}_{1}} = r_{1}(\dot{\theta}S^{2}\sin\theta\cos\theta + S\dot{e}\sin^{2}\theta + \frac{ps}{q\beta}\dot{e}^{\frac{p}{q}-1}g\sin\theta\cos\theta) \\
\dot{\overline{\phi}_{2}} = r_{2}(\frac{ps}{q\beta}\dot{e}^{\frac{p}{q}-1}\dot{\theta}^{2}\sin\theta)
\end{cases} (11)$$

Where  $r_1, r_2 \in \mathbb{R}^+$  are positive gains.

Under the action of the control rate F and the adaptive rate, the trolley of the overhead crane system can achieve precise positioning in finite time. Next, the corresponding analysis is given to the above conclusions by means of proof.

#### 3.3 Stability analysis

The controller given in (10) and adaptive law in (11) ensures that the position/velocity of the trolley tracks the desired trajectory/velocity asymptotically fast in the sense that:

$$\lim_{t \to \infty} (x(t) \,\dot{x}(t))^T = (x_{dx}(t) \,\dot{x}_{dx}(t))^T \tag{12}$$

First, bring Equation (2) into Equation (1), the second derivative of the swing angle can be expressed as:

$$\ddot{\theta} = \frac{-\ddot{x}cos\theta - gsin\theta}{L} \tag{13}$$

Bringing Equation (13) into Equation (1), the following differential equations can be obtained:

$$(M+m)\ddot{x} - mg\sin\theta\cos\theta - m\ddot{x}\cos^2\theta - mL\dot{\theta}^2 \cdot \sin\theta = F$$
 (14)

According to formula (14), the system state space equation can be obtained:

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 g(\theta) = mL\dot{\theta}^2 \cdot \sin\theta + mg\sin\theta\cos\theta + F - dt \end{cases}$$
 (15)

where dt is the disturbance to the system and  $dt \leq D$ .

Let Lyapunov function  $V_1$  as follow:

$$V_1 = \frac{1}{2}g(\theta)S^2 \tag{16}$$

The derivative of Lyapunov function  $V_1$  with time is:

$$\dot{V}_{1} = \frac{1}{2}\dot{g}(\theta)S^{2} + g(\theta)S\dot{S} = ms^{2}\dot{\theta}\sin\theta\cos\theta + g(\theta)S\dot{e} + \frac{pS}{q\beta}\dot{e}^{\frac{p}{q}-1}(mL\dot{\theta}^{2}\cdot\sin\theta + mg\sin\theta\cos\theta + F - dt)$$
(17)

Taking equation (10) into equation (17), and simplifying, we can get:

$$\dot{V}_{1}=\dot{e}(M+\phi_{1}sin^{2}\theta)\left(e+\frac{1}{\beta}\dot{e}^{\frac{p}{q}}\right)+\frac{pS}{q\beta}\dot{e}^{\frac{p}{q-1}}(mL\dot{\theta}^{2}\cdot sin\theta+mg\sin\theta\cos\theta+F-dt)+\phi_{1}S^{2}\dot{\theta}\cdot sin\theta\cos\theta$$
 (18)

According to the formula (18), the control law (10) can be obtained.

The Lyapunov function is designed as in Eq. (10). The derivative of the Lyapunov function with respect to time is as follows:

$$\dot{V} = \dot{V}_1 - \frac{1}{r_1} (\phi_1 - \widehat{\phi}_1) \cdot \dot{\widehat{\phi}}_1 - \frac{1}{r_2} (\phi_2 - \widehat{\phi}_2) \cdot \dot{\widehat{\phi}}_2$$
 (19)

The adaptive law is designed as in Equation (11), using the designed adaptive law and control law the Equation (19) can be written as:

$$\dot{V} = (-\eta S \cdot \tanh(S) - S \cdot dt) \frac{p}{q\beta} \dot{e}^{\frac{p}{q} - 1}$$
(20)

When state variables error  $\dot{e} \neq 0$ , it gives  $\frac{p}{q\beta} \dot{e}^{\frac{p}{q}-1} > 0$ , then we obtain  $\dot{V} < 0$ ;

When state variables error  $\dot{e} = 0$ , we obtain  $\dot{V} = 0$ ;

Therefore, according to the Lyapunov stability principle, the system is asymptotically stable .

Through the design of the adaptive controller in this section, it can be found that the control method can achieve the task of accurate positioning of the overhead crane in a limited time without linearizing the system model.

## 4. Experimental Results

Fig.2 shows the experimental test setup, it consists of four primary components of a mechanical system, drivers, measurement sensors, and a kernel control system.

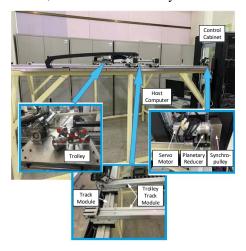


Fig. 2: Experimental test setup

The adaptive non-singular terminal sliding mode control method proposed in this paper is simulated by MATLAB/Simulink R2017a. The trajectory reference is as equation (5). To demonstrate the adaptive ability of the proposed method, three different combinations of payload mass and rope length are set respectively, and different s curve trajectory parameters are used for experiments, as shown in Figure 3-5:

① 
$$L = 1.5m, m = 2kg, p_{dx} = 2, k1 = 0.5, k2 = 0.2, \varepsilon = 3;$$

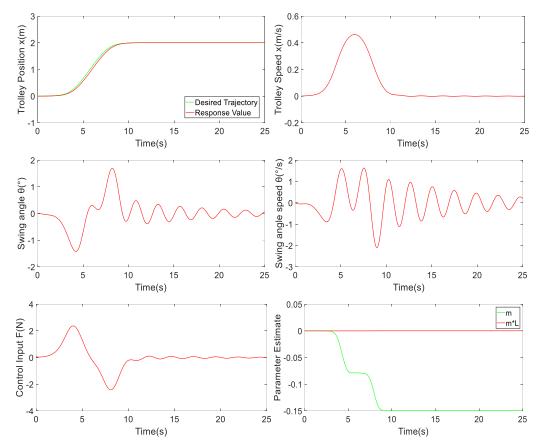


Fig. 3: Results of experiment 1

②
$$L = 1m, m = 1kg, p_{dx} = 2, k1 = 1, k2 = 0.5, \varepsilon = 2;$$

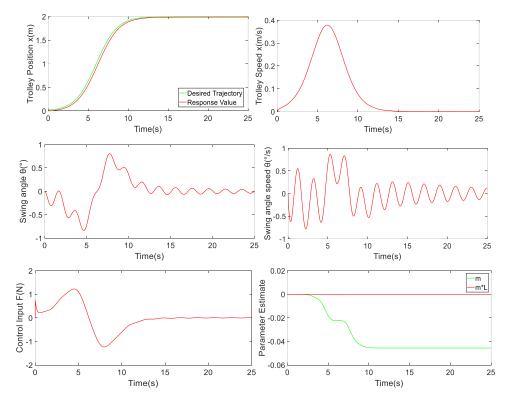


Fig. 4: Results of experiment 2

$$3L = 1.8m, m = 5kg, p_{dx} = 2, k1 = 1.5, k2 = 0.7, \varepsilon = 4;$$

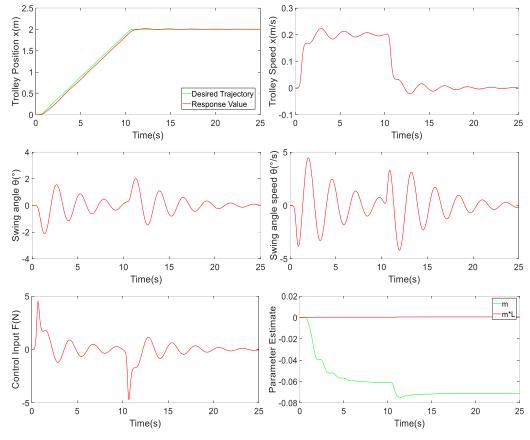


Fig. 5: Results of experiment 3

By comparing the above three experiment, tracking three s curve trajectories with different payload masses and rope length, the following conclusions can be easily found:

- 1) Using the control method proposed in this paper, the control performance is not significantly reduced due to the change of system parameters;
- 2) With the change of system parameters, neither the gains of the controller nor the adaptive law need to be adjusted, which also shows that the algorithm has strong robustness;
- 3) The method proposed in this paper uses the hyperbolic tangent function as the switching item in the sliding mode control, which significantly improves the system chattering phenomenon, thus ensuring the safety and reliability of the method in practical use.

#### 5. Conclusion

In this paper, an adaptive control strategy based on non-singular terminal sliding mode is proposed for underactuated overhead crane systems. This method designs an adaptive control strategy that does not require system model parameters, which greatly improves the scope of application of the method. By combining the non-singular terminal sliding mode technology, the system state has a faster convergence rate. As described in the previous section, these merits bring much convenience for the application of the designed control method in practical overhead cranes and the algorithm design process is relatively simple, easy to promote. Thus, has a strong practical application value.

#### References

[1] Reyhanoglu, M., Van, d. S. A., Mcclamroch, N. H., & Kolmanovsky, I. . (2017). Dynamics and control of a class of underactuated mechanical systems. IEEE Transactions on Automatic Control, 44(9), 1663-1671.

- [2] Wen, Changyun, Wang, Wei, Huang, & Jiangshuai, et al. (2015). Global stable tracking control of underactuated ships with input saturation. Systems & Control Letters.
- [3] MD Hua, Hamel, T., Morin, P., & Samson, C. (2009). A control approach for thrust-propelled underactuated vehicles and its application to vtol drones. IEEE Transactions on Automatic Control, 54(8), 1837-1853.
- [4] C P radip, Saju, N., Sangeetha, S., & Raj, M. G. (2021). Wind energy generation based for grid connected system using zeta converter and terminal sliding mode control. Journal of Physics: Conference Series, 1964(5), 052011 (7pp).
- [5] Li, Y., An, A., Wang, J., Zhang, H., & Meng, F.. (2020). Adaptive robust of rbf neural network control based on model local approximation method for upper limb rehabilitation robotic arm. Journal of Physics Conference Series, 1453, 012086.
- [6] Dong, J., & Yang, G. H. (2017). Observer-based output feedback control for discrete-time t-s fuzzy systems with partly immeasurable premise variables. IEEE Transactions on Systems Man & Cybernetics Systems, 47(99), 98-110.
- [7] He, Z., Liu, C., Zhan, Y., Li, H., Huang, X., & Zhang, Z. (2014). Nonsingular fast terminal sliding mode control with extended state observer and tracking differentiator for uncertain nonlinear systems. Mathematical Problems in Engineering, 2014, (2014-10-27), 2014, 1-16.
- [8] Che, X., Tian, D., Jia, P., Y Gao, & Y Ren. (2020). Terminal sliding mode control with a novel reaching law and sliding mode disturbance observer for inertial stabilization imaging sensor. Sensors (Basel, Switzerland), 20(11).
- [9] Lu, B., Fang, Y., & Ning, S.. (2017). Sliding mode control for underactuated overhead cranes suffering from both matched and unmatched disturbances. Mechatronics, 47, 116-125.
- [10] Fang, Y., Dixon, W. E., Dawson, D. M., & Zergeroglu, E. . (2003). Nonlinear coupling control laws for an underactuated overhead crane system. IEEE/ASME Transactions on Mechatronics, 8(3), 418-423.
- [11] Yongchun, Fang, Bojun, Ma, Pengcheng, & Wang, et al. (2012). A motion planning-based adaptive control method for an underactuated crane system. Control Systems Technology, IEEE Transactions on, 20(1), 241-248.