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Testing and Application for variance Change Points in Long Memory time series

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ABSTRACT. *In this paper, we propose a new statistic to detect for variance change point under long memory, the null distribution of the test statistic is derived and the consistency is proved under the alternative hypothesis. In order to facilitate the practical application, we present a sieve bootstrap procedure which can give asymptotic correct critical value. Simulations show that the proposed method performs well both for small to large and large to small variance change point in long memory time series, Further more, we illustrated our method by a set of IBM stock data.*

KEYWORDS: *variance change point; long memory; Sieve Bootstrap*

1 Introduction

Many hydrological, meteorological, economic, financial and other time series have a long-term dependence phenomenon, that is, the current observation data is correlated with the data observed over a long period of time, although the time interval increases , This correlation will gradually decrease, but it cannot be ignored, and its autocorrelation function decreases slowly in a hyperbolic manner. This phenomenon is called long-memory. It is generally not possible to use some traditional short-term data for time-series data with long memory. Memory time series model to describe. When modeling time series data, not only should the data be considered a short memory process or a long memory process, but sometimes it is necessary to consider whether there is a change point in the data, that is, whether the data is tested before modeling There is a change point, which is a major preparation that needs to be done before modeling.

Variance change point means that the variance of the data before and after a certain time is significantly different. The occurrence of variance change points in the data can easily lead to errors in the results of conventional time series analysis, so it is very important to detect and locate these change points. The earliest Hsu, Miller and Wichern [1] studied the problem of variance change points in time series. Since then, a series of literatures have appeared to study the variance change points

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of independent sequences. For example, Inclan and Tiao [2] have studied the number of independent observation samples. The change point test problem, Chen [3] et al. Proposed a square CUSUM method to detect the variance change point problem in nonparametric regression models, and Gombay [4] et al. Studied the detection and estimation of the variance point of independent and identically distributed samples. In practice, most of the data is independent, that is, it has a certain degree of dependence. The research on the variance change point of dependent data can be traced back to Wichern [5] and other tests on the variance change point in the process of first-order autoregression. Wang [6] used the squared CUSUM test to detect the possible variance points in the linear process of long memory. Horváth and Steinebach [7] studied the mean change of random processes that satisfy the weak invariance principle. Point and variance change point. Other research results on the test point of variance change point can be found in the literature.

2. Models and assumptions

$Y_1,...,Y_n$ is a set of time series observations and is described as follows

$$Y_t = \sigma_t X_t, \quad (1-L)^d X_t = e_t \quad t = 1,...,n,$$

Where L is the lag operator , n is the sample size, σ_t >0 is an unknown and time-dependent parameter, e_t is an independent, identically distributed random variable with mean 0 and variance 1,Memory parameters $0 \le d < 1/2$.

According to the above model, The variance of Y_t is σ_t^2 .This chapter examines the variance change point problem of long-memory time series, that is, the possible change points of the test parameters,

The original assumption is

$$H_0 : \sigma_t = \sigma_0, \qquad t = 1,...,n$$

The alternative hypothesis is

$$H_1 : \sigma_t = \cdots \sigma_{k*} = \sigma_0, \qquad \sigma_{k*+1} = \cdots = \sigma_n = \sigma_1$$

The parameters σ_0, σ_1 and k^* change points are unknown, $\sigma_0 \ne \sigma_1$.

To test the above hypothesis testing problem, the following ratio statistics are proposed:

$$V_n(k) = \frac{\sqrt{n}\left|\varepsilon_{1,k} - \varepsilon_{k+1,n}\right|}{n^{-1}\min\left\{\sum_{t=1}^{k} S_t^2(1,k), \sum_{t=k+1}^{n} S_t^2(k+1,n)\right\}^{1/2}}, \qquad 1 \le k \le n.$$

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$$\varepsilon_{j,k} = (k-j+1)^{-1}\sum_{t=j}^{k} Y_t^{\,2}\ , S_t(j,k) = \sum_{h=j}^{t}(X_h^{\,2} - \varepsilon_{j,k})\,, (\tau_1,\tau_2) = (0.15, 0.85)\,.$$

For a given interval: $\upsilon = [n\tau_1, n\tau_2]$,

$$V_n^{\,*}(k) = \int_{\tau\in\upsilon} V_n(k)d\tau\,.$$

When the value of the test statistic $V_n^{\,*}(k)$ is greater than the given critical value H_0, the null hypothesis is rejected, and there is a point of variance in the data

3. Asymptotic property and Sieve Bootstrap approximation

3.1Asymptotic property

This section gives the test statistics $V_n^{\,*}(k)$ Limit distribution under the null hypothesis and prove the consistency of the test method.

Theorem 1: If the original hypothesis H_0 holds, $n \to \infty$,the limit distribution of $V_n^{\,*}(k)$ is as follows:

$$V_n^{\,*}(k) \to_D \int_{\tau\in\upsilon} \frac{\xi_1(r)}{\xi_2(r)} d\tau$$

$$\xi_1(r) = \left| W_d(r)/r + (W_d(1) - W_d(r))/(1-r) \right|$$

$$\xi_2(r) = \min\left\{ \int_0^r (W_d(s) - \frac{s}{r}W_d(r))^2 ds, \int_r^1 [W_d(1) - W_d(s) - \frac{(1-s)}{1-r}(W_d(1) - W_d(r))]^2 ds \right\}^{1/2}$$

$k = [nr], t = [ns]$,

$$V_n([nr]) = \frac{n^{-1/2-d}\left| \frac{n}{[nr]}\sum_{t=1}^{[nr]} Y_t^{\,2} - \frac{n}{n-[nr]}\sum_{t=[nr]+1}^{n} Y_t^{\,2} \right|}{n^{-1-d}\min\left\{ \sum_{t=1}^{[nr]}[\sum_{h=1}^{t} Y_h^{\,2} - \frac{n}{[nr]}\sum_{t=1}^{[nr]} Y_t^{\,2}]^2,\ \sum_{t=[nr]+1}^{n}[\sum_{h=[nr]+1}^{n} Y_h^{\,2} - \frac{n}{n-[nr]}\sum_{t=[nr]+1}^{n} Y_t^{\,2}]^2 \right\}^{1/2}}$$

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$$=\frac{n^{-1/2-d}\left|\frac{n\sigma_0^2}{[nr]}\sum_{t=1}^{[nr]}(X_t{}^2-1)-\frac{n\sigma_0^2}{n-[nr]}\sum_{t=[nr]+1}^{n}(X_t{}^2-1)\right|}{n^{-1-d}\sigma_0^2\min\{A_1,A_2\}^{1/2}}$$

$$A_1=\sum_{t=1}^{[nr]}[\sum_{h=1}^{[ns]}(X_h{}^2-1)-\frac{[ns]}{[nr]}\sum_{t=1}^{[nr]}(X_t{}^2-1)]^2$$

$$A_2=\sum_{t=[nr]+1}^{n}[\sum_{h=[ns]+1}^{n}(X_h{}^2-1)-\frac{n-[ns]}{n-[nr]}\sum_{t=[nr]+1}^{n}(X_t{}^2-1)]^2$$

$$C_d^{'}>0\,,n\to\infty$$

$$n^{-1/2-d}\sum_{t=1}^{[nr]}(X_t^2-1)\Rightarrow C_d^{'}W_d(r)\,.$$

$$V_n([nr])\Rightarrow_D\frac{\left|W_d(r)/r+(W_d(1)-W_d(r))/(1-r)\right|}{\min\left\{\int_0^r(W_d(s)-\frac{s}{r}W_d(r))^2ds,\int_r^1[W_d(1)-W_d(s)-\frac{(1-s)}{1-r}(W_d(1)-W_d(r))]^2ds\right\}^{1/2}}$$

$$V_n{}^*\Rightarrow_D\int_{\tau\in\upsilon}\frac{\left|W_d(r)/r+(W_d(1)-W_d(r))/(1-r)\right|}{\min\left\{\int_0^r(W_d(s)-\frac{s}{r}W_d(r))^2ds,\int_r^1[W_d(1)-W_d(s)-\frac{(1-s)}{1-r}(W_d(1)-W_d(r))]^2ds\right\}^{1/2}}d\tau$$

Theorem2: If the alternative hypothesis H_1 is set up, $k^*=[n\tau]\in[\tau_1 n,\tau_2 n]$,IF $\sigma\neq 0$ while $n\to\infty$, $\sigma^2=n^{d-1/2}L\ (L\neq 0)$,

$$\lim_{|L\to\infty|}\lim_{n\to\infty}V_n^*\to_P\infty$$

$$V_n(k^*)=\frac{n^{1/2-d}\left|\varepsilon_{1,k^*}-\varepsilon_{k^*+1,n}\right|}{n^{-1-d}\min\left\{\sum_{t=1}^{k}S_t^2(1,k^*),\ \sum_{t=k^*+1}^{n}S_t^2(k^*+1,n)\right\}^{1/2}}$$

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$$\geq \frac{n^{1/2-d}\left|\sigma_1{}^2\right|}{n^{-1-d}\min\left\{\sum_{t=1}^{k} S_t^2(1,k^*),\ \sum_{t=k^*+1}^{n} S_t^2(k^*+1,n)\right\}^{1/2}}$$

$$-\frac{n^{1/2-d}\left|(\varepsilon_{1,k^*}-\sigma_0{}^2)-(\varepsilon_{k^*+1,n}-\sigma_0{}^2-\sigma_1{}^2)\right|}{n^{-1-d}\min\left\{\sum_{t=1}^{k} S_t^2(1,k^*),\ \sum_{t=k^*+1}^{n} S_t^2(k^*+1,n)\right\}^{1/2}} = I_{1n}+I_{2n}$$

$$n^{-1-d}\min\left\{\sum_{t=1}^{k} S_t^2(1,k^*),\ \sum_{t=k^*+1}^{n} S_t^2(k^*+1,n)\right\}^{1/2} \to_D C_d^{'}W_d(\tau,d),$$

$$C_d^{'}W_d(\tau,d)=\min\left\{\int_0^{\tau}(W_d(s)-sW_d(\tau)/\tau)^2\,ds,\int_{\tau}^{1}[W_d(s)-W_d(\tau)-\frac{(s-r)(W_d(1)-W_d(\tau))}{1-\tau}]^2\,ds\right\}^{1/2}$$

$$n^{1/2-d}\left|(\varepsilon_{1,k^*}-\sigma_0^2)-(\varepsilon_{k^*+1,n}-\sigma_0^2-\sigma_1^2)\right| \to_D C_d^{'}Z(\tau,d)$$

$$Z(\tau,d)=\left|W_d(\tau)/\tau+(W_d(1)-W_d(\tau))/(1-\tau)\right|.$$

$$0\leq d<1/2,\ n\to\infty,\ I_{1,n}\to\infty,\ I_{2,n}\to_D Z(\tau,d)/W(\tau,d).$$

$$k\in\upsilon,\ V_n^*(k)>V_n(k^*),\ V_n^*(k)\to\infty.$$

3.2 Sieve Bootstrap approximation

This section uses the following Sieve Bootstrap method to determine the critical value of the statistics, the specific steps are as follows:

(1): Estimate long memory parameters： d ， The estimated amount is $\hat{d}$.

(2): Make: $Y_1,\ldots,Y_t$ the order $\hat{d}$ difference,, get the sequence. $Z_1,\ldots,Z_t$,, and record it Z_t .

(3): Make Z_t the fitting autoregressive process $AR(n)$, $Z_t=a_1Z_{t-1}+a_2Z_{t-2}+\ldots+a_{pn}Z_{t-pn}+\varepsilon_t$,Let the fitting residual

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is $\hat{\varepsilon}_1,...,\hat{\varepsilon}_p$,And extract the autoregressive order and autoregressive coefficients, p ,he autoregressive coefficient is recorded as $\hat{\beta}_1,...,\hat{\beta}_p$.

(4): Make residuals $\hat{\varepsilon}_1,...,\hat{\varepsilon}_p$ resample to get the sequence, And based on $\hat{\varepsilon}_1^*,...,\hat{\varepsilon}_{n+p}^*$, rebuild $AR^*(n)$ process：. $\hat{\varepsilon}_1^*,...,\hat{\varepsilon}_{n+p}^*$

$$\tilde{\varepsilon}_t^* = \hat{\beta}_1\tilde{\varepsilon}_{t-1}^* + \hat{\beta}_1\tilde{\varepsilon}_{t-2}^* + ... + \hat{\beta}_p\tilde{\varepsilon}_{t-p}^* \ , \quad t = 1,...,n.$$

(5): Extract $AR^*(n)$ the process information is fitted to the memory parameters as $\hat{d}$ get Sieve

Bootstrap sample Z_t^*.

(6): Calculate statistics $V_n^*(k)$, Repeat steps4-6 B times, The quantile is taken as the critical value at a significant level.

4 Numerical simulation

This section analyzes the test statistics by means of numerical simulation $V_n^*(k)$ and Sieve Bootstrap the finite sample nature of the method, Sampling cost n is 200and 500, $(\tau_1,\tau_2) = (0.15,0.85)$, All simulations are cycled 2000 times, consider the following data generation process:

$$Y_t = \begin{cases} X_t, & 1 \le k \le k^*, \\ \sigma X_t, & k^* < k \le n, \end{cases}$$

Table 1: Experience level（%）

d	n	Level		
		0.1	0.05	0.01
0.3	200	7.2	3.2	0.5
	500	8.6	3.6	0.8
0.4	200	7.9	4.1	0.6
	500	8.8	4.2	1.1

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Table 2: Empirical potential (%)

σ	τ	n	Level			σ	τ	n	Level		
			0.1	0.05	0.01				0.1	0.05	0.01
2	0.25	200	79.7	66.6	36.9	1/2	0.25	200	76.9	64.0	35.8
		500	92.8	89.6	62.7			500	92.7	83.8	57.2
	0.5	200	91.6	86.1	60.9		0.5	200	90.6	79.4	61.3
		500	100	97.4	86.4			500	99.2	94.3	82.4
	0.75	200	77.6	64.7	32.8		0.75	200	80.7	66.2	43.5
		500	90.6	84.8	60.5			500	95.1	84.9	59.6
3	0.25	200	100	98.0	89.6	1/3	0.25	200	96.7	93.7	84.2
		500	100	100	98.3			500	100	98.8	92.3
	0.5	200	100	100	97.2		0.5	200	99.6	99.1	96.7
		500	100	100	100			500	100	100	98.5
	0.75	200	98.6	95.2	87.4		0.75	200	98.5	96.4	85.9
		500	100	100	95.9			500	100	99.7	95.1

Table 3: $d = 0.4$ *Experience potential（%）*

σ	τ	n	Level			σ	τ	n	Level		
			0.1	0.05	0.01				0.1	0.05	0.01
2	0.25	200	54.7	35.4	16.5	1/2	0.25	200	48.7	30.8	14.3
		500	69.9	56.1	26.1			500	62.3	45.1	24.1
	0.5	200	69.1	60.2	33.7		0.5	200	74.5	54.7	26.5
		500	79.6	66.8	36.2			500	73.6	66.7	34.8
	0.75	200	49.3	32.6	14.3		0.75	200	58.9	37.0	16.7
		500	61.8	45.2	21.6			500	64.3	51.2	26.4
3	0.25	200	89.8	83.0	66.4	1/3	0.25	200	84.4	75.2	55.7
		500	96.6	91.8	70.3			500	93.3	85.6	64.1
	0.5	200	96.9	87.7	77.9		0.5	200	93.7	89.6	78.4
		500	99.0	93.4	83.2			500	97.2	91.5	82.3
	0.75	200	85.7	75.1	62.5		0.75	200	88.6	86.6	57.6
		500	91.4	89.5	68.4			500	95.3	88.3	67.9

Among them is a process, taking 0.3 and 0.4, = 0.25, 0.5, 0.75. The inspection level is taken 10%, 5%, 1%.Table 1 lists the simulation results of the experience level. It can be seen that the experience level is slightly lower than the given inspection level as a whole, but as the sample size increases, the experience level gradually approaches the inspection level, and this slight effect is it can be accepted. This shows that the Sieve Bootstrap method proposed in this chapter can approximate the critical value of the test statistic.

The following analyzes the empirical potential of the test statistic under the alternative hypothesis. Table 2 and Table 3 list the long memory parameters, $d = 0.3$ and $d = 0.4$ respectively. It can see the parameters d And the size of the

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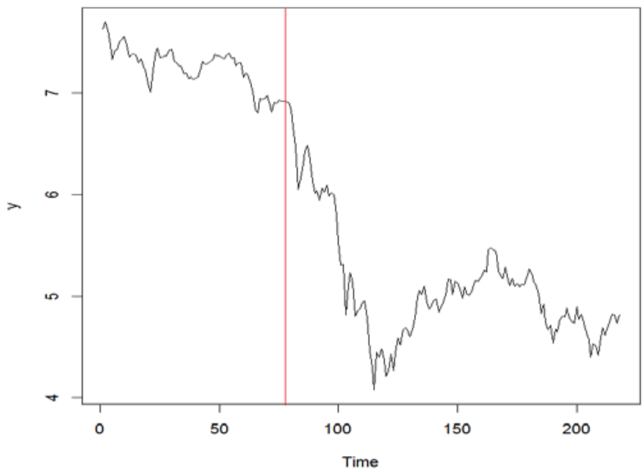
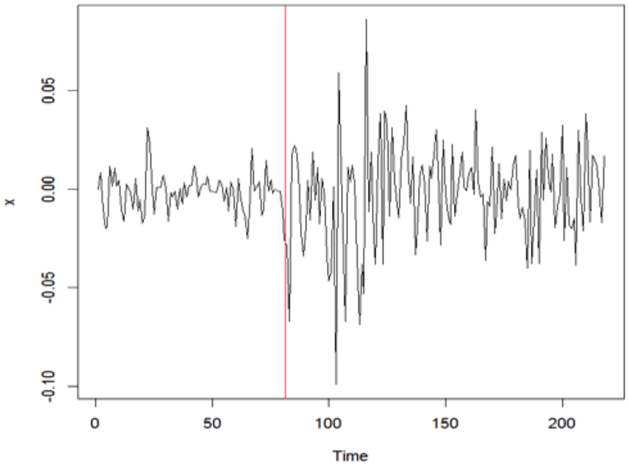
sample size and the change of the change point position have a great influence on the test potential, Under various conditions $d = 0.3$ Empirical potential $d = 0.4$ is high, As the sample size increases, the empirical potential will also increase to a certain extent, and it will be easier to detect when the change point appears in the middle of the sample., When the variance changes from large to small, the position of the point changes $k^* = 0.75n$, The test potential at time is greater than the position of the change point when $k^* = 0.25n$,Conversely, when the variance changes from small to large, the change point is at $k^* = 0.25n$.

5. Case analysis

Consider the data series of IBM stock closing prices from January 1, 1962 to November 2, 1962 $\{Y_{0,k}\}$, a total of 218 observations, see Figure 1. Perform log-order difference transformation on the original data to obtain the sequence $\{Y_k\}, Y_k = \log(Y_{0,k} / Y_{0,k-1})$ The daily logarithmic return of IBM stock is shown in Figure 2.Many scholars have studied this group's rate of return data from different angles. Wichern, Miller and Hsu (1976) are the first to analyze this group of data from the perspective of variance change point. Use the method of this paper to detect whether there is a variance change point in the yield. The value $V_n^*(k)$ of the statistic at the 103rd sample is 48.65, which is greater than the Sieve Bootstrap approximate critical value of the statistic at the level of 0.05, 23.34.Therefore, at the 0.05 level, the original hypothesis of no variance variance point is rejected, that is, there is a variance variance point in the set of data. The position of the variance point given by the vertical line in Figures 1 and 2 is used in this example to estimate the location of the change point is very close to the existing conclusions, which shows that the method in this paper is effective.



Figures 1



Figures 2

6. Conclusion

The estimation of long memory parameters is a very important and complicated problem. This paper proposes a Sieve Bootstrap method of fractional order difference to approximate the critical value of the statistic, which overcomes the traditional method of listing the critical value table and approximates the critical value in the Sieve Bootstrap method. During the process, the accuracy of parameter estimation is low, and the workload is reduced. The simulation results show that the method in this paper can not only control the experience level very well, but also overcome the problem that the traditional variance change point test method cannot simultaneously detect the two cases of variance from large to small and variance from large to small in long memory time series. Compared with the Ratio method and the square CUSUM method in the literature, the test potential has also been significantly improved in some cases. Finally, through a set of IBM stock data to illustrate the practicality of this method.

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