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A Comparative Study of Chinese and Australian Senior Secondary Mathematics Teaching Materials—Exponential, Logarithmic Functions as an Example

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Abstract: *This paper compares and analyzes the content of the Australian VCE mathematics course "Mathematical Methodology" textbook with that of the Chinese version of the exponential and logarithmic functions. A quantitative model of the difficulty coefficient of the course is used. The main aspects of the comparative analysis are the distribution of knowledge points and the order of arrangement, the difficulty of the course, and the setting of examples and exercises. Keywords: exponential logarithmic functions; high school mathematics textbooks; VCE courses; comparative analysis.*

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1. Introduction

With the trend of economic globalization, the internationalization of education has become more and more prevalent, and the international exchange and cooperation in mathematics education has become more and more concerned, and the international comparison of mathematics teaching materials has become a hot issue in educational research. Australia, which is located in Oceania and adjacent to Asia, is rich in educational resources and has a long history and high level of mathematics education, which is worthy of in-depth study. This paper selects exponential and logarithmic functions as the objects of comparison between Chinese and Australian textbooks, and compares the arrangement and difficulty of the two textbooks in order to provide useful reference for the reform and preparation of high school textbooks in China.

2. Study design

2.1 Research Subjects

In this paper, we compare and contrast high school mathematics textbooks in China and Australia. The Chinese textbook is selected from the Renminbi A version of the high school mathematics textbook (hereinafter referred to as the Renminbi version), and the specific chapter is Chapter 4 "4.1 Exponents, 4.2 Exponential Functions, 4.3 Logarithms, 4.4 Logarithmic Functions" in the Compulsory 1 of high school mathematics; the Australian textbook is selected from the VCE mathematics textbook "Essential Mathematical Methods Units 1&2" published by Cambridge University Press (hereinafter referred to as the Cambridge University Press). The Australian textbook is "Essential Mathematical Methods Units 1&2", published by Cambridge University Press, UK, which is translated as "Mathematical Methodology" (hereinafter referred to as (EMM). The textbook is representative and authoritative in Australia, mainly for high school students in Victoria, and the specific content of the comparison is Chapter 12 "Exponential Functions and Logarithms".

2.2 Research Methodology

In this paper, the quantitative model of course difficulty coefficients by Professor Ningzhong Shi is used.[1] In this paper, we analyze the content of the "Exponential and Logarithmic Functions" section of the Chinese and Australian textbooks qualitatively and quantitatively, and derive the course difficulty

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coefficients of the course content of the two textbooks. The quantitative model of course difficulty coefficients is: $N = a\frac{S}{T} + (1-a)\frac{G}{T}$

Where: G represents the breadth of the course, quantified by the number of knowledge points in the textbook; T represents the duration of the course, quantified by the number of class hours used for teaching; S represents the depth of the course; N represents the difficulty factor of the course; $\frac{S}{T}$ represents comparable depth; $\frac{G}{T}$ The weighting factor is a (0 < a < 1), which reflects the emphasis of course difficulty on comparable depth and comparable breadth.[2] The weighting coefficients reflect the emphasis of course difficulty on comparable depth and comparable breadth.

This paper quantifies the content of the Chinese and Australian textbooks to different degrees by the four objectives of "understanding, comprehension, mastery, and flexible application" in the Chinese curriculum standards, and assigns different values according to different objective levels.[3] The different values are assigned according to different target levels, as shown in Table 1.

Table 1: Course depth level assignment

Assignment	Knowledge Skills Objectives	Process Objectives
1	understanding	Experiences(feelings)
2	understanding	Experiences(experience)
3	Mastery	Explore
4	Flexible use	—

3. Research results

3.1 Comparison of Chinese and Australian high school mathematics textbooks

3.1.1 Comparison of the distribution of knowledge points and the order of arrangement

The exponential and logarithmic functions, which are arranged in the Compulsory 1 textbook in the Hanyu version, are as follows:

Table 2: Order of arranging knowledge points in different versions of textbooks

Chinese version and content of the version (human education version)[4]	Australian version and version content (EMM version)[5]
Chapter Knowledge	Chapter Knowledge
4.1 Indices	Chapter 12 Exponential Functions and Logarithms
4.1.1 Roots of nth order and fractional exponential powers	12.1 Graphs of exponential functions (Exponential function graph)
4.1.2 Exponential powers of irrational numbers and their operational properties	12.2 Reviewing rules for exponents (indices)
4.2 Exponential functions	12.3 Rational exponents (rational indices)
4.2.1 The concept of exponential function	12.4 Solving exponential equations and inequations
4.2.2 Graphs and properties of exponential functions	12.5 Logarithms (logarithms)
4.3 Logarithm	12.6 Using logarithms to solve exponential equations and inequations (Solving exponential equations and inequalities with logarithms)
4.3.1 The concept of logarithm	12.7 Graph of y= ,where a >1 (graph of the function of (a > 1))
4.3.2 Operations on logarithms	12.8 Exponential models and applications
4.4 Logarithmic functions	
4.4.1 The concept of logarithmic function	
4.4.2 Graphs and properties of logarithmic functions	
4.4.3 Differences in the growth of different functions	

After comparing the corresponding contents of the two versions of the textbooks, it can be seen that the exponential and logarithmic functions of the two countries' high school mathematics textbooks are

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mainly to study the concepts, properties and graphical characteristics of functions. The general content of the textbooks is basically the same, with a strong focus, and the overall order of arrangement is the same, from exponential to logarithmic, but there are also obvious differences in the curriculum and other aspects of the two textbooks, as follows:

In terms of unit chapter arrangement: the chapter arrangement of the Chinese textbook shows a parabolic pattern, emphasizing the process of knowledge occurrence from shallow to deep in the early part; in the later part, more emphasis is placed on returning to intuitive reality, emphasizing the cultivation of students' ability to return abstract mathematical knowledge to practical problems. The EMM mathematics textbook, on the other hand, focuses more on cultivating students' ability to learn mathematical theory, hoping that through the study of mathematics in high school, the foundation for studying more abstract and systematic mathematical theory in college will be laid, focusing on students' academic development.

The EMM version of the textbook is more decentralized, splitting a large content into different chapters, mainly through the change of function graphs to introduce the definition, image and properties of exponential and logarithmic functions, while the EMM version of the textbook first introduces the model of exponential and logarithmic functions through specific life examples. The textbook of the Human Education version first introduces the model of exponential and logarithmic functions through concrete life examples, and then studies the characteristics and properties of the graphs.

Knowledge selection: The EMM textbook involves the study of the rules of exponential functions, which belongs to the category of junior high school mathematics in our mathematics textbook and lays the foundation for the study of high school mathematics. At the same time, the rational exponents in the EMM textbook correspond to the nth root and fractional exponential power of the human education version of the textbook, the EMM version of the solution of exponential equations and inequalities in a separate section, the human education version is not arranged in the main course, reflected in the exercises to solve the problem.[5]

3.2 Comparison of course difficulty

It is difficult to measure the difficulty of courses in China and Australia, and it is not convincing to portray it by experience alone. In this paper, we analyze qualitatively and quantitatively through three factors: course breadth, course depth, and course duration, with the help of Professor Shi Ningzhong's course difficulty model.

3.2.1 Quantitative basis

The quantification of the breadth of the course directly counts the number of knowledge points in the textbook. In terms of course time, our textbook has given the prescribed time in the curriculum standards, while the EMM textbook needs to be determined according to the actual teaching situation, because the content of each chapter has been divided by the textbook, so according to the actual teaching progress, each subsection of the content is calculated according to the actual 1-2 hours.

The depth of the course, which corresponds to different levels of abstraction of concepts according to the depth of learning thinking, has been classified and assigned as follows: Understanding level means being able to remember the learned knowledge points, including concepts, definitions, formulas, axioms, and operational activities, which is the minimum requirement for knowledge points. Understanding refers to having a holistic understanding of the context of a concept, being able to derive formulas and prove theorems, and being aware of the role of the concept in the whole chapter and section. Mastery refers to the ability to use the concept and to master the ideas and steps of using axioms and theorems to solve problems, which is the role of most theorems, or the common level of students in solving problems. Flexibility level is an advanced level, requiring proficiency in using known knowledge to solve complex problems, which may use multiple knowledge points, from the height of mathematical thinking, using certain mathematical skills in order to solve. The EMM textbook introduces the learning objectives of the chapter before each chapter is studied. Specific behavioral verbs such as: define, sketch, understand, solve, evaluate, apply, etc. divide the mastery level of knowledge points. In order to unify the standards, the depth of knowledge points in the EMM textbook is also divided into understanding, comprehension, mastery and flexible application, taking into account the requirements of actual teaching and examinations. And they are recorded as level 1, level 2, level 3, and level 4, and the assigned weights are 1,2,3,4 respectively, as shown in Table 2 above.$S = \frac{\sum_{i=1}^{4} nidi}{n}$ (ni represents the number of knowledge points of the ith level, di represents the level of hierarchy, n represents the total number of knowledge

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points) to calculate the depth of the course.

3.2.2 Quantification of results

The human education version of the textbook refers to the logarithmic function part of the knowledge points are: n roots and fractional exponential powers (understanding), irrational exponential powers and their properties of operations (understanding), the concept of exponential functions (mastery), the graph and properties of exponential functions (flexibility), the concept of logarithms (understanding), the operation of logarithms (understanding), the concept of logarithmic functions (mastery), the graph and properties of logarithmic functions (flexibility), the The difference between the growth of different functions (understanding) a total of 9 knowledge points. Note that the breadth of 9. [6]

The EMM textbook refers to the logarithmic function part of the knowledge points are: the graph of the exponential function (master), exponential function rules (understand), rational exponents (understand), solving exponential equations and inequalities (flexible use), logarithms (understand), using logarithms to solve exponential equations and inequalities (flexible use), logarithmic function graphs (master), exponential models and applications (understand) a total of eight knowledge points. The breadth is recorded as 8.

A comparison of the depth of the course, dividing the 8 knowledge points of the HRE textbook and the 8 knowledge points of the EMM textbook according to the level of knowledge:

Table 3: The level of knowledge points of different versions of textbooks

	Humanities Education Edition		EMM version	
	Number	Percentage	Number	Percentage
Level 1 (Understanding)	2	22.2%	2	25%
Level 2 (Understanding)	3	33.3%	2	25%
Level 3 (mastery)	2	22.2%—	2	25%
Level 4 (Flexibility)	2	22.2%	2	25%
Total	9	100%	8	100%

As can be seen from the above table 3, the number of knowledge points between the two countries is not very different and the level of knowledge required is relatively equal. In terms of the operation of exponential powers, our textbook has gone into the level of fractional exponential powers and irrational exponential powers, while the EMM version of the textbook involves relatively simple and shallow rules of exponential operations. However, the EMM textbook does not have separate chapters for solving exponential equations and inequalities, solving exponential equations and inequalities with logarithms, and exponential models and applications, which shows that the two countries' textbooks have different emphases on knowledge content.

The average depth is 2.5609 and 2.625, which are close to each other. According to the curriculum standards, the Human Education version refers to a total of 15 hours of lessons on functions, and the EMM textbook is based on the actual teaching, and each subsection is calculated at an average of 1.5 hours for a total of 12 lessons. Based on the above data, we can calculate the difficulty coefficient of the Human Education version to be 0.4603 and the difficulty coefficient of the EMM textbook to be 0.4426. (Here a is taken as 0.5.) It can be seen that although the difference between the breadth and depth of the courses of the two countries is not great, the depth of the Human Education version is slightly greater and the difficulty coefficient is higher.

3.3 Comparison of the difficulty of the examples and exercises

3.3.1 Comparative analysis of the number of example problems

The design of the chapters and the order of the knowledge points are not the same between the Human Education textbook and the EMM textbook, but in general, the four key knowledge modules include the concept of exponential function, the graph and properties of exponential function, the concept of logarithmic function, and the graph and properties of logarithmic function, etc. From the total number of sample problems, we can see that there are 17 sample problems in the Human Education textbook and 28 sample problems in the EMM textbook.

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Table 4: Percentage of each knowledge module in the total number of examples in the book for both editions

Knowledge Modules	The number of examples of each knowledge module as a percentage of the total number of examples in the book	
	Humanities Education Edition	EMM version
(a) Exponential power operation rules	2.803%	0.763%
(b) the graph and properties of the exponential function	3.738%	1.526%
(c) the concept of logarithms and operations	4.807%	0.763%—
(d) The graph and properties of logarithms	1.869%	0.763%

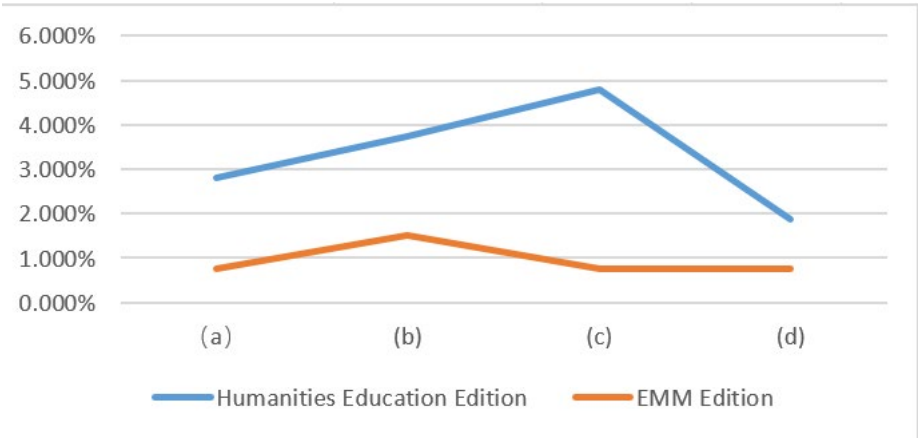

Figure 1: Percentage of each knowledge module in the two

According to Table 4 and Figure 1, the Human Education version has more examples on the concept and operation of logarithmic functions, and both versions have more examples on the graphical part of exponential functions. In comparison, the EMM version focuses more on the graphing calculator to deepen the understanding of function graphs.

3.3.2 Comparative analysis of the difficulty of the example problems

The structure of the examples in the two textbooks has commonalities and differences. The similarities lie in the fact that each section of the textbook basically involves examples, while the differences lie in the fact that the Human Education version of the textbook sets up an inquiry, which introduces a specific situation by asking a question and using the knowledge of the section to solve the problem in combination with the explanation of the example to stimulate students' interest in the inquiry. The examples and exercises in the textbook have appropriate connection and linkage, but not one-to-one correspondence, while the EMM textbook handles this point meticulously, each example has a detailed analysis and solution process, the practice problems are appropriately difficult with the examples, and humanely marked with the example number corresponding to the exercise, that is, for each detailed example, you can find the matching exercise for practice and timely consolidation. It is not only convenient for students to consolidate their knowledge, but also helps teachers to assign homework.

In terms of the difficulty of the examples, the EMM textbook shows a complex and large amount of arithmetic, but it is just suitable for solving problems with the graphing calculator. The example problems in the Human Education version are moderately difficult and meet the cognitive development level of students.

Two example problems from each of the two versions of the textbook are taken:

Index component:

HRE: (Example 1) The known exponential function$f(x) = a^x$ (a>0 and a≠1), and $f(3) = \pi$, find the values of $f(0), f(1), f(-3)$.

EMM Version: (Example 6) Sketch the graphs of each of the following.

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a.$y = 2 \times 3x$ b. $y = 32x$ c. $y = \frac{3x}{2}$ d. $y = -3^{2x} + 4$

Logarithmic component: Humanities: (Example 2) Find the value of x in the following equations

(1) $log64\, x = -\frac{2}{3}$ (2) $logx\, 8 = 6$

(3) $log10\, 100 = x$ (4) $-ln\, e^2 = x$

EMM version: (Example 16) Without the aid of a calculator evaluate the following.

a. $log2^{32}$ b. $log3^{81}$

This part of the examples pay particular attention to the development of students' abstract thinking, through the observation and mastery of the graph of the exponential function, logarithmic function, to improve the students' core literacy in mathematical abstraction and mathematical operations.

3.3.3 Comparative analysis of exercise settings and difficulty

There are three types of exercises in the chapter on exponential and logarithmic functions in the Hanyu textbook: section exercises, post-section exercises, and chapter review reference questions. The post-section exercises and chapter review reference questions are divided into three sections: review and consolidation, comprehensive application and broadening exploration. The specific numbers are as follows: 27 exercises in the section, 43 post-section exercises, 19 review and consolidation, 16 comprehensive application, and 8 broadening exploration; 13 review reference questions, 4 review and consolidation, 6 comprehensive application, and 3 broadening exploration.

There are 2 types of questions in the EMM textbook: post-section practice questions and chapter review questions. The number of questions is as follows: 47 practice questions after the section. There are 31 chapter review questions, 10 multiple choice questions, 11 short answer questions, and 10 extended answer questions; the classification of the questions is more detailed in the Chinese textbook, which is helpful for teachers and students to choose exercises of suitable difficulty and quantity, while the exercises in the Australian textbook require teachers' grasp of the textbook and knowledge and understanding of students' situation. In terms of the content of the exercises, the Chinese textbook exercises are divided into two categories: those involving mathematical knowledge training and those involving practical problems.

Most of the exercises involving practical problems are arranged in the post-section exercises and chapter review reference questions, accounting for about 50%-60% of the total number of questions, most of which are related to real life and production, reflecting the new curriculum reform's emphasis on the application of mathematical knowledge and the need to relate to practical aspects. Most of the questions are related to real life and production, which reflects the new curriculum reform's emphasis on the application of mathematical knowledge and the need to link it to the real world. In addition, the exercises in the EMM textbook are more difficult and require students to apply their knowledge in a comprehensive manner. Therefore, it is better than the Chinese textbook in training students' mathematical thinking skills and comprehensive application of mathematical theory knowledge, which is more beneficial to students' further study.

4. Suggestions for reflection (conclusions and insights)

This study compares the content of exponential and logarithmic functions in the textbook of the Human Education Edition and the textbook of the Australian VCE course EMM (Mathematical Methodology), starting from the distribution and order of knowledge points, the difficulty of the course, the difficulty of examples and exercises, etc. Both textbooks have their own characteristics. In addition, there are many shortcomings in this study, such as the definition of the level of knowledge points should be related to other knowledge, and should be related to other factors besides depth, breadth, and time, and the comparative analysis of the difficulty of the examples and exercises in the textbook is not deep enough, and the background of the topic, the level of inquiry, the degree of arithmetic, the degree of reasoning, and the knowledge content are not considered.

Therefore, through the comparison of the textbooks of the two countries, it enlightens the textbook writers that they should go deep into the front line of teaching when writing the textbooks, make dynamic adjustments based on the feedback from teaching through actual surveys of teachers and students, refer to academic papers on the comparison of the difficulty of mathematical problems at home and abroad, and draw on the examples of other versions of textbooks to write mathematical problems and even

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mathematics textbooks with Chinese characteristics for the preparation of "icing on the cake".

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