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Design and Reflection on High School Mathematics Proposition Teaching Based on the CPFS Structure: A Case Study of "Graphs and Properties of Exponential Functions"

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Abstract: *Mathematical propositions are fundamental building blocks of the knowledge system in high school mathematics. To advance competency-oriented curriculum reform, align proposition-based instruction with students' cognitive experiences, and achieve deep integration of teaching and learning, this study takes the "Graphs and Properties of Exponential Functions" as a specific case to explore how to construct students' CPFS structure (a theoretical framework of cognitive structure) in proposition-based teaching. The paper begins by outlining the knowledge system of exponential functions' graphs and properties, clarifying core concepts and their intrinsic connections. It then establishes learning objectives for the lesson, and finally proposes an instructional process design based on the CPFS structure. This aims to provide practical teaching insights for educators, promote the systematization of students' cognitive structures, and optimize the teaching reform of mathematical proposition instruction in high school.*

Keywords: *CPFS Structure, Proposition Teaching, Exponential Functions*

1. Introduction

Mathematics Curriculum Standards for Ordinary High Schools (2017 Edition, Revised in 2020) emphasizes the selection of core disciplinary content and structuring the curriculum around key concepts to develop students' mathematical core competencies[1]. The implementation of the new curriculum standards has prompted a gradual transformation in high school mathematics teaching models, calling for greater emphasis on students' independent inquiry and deep understanding.

In particular, learning mathematical propositions in high school involves three stages: acquisition, proof, and application. However, in practice, some teachers tend to merely list isolated knowledge points, focusing heavily on the application of propositions while neglecting the acquisition process. This approach hinders students' understanding of the overall knowledge architecture, limits their ability to establish broad connections between concepts, and results in a fragmented cognitive structure, ultimately constraining their capacity for knowledge transfer and problem-solving.

How can proposition-based instruction be better aligned with students' cognitive experiences, and how can teaching and learning be deeply integrated? Research suggests that the CPFS structure theory offers a new perspective for propositional teaching. From a cognitive psychology standpoint, this theory reveals the essence of mathematical learning: forming a well-connected and robust knowledge network. The CPFS structure is a unique cognitive framework in mathematics learning, encompassing not only the knowledge structure within an individual's mind but also the intricate connections between mathematical concepts-especially relationships such as equivalence, strong abstraction, weak abstraction, and generalized abstraction among mathematical concepts and propositions[2].

Therefore, this theory provides a foundation for teachers to help students better understand the nature and internal connections of mathematical propositions. By designing instruction centered on the CPFS structure, teachers can guide students to gradually uncover the logical chains between knowledge points, promoting the systematization of their cognitive structure. Based on this approach, this study uses the "Graphs and Properties of Exponential Functions" as a practical case to explore how to construct students' CPFS structure in proposition-based teaching.

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2. Instructional Preparation for Teaching the Graphs and Properties of Exponential Functions within the CPFS Structure

This lesson is drawn from Section 4.2.2 of the Compulsory Mathematics Textbook for Senior High Schools, Volume 1 (People's Education Press Edition A), and plays a pivotal role in the unit and even within the broader function-focused curriculum of high school mathematics. As the exponential function is the next complex function students encounter after power functions, this lesson not only delves into an in-depth exploration of the properties of exponential functions, but also serves as a guiding model within the larger context of function learning. It lays a solid foundation and accumulates methodological experience for students' subsequent study of logarithmic and trigonometric functions.

2.1 Analysis of Learning Content and the CPFS Structure

Prior to this lesson, students have already learned the concept of exponential functions and their analytic expressions, and have been introduced to two types of exponential function models: exponential growth and exponential decay (concept field). Furthermore, building on their previous study of power functions, students have a preliminary understanding of the general approach to studying functions, namely "background → concept → graphs and properties → application" (concept system). That is, they are able to plot function graphs using the method of plotting points based on the analytic expression, and use the graphs to summarize the properties of functions. At this stage, students' abstract thinking abilities have improved, but their skills in exploring problems, as well as their mathematical modeling and abstraction capabilities, remain underdeveloped.

2.2 Learning Objective Design Based on the CPFS Structure

Based on the requirements of the new curriculum standards and students' current level of CPFS structure development, the main instructional thread of this lesson is determined as follows: starting from concrete examples and integrating prior knowledge of power functions, outline the general approach and methods (concept system) for studying exponential functions → plot specific function graphs → summarize common features of the graphs, and describe properties of exponential functions (such as monotonicity and special points) in multiple ways (verbal, algebraic, graphical, etc.) (proposition field) → through solving problems like "comparing the size of numerical values," independently reveal the logical relationships between properties and graphs, between different properties, and between old and new knowledge (e.g. using monotonicity to solve inequalities) (proposition system), thereby deeply understanding the process of mathematical knowledge development and enriching the function learning system (CPFS structure). Simultaneously, during this process, students will experience mathematical thought and method such as transformation and classification, and the integration of numbers and shapes, enhance their observational, analytical, and inductive abilities, and cultivate core mathematical competencies like mathematical abstraction and intuitive imagination.

2.3 Teaching Activity Design Based on Cognitive Progression

Based on the above analysis, a flowchart of teaching activities for learning the graphs and properties of exponential functions under the CPFS structure has been designed, building on previous research (as shown in Figure 1) [3].

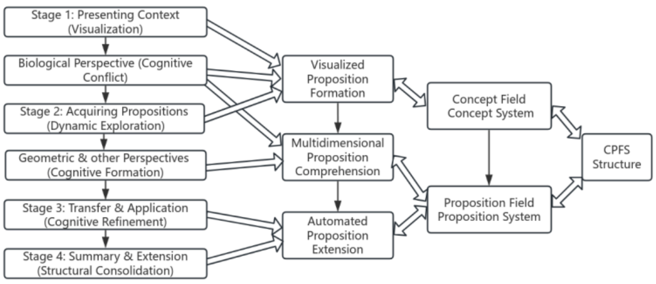

Figure 1: Instructional Activity Flowchart under the CPFS Structure.

3. Instructional Process Design for "Graphs and Properties of Exponential Functions" within the CPFS Structure

To effectively achieve the teaching objectives, suitable instructional scenarios are designed based on the CPFS structure, accompanied by carefully formulated mathematical questions. Using interactive teaching methods, students are guided to learn through independent inquiry and collaborative communication, enabling them to learn how to learn and how to think.

3.1 Effective Review to Activate Prior Knowledge

[Question 1] What is the definition of an exponential function?

The teacher guides students to recall the definition of an exponential function and highlights the widespread application of exponential function models in real life. Through the example of an exponential growth model based on the “rabbit plague in Australia,” students gain an intuitive understanding of the characteristics of exponential growth.

[Introductory Example] Let us consider two breeds of rabbits, Breed A and Breed B, which possess different reproductive capabilities, and assume that both start with an initial population of 1 unit.The population of Breed A doubles every two months, while the population of Breed B triples every three months. Which breed will have a larger population after five years?

Students think independently and transform the practical problem into a mathematical task of comparing the sizes of two values, $2^{30}$and 3^{20}.

Design Intent:

Through review and reflection, this activity aims to reactivate the concept system within students' existing CPFS structure related to exponential functions. The “rabbit plague” problem is used to create cognitive conflict—students realize that using previously learned properties of power functions is insufficient to compare exponential expressions with different bases and exponents. This paves the way for enriching the students' CPFS structure in this lesson. Throughout the process, students develop the ability to observe the world through a mathematical lens, think about the world with mathematical reasoning, and describe the world in mathematical language.

[Question 2] To clarify that this lesson will continue to explore the "graphs and properties of exponential functions," students review the "general approach to function study." By further recalling the study of power functions, consider: How should we investigate the graphs and properties of exponential functions?

Teacher-Student Interaction: Students independently outline the exploratory approach for studying the graphs and properties in this lesson, with the teacher providing guidance and corrections: starting from the analytic expression, plotting specific function graphs, observing common features of the graphs, and summarizing the function’s properties such as domain, range, monotonicity, parity, and common points.

Design Intent:

Reviewing the general function study approach helps clarify the learning focus of this lesson. By drawing analogies with the learning process and content of power functions, students further activate their existing CPFS structure related to functions. This prepares them for building the proposition field of "properties of exponential functions" and expanding the proposition system of "function properties" in this lesson.

3.2 Exploring New Knowledge and Building the Proposition Field

[Exploratory Activity 1] Group collaboration to plot specific graphs.

Student Activity: The class is divided into three groups. Each group completes the plotting of six function graphs (such as $y = 2^x$, $y = (\frac{1}{2})^x, y = 3^x$, $y = (\frac{1}{3})^x, y = 4^x, y = (\frac{1}{4})^x$), shares their results, and discusses the process and key points of graphing using the point-plotting method.

[Exploratory Activity 2] Observing function graphs to identify common characteristics.

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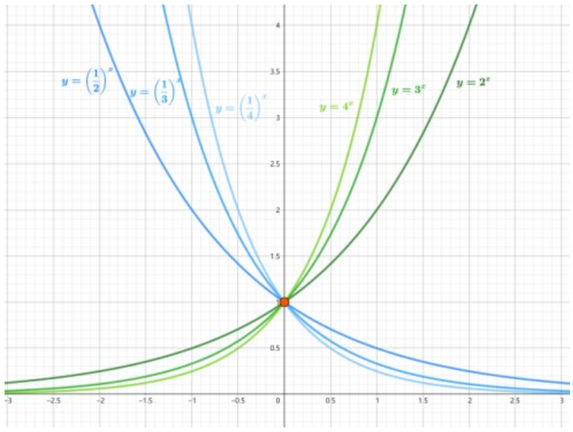

Figure 2: Graphs of Six Specific Exponential Functions.

Teacher-Student Interaction: Using GGB software, the teacher displays the six function graphs in the same coordinate system (as shown in Figure 2). Students observe the position, common points, and trends of the six graphs and consider: What common features do they share? After discussion, students exchange viewpoints, and the teacher guides them to explain the different trends of the functions.

[Question 3] How is the graph of an exponential function related to the base a?

Student Activity: In groups, students propose conjectures based on the six function graphs and independently verify them.

[Exploratory Activity 3] We summarize function properties and generate the proposition field based on graph characteristics.

Student Activity: Based on the common features of the graphs identified above, students describe the summarized function properties in multiple equivalent forms—such as verbal, graphical, and symbolic representations—from various perspectives including geometric representation, algebraic expression, and existing CPFS structure. This generates the proposition field for each property of exponential functions. For example, the proposition field for the monotonicity of exponential functions is shown in Table 1.

Table 1: Proposition Field for Monotonicity of Exponential Functions.

Equivalent Proposition 1	If $a > 1$, the graph rises from left to right；If $0 < a < 1$, the graph falls from left to right.	Geometric Representation
Equivalent Proposition 2	If $a > 1$, the function $y = a^x$is monotonically increasing;If $0 < a < 1$, the function $y = a^x$is monotonically decreasing.	Extension of the Monotonicity Concept System
Equivalent Proposition 3	If $a > 1$, when $x_1 > x_2$, $a^{x_1} > a^{x_2}$; If $0 < a < 1$, when $x_1 > x_2$, $a^{x_1} < a^{x_2}$.	Algebraic Comparison

Design Intent:

Starting with specific functions, the teacher guides students through the learning process of "collaborative graphing→observing graphs→summarizing features→discovering invariants," allowing them to personally experience the exploration of exponential function properties. Comprehensively summarizing function properties from multiple angles is the first step in constructing the CPFS structure for the graphs and properties of exponential functions. The teacher should allocate sufficient time for students to independently undergo the process of "proposing conjectures-verifying conjectures," using the mathematical thought of moving from specific cases to general conclusions to derive accurate results. This enhances rigorous thinking, fosters an exploratory spirit, and develops competencies such as intuitive imagination, integration of numbers and shapes, and transformation and classification.

3.3 Emphasizing Reflection and Practice to Expand the Proposition System

[Example] Students compare the values in each of the following problems (Table 2) and independently summarize the method for solving problems of "comparing the size of powers". We summarize function properties and generate the proposition field based on graph characteristics.

Table 2: Example Design Analysis under the CPFS Structure.

Specific Example Problems	Question Type	Core Focus	Value in Refining CPFS Structure
Comparing the values $1.7^{2.5}$ and 1.7^3	Same Base, Different Exponents	Monotonicity of Exponential Functions	Strengthens the proposition field: Concretizes the propositional field of "exponential function monotonicity" into an operational tool, enriching the understanding of properties. Reinforces conceptual connections: Strengthens the logical chain among "value of base a" "monotonicity of function" "size comparison of powers".
Comparing the values $(\frac{6}{5})^7$ and $(\frac{3}{2})^7$	Same Exponent, Different Bases	Monotonicity of Power Functions, "Larger Base, Higher Graph" Property of Exponential Functions	Constructs cross-chapter propositional systems: Breaks down barriers between exponential and power functions, establishing a new strong propositional link connecting the two conceptual systems under the overarching concept of "functions". Deepens the conceptual field: Students are compelled to consider the dual identity of the expression—both as a value of an exponential function and as a value of a power function—deepening the understanding of power from different perspectives.
Comparing the values $1.7^{0.3}$ and $0.9^{3.1}$	Different Exponents, Different Bases	Transformation Thought, Computational Proficiency, Strategic Decision-Making	Activates and integrates the broadest proposition system: Requires students to flexibly mobilize prior knowledge (e.g. rules of exponents) or build upon existing knowledge to generate mathematical methods such as constructing intermediary values, thereby enhancing knowledge transfer and problem-solving skills.

Design Intent:

The design of the three problems follows a cognitive progression from simple application to comprehensive correlation, and then to strategic creation. Through this scaffolded learning, students experience a complete cognitive construction process—from "building knowledge points" to "weaving a knowledge network," and finally to "flexibly applying the knowledge network." This connects seemingly isolated "islands" in their minds related to exponential functions, power functions, and even the entire functions module into a complete and systematic "knowledge continent" via multiple bridges (such as the methodological bridge of "comparing sizes").

3.4 Internalization and Transfer to Refine the CPFS Structure

Teacher-Student Interaction: The teacher guides students to independently summarize the insights and gains from the lesson in the form of questions, focusing on methodological reflection and the construction of an exponential function knowledge system.

Open-ended assignment: Independently create three "comparison" problems that align with the three types mentioned above.

Design Intent:

Deeply integrate teaching and learning. By assigning open-ended tasks, students' cognitive development and knowledge network construction are genuinely promoted, refining the CPFS structure of exponential functions.

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4. Implications of the CPFS Structure for Teaching Mathematical Propositions in High School

Mathematical propositions are important content in high school mathematics. Forming a well-developed CPFS structure is highly beneficial for enhancing students' understanding of mathematical propositions.

4.1 Focus on the Process of Proposition Acquisition to Establish a Cognitive Starting Point

Teaching new propositions should begin with visualizing the formation of the proposition, paying attention to the instructional starting point, and constructing the CPFS structure. First, teachers must clarify that the instructional purpose of a new lesson should shift from "teaching knowledge" to "building structure." The core task of lesson preparation is no longer listing knowledge points, but accurately analyzing the position of the new proposition within the entire mathematical knowledge network. Teachers should consider, based on students' existing CPFS structure, "Which concept fields, concept systems, proposition fields, and proposition systems should I help students enrich or construct through this lesson?" and design connecting "anchor points" accordingly. This means that instructional design must consciously preset connection points, select examples that activate prior knowledge while introducing new knowledge, and guide students to actively weave knowledge networks through vertical and horizontal comparison of propositions, thereby building a sound CPFS structure.

4.2 Analyze Propositions from Multiple Perspectives to Establish Connections Between Old and New Knowledge

The teaching of mathematical propositions in high school should reflect the essence and extensions of propositions. In traditional teaching, some students are only satisfied with memorizing formulas and problem types, lacking a deep understanding and flexible application of propositions. Research shows that cognitive structure is the entire content and organizational form of a concept in the learner's mind [4]. Advocating a multi-angle, deep-level revelation of the mathematical essence through the CPFS structure is an effective way to promote students' understanding of the essence of propositions. First, in teaching, teachers should guide students to examine and express propositions from multiple dimensions such as algebra and geometry, forming broad proposition fields in different structures. Second, teachers should carefully select examples and variant training to enable students to flexibly mobilize previously constructed proposition fields and proposition systems in different contexts, deepen their understanding of the connotations and extensions of propositions, enrich the CPFS structure, and form a systematic knowledge network.

4.3 Emphasize Thematic Summaries to Refine the CPFS Structure

New lessons are the initial construction period of the CPFS structure, not the completion phase. Rich and effective classroom activity design is key for students to independently acquire propositions and initially establish concept systems and proposition systems. However, limited by class hours and objectives, the networks constructed in new lessons are often partial and fragile. The depth of students' understanding of new propositions and their connection strength with other knowledge nodes (e.g., exponential functions and subsequent logarithmic functions, derivatives, etc.) are far from sufficient. Therefore, the enrichment, consolidation, and refinement of the CPFS structure must be strengthened and elevated in subsequent review lessons. Review lessons should play an important role in integrating knowledge networks. Their core value lies in breaking down chapter barriers, constructing cross-domain, cross-chapter proposition systems, enabling students' CPFS structure to evolve from partial to whole, from fragile to stable, and ultimately forming a systematic mathematical thinking framework [5].

5. Conclusion and Outlook

The CPFS structure is a unique cognitive framework in mathematics learning, encompassing not only the knowledge structure within an individual’s mind but also the intricate connections among mathematical concepts—especially relationships such as equivalence, strong abstraction, weak abstraction, and generalized abstraction that exist between mathematical concepts and propositions. Forming a well-developed CPFS structure is of great significance in helping students understand the essence and internal connections of mathematical propositions and apply them flexibly.

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In teaching, priority should be given to the process of proposition formation. Carefully designed sequences of questions should be used to gradually guide students in autonomously establishing knowledge connections, laying a solid cognitive foundation. Furthermore, through multi-faceted observation, interpretation, and varied practice, students' breadth of understanding of propositions can be expanded, strengthening the links between new and prior knowledge and constructing a robust network for mathematical learning.

Due to practical constraints such as limited class hours, teachers can break through chapter boundaries during review phases to further systematically integrate knowledge networks. This promotes a more stable cognitive structure in students, ultimately fostering a deep understanding that supports flexible transfer of knowledge.

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