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Exploring the "5E" Teaching Model of High School Mathematics Pointing to Problem Solving

Hou Yaxin[1,a], Kang Jinmei[1,b], Long Bingsong[1,c,*]

[1]Faculty of Mathematics and Statistics, Huanggang Normal University, Huanggang, Hubei, China
[a]3089770661@qq.com, [b]jinmeiKang23@163.com, [c]bslong20@163.com
[*]Corresponding author

Abstract: *Improving students' problem-solving skills is key to implementing core literacy. Taking "Bayes' formula" as the teaching content, with the help of the "5E" teaching model, and combining with the actual problems related to artificial intelligence, a class teaching design pointing to problem solving has been constructed, in order to provide a reference for the teaching of front-line teachers.*

Keywords: *Problem solving ability, "5E"teaching model, Bayesian formula, Artificial intelligence*

1. Introduction

Compared with the old college entrance examination, the new college entrance examination has an obvious trend of anti-brushing and de-modeling, which indicates that the current high school education pays more attention to the cultivation of students' problem solving ability [1]. Therefore, after the reform of the college entrance examination, what kind of teaching mode teachers use to teach effectively, and then improve the students' problem-solving ability has become an urgent problem to be solved.

5E teaching mode is a teaching mode of scientific inquiry based on constructivist theory [2]. The model is committed to improving students' learning interest, cultivating students' inquiry ability and problem solving ability, and these features are in line with the current requirements of high school education. At present, this model is widely used in chemistry, biology, physics and other courses, but less involved in mathematics courses [3]. Therefore, using the "5E" teaching model to improve students' problem-solving ability in the mathematics curriculum is a key step in the implementation of core literacy.

The rapid development of artificial intelligence has not only changed our way of life, but also had a great impact on the change of education. The Planning and Curriculum Standards issued by the State Council and the Ministry of Education successively in 2017 pointed out that artificial intelligence has brought positive impacts on social life, and called for cross-fertilization of artificial intelligence with secondary school curriculum [4,5]. In the field of artificial intelligence, a large number of problems belong to classification problems, and solving classification problems is exactly a typical application of Bayes' formula [6].

Therefore, this paper takes improving students' problem solving ability as the main line, combines the 5E teaching mode, develops the teaching around "Bayes' formula", effectively cultivates students' core literacy such as mathematical modeling. And to provide a proven way for the cross-fertilization of artificial intelligence and high school mathematics.

2. Basic connotation of the 5E teaching model

The 5E teaching model was first proposed by the American Society for Curriculum Studies in Biology based on the "learning loop". It consists of five stages: introduction, inquiry, explanation, refinement and evaluation. The five stages are explained as follows[7] (see Table 1):

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Table 1: Explanation of the five links of the 5E teaching model.

Stage Name	Explanation in Detail
Introductory stage	Beginning stages of teaching. Creating situations that engage students, relate curriculum content to learning tasks, and stimulate interest in inquiry.
Inquiry stage	The core stage of teaching. Students try to carry out research alone or in small groups based on their prior knowledge and experience, and the teacher plays the role of "scaffolding" in this process.
Interpretation phase	The key stage of teaching. Students present the results of the "inquiry" session, the teacher should guide students to summarize the understanding of the new concept, based on which the teacher needs to give standardized terminology. This part is teacher-led, focusing on the integration of interdisciplinary knowledge.
Migration phase	The stage of applying knowledge. Students will use the new concepts constructed to explain new problems and situations, and deepen their understanding of the new concepts through their ability to solve real-world problems. Attention needs to be paid to the use of standardized mathematical language or symbols for descriptions and answers in this segment.
Evaluation phase	The evaluation stage runs through the whole teaching process. The evaluation process includes process evaluation and summative evaluation, and the evaluation methods include self-assessment and mutual evaluation, teacher evaluation and so on. Its goal is to help teachers and students reflect on the teaching and learning process in time. At the same time, teachers can test students' understanding of new knowledge and assess students' research ability through formal or informal methods.

3. Instructional design based on the 5E teaching model

3.1. Teaching content analysis

The Curriculum Standard adds the full probability formula and Bayes' formula to Probability and Statistics. Students are required to be able to combine the classical probability model using the full probability formula to calculate probability, and understand the Bayes formula [5].

3.2. Analysis of teaching objectives

Bayes' formula is not only widely used in mathematics and life, but also plays an important role in today's state-of-the-art technology [8]. Mastering Bayes' formula can help students naturally extend and rationally improve the knowledge (algorithm) system of the probability of random events, and understand the importance of Bayes' formula in uncertainty decision-making [9]. Now the "Bayesian formula" and 5E teaching model, set the following teaching objectives (see Table 2).

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Table 2: Analysis of Teaching Objectives.

Mathematics Core Literacy Dimensions	Design of Instructional Objectives Based on the 5E Instructional Model
Mathematical Abstraction Dimension	On the one hand, students need to abstract mathematical models and concepts from the background of life problems; on the other hand, they will experience mathematical thinking in the opposite logical sense of “cause to effect” and “effect to cause” in this process, which has a certain degree of abstraction.
logical reasoning dimension	In the actual teaching process, according to the order of knowledge occurrence, students need to go through the process from conditional probability to full probability to Bayes formula, and derive the Bayes formula from the relationship between the three.
Mathematical modeling dimensions	The content of this lesson is all about abstracting mathematical problems from real-world problems and is taught with the aim of solving real-world problems using Bayes' formula. The idea of mathematical modeling is used throughout.

3.3. Analysis of learning situation

Students have completed the study of conditional and full probability formulas and mastered the laws of probability operations. Cognitively, sophomores are at the stage of rapid development of logical and abstract thinking skills, but may encounter challenges in dealing with complex conditional probability problems; learning attitudes show that most students are interested in AI-related problems, but may have problems understanding the logic behind the formulas.

3.4. Teaching priorities and difficulties

The following teaching points have been identified in relation to the teaching objectives and the learning situation.

Focus: master the derivation process of Bayes' formula, and will apply Bayes' formula to solve practical problems.

Difficulty: understanding the relationship and practical significance of prior and posterior probabilities in Bayes' formula.

4. Design of the teaching process

Creating problematic situations to engage students

Teacher Behavior: Currently, many cell phones have the ability to filter spam. Suppose there are two types of message sets available, spam and normal messages. If it is stipulated that a message will be recognized as spam if it contains “a”. The probability of identifying a message as spam under normal conditions is 0.9 for a certain model of cell phone, the probability of “a”appearing when the message is a normal message is 0.9, and the probability of “a”appearing when the text message is normal is 0.9. Determine whether a message appears as spam or “a”when it is a normal message by building a mathematical model. “a”when it is a spam message or a normal message.

[Design Intent] The problems faced by students in their lives as the introduction material can attract students' attention, give the data needed in the process of problem solving, cultivate the ability of students to abstract life problems into mathematical problems, and at the same time stimulate students' desire to explore.

Guided group work to explore models

Teacher Activities: Lead students through the logic of the problem by reviewing the conditional

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probability formula and the full probability formula. Then guide students to explore in groups and build their own mathematical models to solve the problem. Teacher visits and solves the confusion encountered by students in the process of group investigation. Problems that students may encounter during this process are listed below:

(1) Inability to observe life through a mathematical lens and to appropriately utilize conditional and full probability formulas.

(2) In the derivation of the formula, it is not possible to find out that $A \cup B$ and $B \cup A$ are the same event in time to complete the modeling process.

The teacher guides the students in solving the model, and the main process can be unfolded in the following steps:

(1) Develop a mathematical model. Let A= "new information contains "a", B_1= "spam", B_2= normal information. Then there are:

$$P(B_1) = 0.9, P(B_2) = 0.1, P(A|B_1) = 0.9, P(A|B_2) = 0.9.$$

According to the conditional probability formula, we can get:

$$P(B_1|A) = \frac{P(B_1)P(A|B_1)}{P(A)} \tag{1}$$

and

$$P(B_2|A) = \frac{P(B_2)P(A|B_2)}{P(A)} \tag{2}$$

Thus it transforms the problem of determining whether a new message is spam or not into a problem of comparing the size of (1) and (2). If $P(B_1|A) > P(B_2|A)$, the message can be categorized as spam, and vice versa for normal messages.

(2) Model simplification. To solve the problem directly using conditional probability would reveal unknowns. Therefore the model needs to be simplified. We found out that $P(B_i)(i = 1,2)$ is known, and $A \cup B$ and $B \cup A$ represent the same event. $P(B_i|A)(i = 1,2)$can beconverted to$P(A|B_i)(i = 1,2)$. Thus the above problem can be transformed into a problem of comparing the size of $P(A|B_1)$and $P(A|B_2)$.

(3) Calculations. Combined with the conditional probability formula and the full probability formula can be obtained:

$$P(A|B_1) = \frac{P(B)P(A|B_1)}{P(B_1)P(A|B_1)+P(B_2)P(A|B_2)} = 0.9 \tag{3}$$

and

$$P(A|B_2) = \frac{P(B)P(A|B_2)}{P(B_1)P(A|B_1)+P(B_2)P(A|B_2)} = 0.1 \tag{4}$$

So, $P(A|B_1) > P(A|B_2)$ 即 $P(B_1|A) > P(B_2|A)$, Therefore there is a higher probability that this message is spam.

[Design Intent] In this session students will experience the process of facing a problem, cooperating to find a solution and building a mathematical model. It can cultivate students' ability to cooperate and exercise their logical thinking and problem solving skills.

Review the reasoning process and explain the model

Teacher's behavior: guide students to use the self-explanatory method to summarize the process of formula reasoning in the inquiry session. Inaccuracies in students' expressions are supplemented by the teacher and detailed and exact Bayesian formulas are given in the context of the inquiry questions. That is:

Let Ω is the sample space, A is an event in Ω, be a partition of Ω, and for any event , then :

$$P(B_i|A) = \frac{P(AB_i)}{P(A)} = \frac{P(B_i)P(A|B_i)}{\sum_{j=1}^{n} P(B_j)P(A|B_j)} \tag{5}$$

It is not difficult to arrive at the Bayes formula through logical reasoning, but it is a challenge for students to gain a deeper understanding of the Bayes formula as well as to master the use case. Formally, the Bayes formula is very similar to the conditional probability formula. In essence, Bayes' formula is conditional probability, just in a different sense.

Simply put, if Adenotes the outcome of an event and $B_i(i = 1,2 \dots n)$denotes the cause of the outcome, then the Bayesian formula is to find the cause of the outcome if the outcome is known, while the conditional probability is to find the outcome if the cause is known. And the full probability formula is to find all the causes of the event, that is, the sum of all the conditional probabilities of the event.

[Design intent] The key link in the "5E" teaching model is explanation, in which the teacher should guide students to independently integrate the information in the inquiry session and give a standardized and logical expression as much as possible, so that students can understand the concepts in their original cognition and master the process of knowledge formation. Finally, the teacher corrects and summarizes the students' expressions and gives standard mathematical expressions to promote students' deep understanding of knowledge.

Guiding practice understanding, applying models

Teacher behavior: Bayes' formula is used to calculate the likelihood that an "outcome" is caused by a "cause" under the condition that the "outcome" is known to occur, that is, the posterior probability. A priori probability, a posteriori probability, conditional probability, what is the relationship between the three of them? Next, let's explore through the case of search and rescue of nuclear submarines.

"The USS Scorpion, a U.S. nuclear-powered submarine, was wrecked and sank on May 22, 1985, and the U.S. Navy later developed a search and rescue plan based on the Bayesian formula, and succeeded in locating the USS "Scorpion" [10]. Based on the above historical events, the following case study is available:

Example Suppose a nuclear submarine sinks in one of the three areas A, B, or C. The submarine technical department determines that the probability of sinking the submarine is $1/2$, $1/3$, $1/6$,respectively, and the probability of finding the submarine when searching for it in these areas is$1/2$, $2/3$, $1/4$, respectively.Based on the above data, how should a search plan be formulated?

Predetermined student answer: Let A event means that the nuclear submarine was not found in search and rescue area A, and the events B_1, B_2, B_3 , mean that the nuclear submarine sank in areas A, B and C respectively. Therefore, there are:

$$P(B_1) = \frac{1}{2}, P(B_2) = \frac{1}{3}, P(B_3) = \frac{1}{6}, P(A|B_1) = \frac{1}{2}, P(A|B_2) = 1, P(A|B_3) = 1.$$

It is obtained according to Bayes' formula:

$$P(B_1|A) = \frac{1}{3} .$$

The same reasoning leads to:

$$P(B_2|A) = \frac{4}{9}, P(B_3|A) = \frac{2}{9} .$$

Therefore, the probabilities of a nuclear submarine sinking in areas A, B, and C without searching for a nuclear submarine in area A are$1/3$, $4/9$, $6/19$.This set of probabilities represents the new predicted probabilities of a nuclear submarine sinking in the three areas. That is, the next search should be for region B. If the search of area B does not find the nuclear submarine, then the new predicted probabilities at this point are$9/19$, $4/19$, $6/19$ respectively. So the next search should be for area A rather than area C. The next search should be for area A, not area C. The next search should be for area A, not area C.

[Design intent] Solving this problem after mastering Bayes' formula is not difficult for students. Through this case, the teacher wants students to understand the role of the prior probability, the influence of the posterior probability on the prior probability and the role of conditional probability in the prior probability and the posterior probability. Among them, the probability of the three regions A, B and C where the nuclear submarine sank in the case is the a priori probability, which determines that we should search and rescue region A first. If the nuclear submarine is searched in region A, then the search and rescue mission is over, on the contrary, we need to re-determine the search and rescue scope according to the calculated probability. The recalculated probability is called the a posteriori probability, this process needs to be used to the conditional probability, the a posteriori probability is the correction of the a priori probability, but also to determine the basis of the search area, which also reflects the idea of iteration.

Analyzing the classroom process and multiple evaluations

Evaluation should be carried out throughout the teaching process of mathematics, and the 5E teaching model evaluates students from various aspects and perspectives by means of process evaluation and expressive evaluation. Students are guided to conduct self-assessment and mutual assessment from multiple dimensions, such as the inquiry process and the explanation process, so as to enhance the quality of students' mathematical thinking and problem-solving ability through evaluation.

5. Pedagogical recommendations

The Curriculum Standards require students to understand Bayes' formula but the textbooks include Bayes' formula as an optional component, which has led to many schools ignoring the importance of Bayes' formula. In this regard, it is suggested that teachers can introduce Bayes' formula appropriately when teaching for the following reasons:

(1) As a statistical inference method, Bayesian formula is widely used in medical diagnosis, speech recognition, search engine and so on. Mastering the Bayesian formula is conducive to helping students solve practical problems and improve their problem-solving ability.

(2) Bayesian formula and the full probability formula are simple events with the operation of complex events, the full probability formula is "from cause to effect", Bayesian formula is "Seeking the cause from the result"[9]. The two formulas are closely linked and should be compared in teaching to help students build a complete knowledge system.

(3) The Bayesian formula is an example of the integration of curriculum knowledge and values education [11]. The key to the Bayesian formula is that new information is constantly acquired and strategies are adjusted through iteration. This emphasizes the importance of updating and decision-making. This is also the philosophy of life: adjusting our understanding and experience through continuous learning is what helps us make the best decisions.

6. Conclusion

Mathematical knowledge is for problem solving. In the context of artificial intelligence, the introduction of artificial intelligence-related problems in the classroom can help students improve their problem-solving ability and enhance their interest in learning. At the same time, the integration of AI-related knowledge in secondary education can identify "seeds" with innovative potential and cultivate innovative talents for the country.

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