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Research on Multistage Production Optimization Based on Hypothesis Testing and Bayesian Inference

Jingtao Huo[1,*], Min Wang[2], Zeyu Chen[3]

[1]Polymer Materials and Engineering, Hubei University, Wuhan, China
[2]Mathematics, Hubei University, Wuhan, China
[3]Artificial Intelligence, Hubei University, Wuhan, China
[*]Corresponding author: 647385671@qq.com

Abstract: *As modern industry evolves, the complexity of production and supply chain management increases, making quality control and decision optimisation crucial for competitiveness. This paper addresses these challenges through mathematical modelling and algorithms, focusing on spare parts quality inspection and multi-stage production optimisation. For defective parts detection, a sampling scheme based on hypothesis testing is proposed, calculating minimum sample sizes using binomial and normal approximations—138 samples at 95% confidence and 108 at 90% confidence—effectively reducing inspection costs while ensuring quality. Additionally, a "01 model" is used to optimise multi-stage production decisions, balancing detection and processing costs to minimise overall expenses and maximise profits. Strategic decision-making, as demonstrated by the model, can significantly cut costs and boost profitability. By incorporating Bayesian inference, the study estimates defective rates from sampling data, further refining multi-stage process optimisation and enhancing decision accuracy. This research provides practical tools for quality control and production optimisation, helping enterprises reduce costs, improve profits, and enhance market competitiveness, offering valuable insights for broader industry applications [1].*

Keywords: *Binomial distribution, dynamic programming, 0-1 programming, Bayesian inference*

1. Introduction

In modern manufacturing, the complexity of production management and quality control has increased due to advances in technology and supply chains. Improving efficiency and reducing defective rates are critical for enterprises to enhance competitiveness and sustainability. Companies need to implement strict quality inspections in spare parts procurement and optimise multi-stage production decisions to balance inspection and non-conformance costs. Traditional models often fail to meet the dynamic needs of modern industry, making effective sampling schemes and multi-stage optimisation essential. This paper uses mathematical modelling and algorithms, including hypothesis testing and Bayesian inference, to address decision-making challenges in quality inspection and production process optimisation. The proposed solutions, tailored for real-world production environments, help reduce costs, improve product quality, and provide strategic insights for industry applications.

2. Hypothetical inspection and testing programme

The core of this paper is to design a sampling and testing programme for a company to determine whether spare parts purchased from a supplier comply with the defective rate requirement, with the objective of having as few tests as possible and giving a basis for decision making at different levels of confidence. The sampling and testing methods we need to consider are usually based on hypothesis testing in statistics [2].

In general, companies need to decide whether to accept spare parts based on the sampling results. For this purpose, we can design the testing programme based on binomial distribution or normal distribution (when the sample size is large, binomial distribution can be approximated as normal distribution) [3,4].

For hypothesis testing, two types of errors need to be controlled:

Type I error (α): wrongly rejected spare parts with a defect rate of up to 10 per cent.

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Type II error (β): wrongly accepting spare parts with a defect rate of more than 10 per cent.

Option 1: It can be verified whether the decision to reject spare parts or to receive spare parts is made by assuming that the sample size n for extraction testing and thus z is calculated:

$$n=\left(\frac{Z_\alpha+Z_\beta}{P_1-P_0}\right)^2 P_0(1-P_0) \tag{1}$$

A proportionality test can be used to deal with sampling results. In a testing programme, let $\hat{P}=\frac{k}{n}$ be the rate of rejects in the sample, and a standardised hypothesis testing formula can be used:

$$z=\frac{\hat{P}-P_0}{\sqrt{\frac{P_0(1-P_0)}{n}}} \tag{2}$$

Option 2: Calculate the minimum sample size required to receive the original hypothesis at a given level of confidence and consult the information to obtain the formula:

$$n=\left(\frac{Z_\alpha\sqrt{P_0(1-P_0)}}{E}\right)^2 \tag{3}$$

where Z_α is the critical value of the standard normal distribution, P_0 is the nominal defective rate (10%), and E is the acceptable margin of error (Order $E=5\%$).

Based on the model constructed above, we can propose the following two specific options:

Option 1: Reject spare parts if the defective rate is found to exceed the nominal value at 95 per cent confidence level.

Here we require the first type of error probability α is 0.05. According to the normal distribution, $\alpha=0.05$ corresponds to the critical value $Z_\alpha\approx1.645$, the enterprise can control the number of unqualified spare parts in the sample k to determine whether to reject, The process is shown in Figure 1:

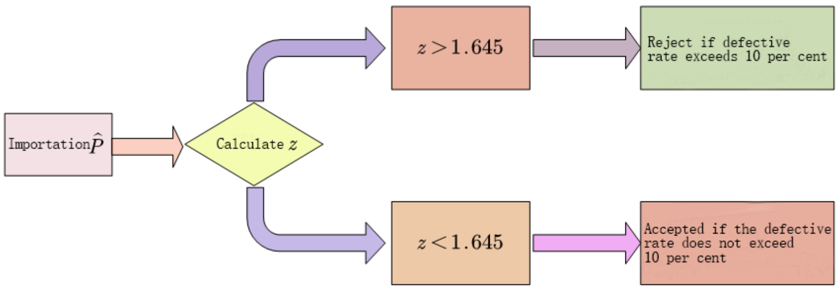

Figure 1: Flowchart of Option 1

Assuming an actual defective rate $P_{practice}=18\%$ and a sample size $n=100$, this can be calculated:

$$z=\frac{0.18-0.1}{\sqrt{\frac{0.1\times0.9}{100}}}\approx2.667>1.645 \tag{4}$$

Therefore, the company could find that the defective rate exceeded the standard value at 95 per cent confidence level and rejected the spare parts.

Option 2, the calculation of the minimum sample size n is carried out directly by building the model

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as described above:

(1) At 95 per cent confidence level:

It can be seen that $Z_\alpha = 1.96$, $P_0 = 0.10$, and $E = 0.05$

$$n = \left(\frac{1.96 \times \sqrt{0.1 \times 0.9}}{0.05}\right)^2 \approx 138 \tag{5}$$

Therefore, the sample size n for sampling and testing should be 138 at 95 per cent confidence level.

(2) At 90 per cent confidence level:

Standard normal distribution critical values $Z_\alpha = 1.65$, $P_0 = 10\%$ and $E = 0.05$

$$n = \left(\frac{1.65 \times \sqrt{0.10 \times 0.90}}{0.05}\right)^2 \approx 108 \tag{6}$$

In summary, the minimum sample size n is 138 at 95 per cent confidence level and 108 at 90 per cent confidence level.

3. Analysis of the benefits of decision-making at all stages of the enterprise

This section involves making decisions about the various stages of production and how to optimise the decisions in production to minimise losses and costs and maximise returns.

This can be solved by the '01 model', which compares the total profit of each scenario and selects the most profitable option, but if the profits of the two scenarios are similar, other factors, such as the reputation of the enterprise, are taken into account.

Before building the model, we make assumptions about the production process:

1) Independent detection: each spare part and finished product are detected independently, i.e. the detection result of one spare part will not affect the detection result of another spare part or finished product;

2) Perfect dismantling: after dismantling the substandard finished product, the spare parts will not be damaged and can be reused;

3) Fixed loss: the replacement loss and dismantling cost of non-conforming products are fixed, and are not affected by the market and other external factors;

Based on the assumptions, the solution can be simulated using the '01 model'. Assume that an equal number of parts 1 and 2 are produced.

Option 1: The costs incurred by the company are not recurring, so a mathematical model can be constructed to compare the cost of testing with the cost of not testing, taking into account the company's reputation and other factors, and the costs of the company in different scenarios can be compared to arrive at the optimal choice.

From the perspective of the optimal choice, only use the concern of whether to detect spare parts, whether to detect the finished product, whether to dismantle the unqualified finished product, so the different steps are listed, can be divided into the following three categories:

1) Classify whether or not to test parts:

(1) The cost of this step is if neither part 1 nor part 2 is tested:

$$C_{Nopartstesting} = C_1 + C_2 \tag{7}$$

(2) The cost of this step is if both part 1 and part 2 are tested:

$$C_{InspectionParts} = C_1 + C_2 + D_1 \cdot (1 - P_1) + D_2 \cdot (1 - P_2) \tag{8}$$

(3) The cost of this step is if part 1 is tested and part 2 is not tested:

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$$C_{InspectionParts} = C_1 + C_2 + D_1 \cdot (1 - P_1) \tag{9}$$

(4) The cost of this step is if part 1 is not tested and part 2 is tested:

$$C_{InspectionParts=} C_1 + C_2 + D_2 \cdot (1 - P_2) \tag{10}$$

2) Classify whether or not to test the finished product:

(1) To test the finished product, the cost of the step is:

$$C_{Inspectionoffinishedgoods} = D_A + D_f \cdot (1 - P_f) \tag{11}$$

(2) The finished product is not tested, so the cost of this step is:

$$C_{Notestingoffinishedproduct} = D_A + P_f \cdot L \tag{12}$$

3) Classify whether or not the finished product is disassembled:

(1) Disassembling the finished product results in a cost for that step:

$$C_{Disassemblycosts} = P_f \cdot T \tag{13}$$

(2) The cost of this step is if the finished product is not disassembled:

$$C_{Disassemblycosts} = 0 \tag{14}$$

Option 2: The objective of the firm is to minimise total cost and maximise profit, and to make the firm reputable with similar profit. If we start from the objective of minimising the total cost of the production process, the objective function can be expressed as:

$$C_{Sum} = C_{Detection} + C_{Assembly} + C_{Discard} + C_{Disassembly} + C_{Adjustment} \tag{15}$$

The specific expressions for each of these costs are given below:

1) Testing costs $C_{Detection}$:

$$C_{Detection} = x_1 \cdot D_1 + x_2 \cdot D_2 + y_f \cdot D_f \tag{16}$$

2) Assembly costs $C_{Assembly}$:

$$C_{Assembly} = D_A \tag{17}$$

3) Purchase costs $C_{Purchase}$:

Purchase cost refers to the total cost of purchasing the part, which can be calculated:

$$C_{Purchase} = C_1 + C_2 \tag{18}$$

4) Disassembly costs $C_{Disassembly}$:

$$C_{Disassembly} = y_c \cdot T \tag{19}$$

5) Loss on exchange $C_{Adjustment}$:

Losses when non-conforming products are switched out after they have entered the market:

$$C_{Adjustment} = (1 - y_f) \cdot P_f \cdot L \tag{20}$$

In summary:

$$C_{Sum} = C_{Detection} + C_{Assembly} + C_{Purchase} + C_{Disassembly} + C_{Adjustment}$$

$$C_{Sum} = x_1 \cdot D_1 + x_2 \cdot D_2 + y_f \cdot D_f + D_A + C_1 + C_2 + y_c \cdot T + (1 - y_f) \cdot P_f \cdot L \tag{21}$$

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It is assumed that after testing the parts, if the part is inferior, it will be directly discarded; if the part is qualified, it will be assembled as a finished product and then directly put into the market.

The proportion of products put on the market is $P_{putonthemarket}$, which represents the ratio of the number of finished products put on the market to the number of finished products assembled and put on the market without parts testing, and can be expressed as.

$$P_{putonthemarket} = \prod_{i=1}^{2}(1-P_i)\cdot x_i \tag{22}$$

Where the finished product put on the market contains undetected defective products, the actual pass rate is set to $P_{Actualqualification}$, while the defective rate of the finished product is P_f, which can be obtained:

$$P_{Actualqualification} = \prod_{i=1}^{2}(1-P_i)\cdot(1-P_f) \tag{23}$$

Assuming that the total selling price of the finished products is all \$56, the profit can be obtained directly from the above formula:

$$w_{Sum} = 56\cdot P_{putonthemarket} - C_{Sum} \tag{24}$$

The above process constructs a specific total profit model, which can be derived by calculating the total profit for each scenario and thus the optimal decision of whether to test the finished product versus the subsequent process that leads to the result.

The optimal choice can be obtained by comparing the costs of different scenarios as shown in Figure 2 and Figure 3:

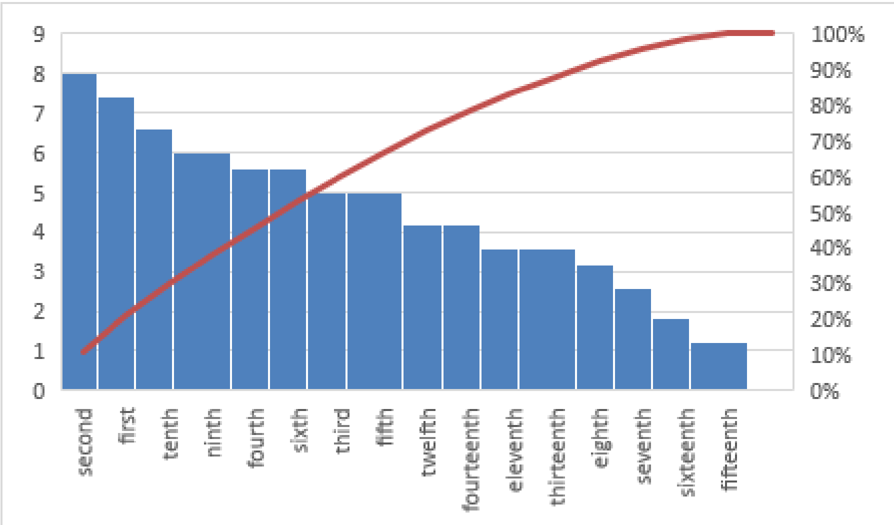

Figure 2: Counterpart decision-making programme

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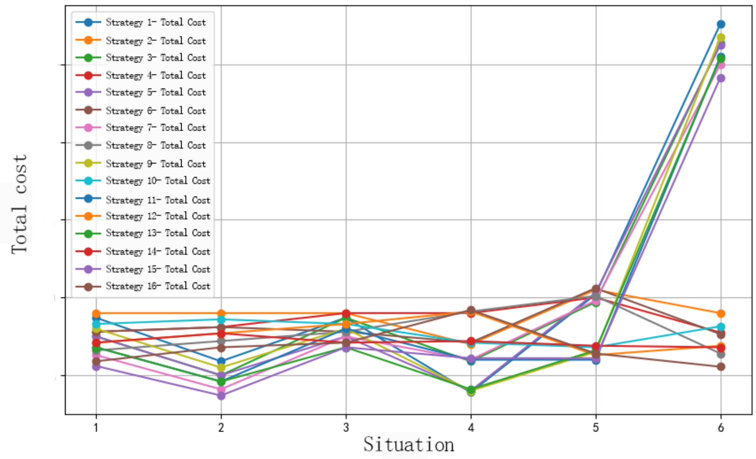

Figure 3: Combined line graphs for multiple scenarios

4. Bayesian inference method

The defective rate is obtained by sampling and testing, so let the defective rate be θ. Enterprises need to estimate the defective rate of spare parts and finished products, and decide whether to carry out testing, dismantle unqualified finished products and other decisions based on the estimated defective rate.

We assume that all the defective rate estimates are obtained through sampling and testing, and processed using the method of Bayesian inference, in which, since they are obtained through sampling and testing, the defective rate of semi-finished products and the defective rate of finished products can be obtained through the calculation of the defective rate of spare parts [5,6].

Bayesian formula:

$$P(\theta/D)=\frac{P(D/\theta)P(\theta)}{P(D)} \tag{25}$$

where $P(\theta/D)$ is the posterior probability, $P(D/\theta)$ is the likelihood function, $P(\theta)$ is the prior probability, is the marginal likelihood, and $P(D)$ let the prior distribution for each inferior rate θ be the Beta distribution $Beta(\alpha,\beta)$.

Let k defective parts be found by sampling n parts, then the likelihood function is:

$$P(D/\theta)=\theta^k(1-\theta)^{n-k} \tag{26}$$

According to Bayes' theorem, the updated posterior distribution is any Beta distribution of the form:

$$\theta/D\sim Beta(\alpha+k,\beta+n-k) \tag{27}$$

where $\alpha \ and \ \beta$ is the parameter of the prior distribution, the number of substandard products observed at k, and n is the total number of samples sampled.

Let the defective rate of different parts be $\theta_{Sparepartsi}$ can follow the mathematical model constructed above.

In building the mathematical model for solving problem two

1) Whether the spare parts are tested:

(1) Testing of spare parts can be obtained:

$$C_{Testedparts}=\sum_{i=1}^{2}D_i\cdot(1-\theta_{Spareparts\ i})+\sum_{j=1}^{2}C_j \tag{28}$$

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(2) The spare parts are not tested and can be obtained:

$$C_{Non-testedparts} = \sum_{j=1}^{2} C_j \quad (29)$$

2) Whether the finished product is tested:

(1) Testing of the finished product is available:

$$C_{Finishedproduct} = D_A + D_f \cdot (1 - P_f) \quad (30)$$

$$P_f = \theta_{Spareparts1} \cdot \theta_{Spareparts2} \quad (31)$$

(2) The finished product is not tested and can be obtained:

$$C_{Finishedproductnottested} = D_A + P_f \cdot L \quad (32)$$

3) Whether to disassemble the unqualified finished product:

(1) Disassembly of non-conforming finished products is available:

$$C_{Dismantlingcost} = P_f \cdot T \quad (33)$$

(2) No dismantling of substandard finished products is available:

$$C_{Nodismantlingcost} = 0 \quad (34)$$

5. Conclusions

This research develops models and algorithms to improve quality control and production decisions in modern industry. By designing an effective sampling and inspection scheme, we determined the optimal sample size for detecting the rate of defective products at different confidence levels, thereby reducing inspection costs while maintaining quality.

We also optimised multi-stage production decisions using the ‘01 model’ to balance inspection and handling costs, thereby significantly reducing production costs and improving profitability. Combined with Bayesian inference, the model can be dynamically adapted to different defect rates, thereby improving decision-making accuracy.

In conclusion, this study provides practical tools for optimising quality control and production strategies, which can help companies to reduce costs, increase profits and improve competitiveness. These methods provide a powerful framework for addressing similar challenges in the industry.

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