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Analysis and Control Strategy of Residual Chlorine Concentration in Swimming Pool Based on Sine and Cosine Function Temperature Prediction Model

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Abstract: *With the progress of society, the safety and health needs of human beings are increasing rapidly, and too high or too low residual chlorine concentration in the pool will affect the quality of pool water and the health of users. Therefore, the analysis and control of residual chlorine concentration in swimming pools is an important problem in swimming pool management. First, a differential equation model is established, the reaction rate constant is calculated, and the numerical solution of the differential equation is solved by the Runge-Kutta method, and the temperature is predicted. Next, the influence of the number of swimmers and temperature on the residual chlorine content was considered, and the corresponding differential equation model was established. Runge-kutta method was also used to solve the numerical solution of the differential equation, and the time of each dosage was calculated. In addition, the influence of dosing time, temperature and number of people on swimming pool operation was also considered, and the weights of each factor were determined by the Topsis evaluation method for decision makers' reference. The method in this paper has high accuracy and universality, and provides a valuable reference for solving similar problems.*

Keywords: *Residual chlorine concentration; Temperature prediction; Runge-kutta method; Topsis evaluation method; Differential equation*

1. Introduction

Chlorine is a common substance widely used to disinfect water, while residual chlorine is the chlorine content remaining after the chemical reaction between chlorine and water when disinfecting water Residual chlorine concentration in swimming pools refers to the concentration of free chlorine ions in the water that has not reacted with the organic substances, which is one of the most important indicators of swimming pool water treatment.

Too high or too low residual chlorine concentration will affect the quality of pool water and the health of users. In pool management, residual chlorine concentration needs to be monitored and controlled to ensure the hygienic quality of pool water. Therefore, analyzing and controlling the residual chlorine concentration in swimming pools is an important issue in pool management.

The residual chlorine concentration in swimming pools is affected by a number of factors, including water temperature, disinfectant, number of swimmers and pool volume. Especially in the high temperature of summer, the residual chlorine concentration needs to be tested and adjusted in time to ensure the safety of water quality. The number and duration of swimmers will also affect the residual chlorine concentration, which needs to be tested and adjusted regularly to maintain the safety of water quality. Pool volume and hours of operation can also affect residual chlorine levels and water safety.

At home and abroad, the study of residual chlorine concentration in swimming pools has received extensive attention. Zhang, J et al[1] reviewed the methods for monitoring and evaluating residual chlorine in drinking water distribution systems, including traditional methods and emerging technologies, and discussed their advantages, disadvantages and applicability. Zhou, Y et al[2-4] evaluated the distribution of residual chlorine in a drinking water distribution system using a computational fluid dynamics model, and explored the effects of piping layouts and disinfectant dosing strategies on it. Wang , J et al [5] evaluated the distribution of residual chlorine in drinking water distribution systems using a hydraulic model and explored the effects of pipe network structure and water quality parameters on it. Pan, J et al [6] proposed a novel dynamic model that can be used to predict the content and distribution of residual chlorine in drinking water distribution systems and evaluated its accuracy and reliability. Li,

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Y et al [7] summarized the Li, Y et al [8] reviewed the degradation pattern and the formation of disinfection by-products in drinking water distribution systems, including the aspects of influencing factors and control measures. Li, Y et al [8] reviewed the monitoring and modeling methods of residual chlorine in drinking water distribution systems, including the research progress and the current application status based on the traditional methods and emerging technologies. Li, Y et al [9] reviewed the modeling and predicting methods of residual chlorine in drinking water distribution systems, including the methods based on statistics, artificial neural networks and machine learning, etc. Wang, H et al [10] reviewed the degradation pattern of residual chlorine in drinking water distribution systems and the formation of disinfection by-products, including the aspects of influencing factors and controlling measures, and discussed its impacts on water quality safety.

The research on the concentration of residual chlorine in swimming pools at home and abroad has made some progress, but there are still some challenges and problems, such as how to effectively control the concentration of residual chlorine in swimming pools and ensure the safety of water quality in practical operation. Therefore, further in-depth research and exploration are needed. The larger problem in the analysis of residual chlorine concentration in swimming pools is the control strategy of residual chlorine content, and the establishment of a suitable mathematical model is particularly important. The aim of this paper is to explore how to control and manage the residual chlorine concentration in swimming pools through weather prediction and mathematical modeling. A differential equation model was developed, the reaction rate constants were calculated, and the numerical solution of the differential equation was solved using the Lunger-Kutta method, and the graph of residual chlorine concentration over time was plotted. The effect of the number of swimmers and temperature on the residual chlorine content was considered, and the corresponding differential equation model was established, the numerical solution of the differential equation was solved using the Lunger-Kutta method, the residual chlorine concentration versus time was plotted, and the moment of each dosage was deduced. The effect of dosing time on the operation was further considered, and using the Topsis evaluation method, the weights of each factor were provided for decision makers' reference and some suggestions were made. The method of this paper has high accuracy and generalizability, which can provide some references for the solution of similar problems.

2. Temperature prediction model with sine-cosine function

For the temperature change problem, in order to build a suitable model to describe the changing pattern of daytime temperature and nighttime temperature with time. Firstly, in order to avoid the complexity of the model, some discrete parameters, such as season, climate, terrain, wind direction, etc., are ignored. The most intuitive maximum and minimum temperatures were selected as the main parameters. As the change rule of temperature with time during the day and night is a cyclic change process. And the image of the sine-cosine function is also a periodic wavy line, which is one of the periodic functions. Therefore, we can use the sine-cosine function to describe the change rule of temperature with time during daytime (6:00-20:00) and nighttime (00:00-6:00,20:00-24:00).

Taking time t as the independent variable and temperature value T as the dependent variable, the formula of the sine-cosine function is used to calculate the temperature value at each time point, and the temperature prediction model of the sine-cosine function is established as follows.

$$\text{daytime: } Td(t)=T_0+\sum A_n*\cos(2\pi\, nt/14+\varphi n) \tag{1}$$

$$\text{evening: } Tn(t)=T_0+\sum B_n*\cos(2\pi\, n(t-14)/10+\varphi n) \tag{2}$$

Subsequently fitted with nonlinear least squares, nonlinear least squares can help us find the most suitable fitting curve to reduce the model error when building the temperature prediction model. By minimizing the gap between individual observations and the fitted values, the nonlinear least squares method can find the optimal solution, making the gap between the fitted curve and the actual data as small as possible. By fitting the curve, the respective amplitude and phase difference between daytime and nighttime can be fitted, so that the change rule of air temperature in a day can be clearly seen, as well as the trend of the daytime and nighttime air temperature over time, and then given the highest and lowest temperatures on any date, the model can describe the change rule of the daytime and nighttime air temperatures over time.

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3. Maximum and minimum temperature prediction models

Since it is necessary to establish a model that can predict the change of temperature for 24 hours on any future date, a sine-cosine function temperature prediction model has already been established in the above text, both the maximum and minimum temperatures of the day are given, and the change of temperature for 24 hours of the day can be obtained through the sine-cosine function temperature prediction model. So it is only necessary to establish a model to predict the maximum and minimum temperature of any future date.

Weather temperatures are time series in nature, i.e., the temperature at each moment is related to the temperature at the previous moment, and may also be affected by seasonal and cyclical variations. Therefore, time series models are a suitable model for weather temperature forecasting, and time series-based models such as ARIMA, LSTM, etc. can be considered. These models are able to capture the long-term dependence and cyclical changes in the time series and are able to better predict future temperatures. Whereas, if a non-time series model is used, the effect of time may be ignored, resulting in poor prediction accuracy.

By looking at the historical temperature data, we can see that it shows periodicity, stochasticity and trend, so the ARIMA model is used to establish the maximum and minimum temperature prediction model.

Since the original time series is not smooth, the smooth time series is first obtained by differencing, and the first-order difference is expressed as:

$$\Delta X_t = X_t - X_{t-1} = X_t - LX_t = (1-L)X_t \tag{3}$$

$$\Delta^2 X_t = \Delta X_t - \Delta X_{t-1} = (1-L)X_t - (1-L)X_{t-1} = (1-L)^2 X_t \tag{4}$$

$$\Delta^d X_t = (1-L)^d X_t \tag{5}$$

For a single integer sequence of order d, $X_t \sim I(d)$:

$$W_t = \Delta^d X_t = (1-L)^d X_t \tag{6}$$

Then W_t is a smooth series, so an ARMA (p,q) model can be built for W_t ,and the resulting model is called $X_t \sim ARIMA(p,d,q)$ and has the model form:

$$W_t = \phi_1 X_{t-1} + \phi_2 X_{t-2} + \cdots + \phi_p X_{t-p} + \delta + u_t + \theta_1 u_{t-1} + \theta_2 u_{t-2} + \cdots \theta_q u_{t-q} \tag{7}$$

$$\Phi(L)\Delta^d X_t = \delta + \Theta(L)u_t \tag{8}$$

Next, the orders of AR and MA, both p and q values, need to be determined, which can be chosen by autocorrelation and partial autocorrelation functions, or by methods such as the information criterion.

After the estimation of the model parameters is completed, we use the historical data and the estimated model parameters to fit the ARMA model through the least squares method, and subsequently, we need to check whether the residuals in the model are white noise.

The model is used to forecast the weather in Shaoxing in the next three years, and the prediction results of the original series are obtained by inverse solving according to the fitted results, and the two indexes of mean square error and mean absolute error are used to evaluate the performance and accuracy of the model, and the model is adjusted and improved according to the evaluation results, so that we can get the model that can more accurately predict the change of the maximum and minimum temperatures of any date in the future model.

Subsequently, the maximum and minimum temperatures predicted by the maximum and minimum temperature prediction model are inputted into the cosine function temperature prediction model in Problem 1, and the 24-hour temperature change curve of the same day can be obtained.

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4. Predictive modeling of integrated changes in residual chlorine concentrations

4.1 Predictive modeling of residual chlorine concentration as a function of temperature

In the absence of swimmers, the residual chlorine reaction rate, v, can be related to the water temperature, T, and residual chlorine concentration, y, using differential equations without regard to human factors.

Since in the absence of swimmers, it is known that the residual chlorine reaction rate in pool water has the following empirical relationship with water temperature and residual chlorine concentration:

$$v \propto 10^{\frac{T_{water}-25}{5}} \tag{9}$$

According to the conditions in the title, the reaction rate is related to the residual chlorine concentration y and T. The following relationship can be derived:

$$v = k * y * 10^{\frac{T_{water}-25}{5}} \tag{10}$$

Therefore, a differential equation can be considered to model the predicted change in residual chlorine concentration with temperature to describe the change in residual chlorine concentration with time, and according to the conditions, the equation can be given as follows:

$$\frac{dy}{dt} = -k * y * 10^{\frac{T_{water}-25}{5}} \tag{11}$$

4.2 Predictive Modeling of Residual Chlorine Concentration as a Function of Temperature and Number of Persons

This model is based on the prediction model of residual chlorine concentration as a function of temperature. The residual chlorine concentration in swimming pools is affected by a number of factors, such as the water temperature and the number of people in the pools, so it is necessary to take into account the effects of these factors on the residual chlorine concentration.

The prediction model of residual chlorine concentration with temperature and number of people is established as follows:

$$\frac{dy}{dt} = -k * y * n(t) - p * y * (T(t) - 25) \tag{12}$$

Where y denotes the residual chlorine concentration, t denotes the moment, n(t) denotes the number of swimmers at that moment, T(t) denotes the water temperature at that moment, and k and p are parameters indicating the extent to which the number of swimmers and the water temperature affect the residual chlorine concentration. The prediction model of residual chlorine concentration with temperature and number of swimmers represents the rate of change of residual chlorine concentration over time, where the first term represents the effect of the number of swimmers on the residual chlorine concentration, and the second term represents the effect of water temperature on the residual chlorine.

4.3 Predictive modeling of integrated changes in residual chlorine concentrations

On the basis of the above model, the Service added the consideration of dosing time, for the sake of swimming pool revenue, it is necessary to establish a comprehensive change prediction model of residual chlorine concentration considering the factors of air temperature, number of people, dosing time point and dosing duration at the same time.

Considering the effects of temperature, number of people, dosing time and other factors on the amount of residual chlorine concentration, a comprehensive change prediction model for residual chlorine concentration was established as follows:

$$\frac{dy}{dt} = -k * y * n(t) - p * y * (T(t) - 25) + l * y * C(t) \tag{13}$$

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where y denotes the residual chlorine concentration, t denotes the moment, n(t) denotes the number of swimmers at that moment, T(t) denotes the water temperature at that moment, C(t) denotes the dosing rate at that moment, and k, p, and l are parameters denoting the extent to which the number of swimmers and the water temperature affect the residual chlorine concentration. The predictive model of residual chlorine concentration with temperature and number of swimmers represents the rate of change of residual chlorine concentration over time, where the first term represents the effect of the number of swimmers on residual chlorine concentration, the second term represents the effect of water temperature on residual chlorine concentration, and the third term represents the effect of dosing rate on residual chlorine concentration.

After that, the revenue strategy of the swimming pool can be analyzed by the Topsis method for decision-making using three weighting parameters k, p, and l. Topsis is a multi-attribute decision analysis method used to help decision-makers to make choices with limited resources.

5. Analysis of results

5.1 Temperature prediction model with the sine-cosine function

The weather temperature of April 6, 2023, in Shaoxing City was obtained from the weather website as shown in Table 1, which gives the maximum temperature of the day as 17°C, the minimum temperature as 11°C, and the average temperature is about 14.375°C.

Table 1: Weather Temperatures for April 6, 2023, in Sao Paulo City

Times	Temperatures	Wind power	Wind speed	Pneumatic	Moisture	Probability of rain
00:00	15°C	3-4	14m/s	1013hPa	61%	49%
01:00	15°C	3-4	13m/s	1013hPa	62%	20%
02:00	15°C	1-2	11m/s	1013hPa	66%	20%
03:00	15°C	1-2	11m/s	1013hPa	63%	20%
		...	...			
20:00	12°C	3-4	13m/s	1014hPa	84%	20%
21:00	12°C	3-4	14m/s	1015hPa	86%	20%
22:00	12°C	3-4	14m/s	1016hPa	87%	20%

Parameters can be fitted based on the sine-cosine function temperature prediction model, as shown in Table 2.

Table 2: Various parameters during daytime and nighttime

Notation	Numerical value
Daytime Amplitude	1.994352147185953
Daytime phase difference	0.49269228508467183
Evening Amplitude	-1.21839573221341
Phase difference at night	0.253976364629642

According to the fitted parameters, the change rule is obtained as shown in Figure 1:

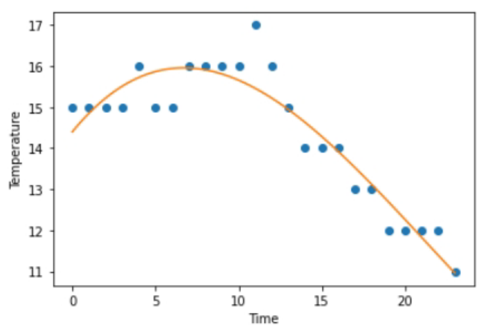

Figure 1: Fitting curve of a temperature prediction model with the sine-cosine function

From the above, it can be seen that by using the sine-cosine function to describe the law of daytime and nighttime temperature change over time, a simple and accurate model can be obtained, which can not only effectively describe the phenomenon of temperature change in Shaoxing on the day of April 6,

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but also can be used to predict the 24-hour temperature change of any date where the maximum and minimum temperatures are known.

5.2 Maximum and minimum temperature prediction models

The meteorological data of Shaoxing from 2011 to 2020 were screened in the weather website, as shown in Table 3.

Table 3: Meteorological data for Shaoxing, 2011-2022

Provinces	Region	Times	Weather	Maximum temperature	Minimum temperature	Wind power
Zhejiang	Shaoxing	2011/01/01	fine	6℃	-1℃	≤3
Zhejiang	Shaoxing	2011/01/02	snow	6℃	2℃	≤3
Zhejiang	Shaoxing	2011/01/03	snow	4℃	-1℃	≤3
Zhejiang	Shaoxing	2011/01/04	cloudy	6℃	0℃	≤4
				...	...	
Zhejiang	Shaoxing	2020/03/17	cloudy	21℃	12℃	3-4
Zhejiang	Shaoxing	2020/03/18	fine	24℃	13℃	3-4
Zhejiang	Shaoxing	2020/03/19	cloudy	23℃	8℃	4-5

The maximum air and minimum temperatures are selected for visualization, as shown in Figure 2, which shows that the data have good periodicity as well as interval stability, which is very suitable for the maximum and minimum temperature prediction model.

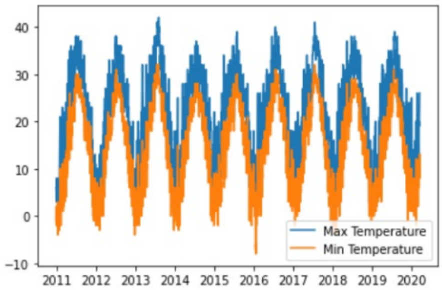

Figure 2: Shaoxing City Temperature Change 2011-2020

The maximum and minimum temperatures were predicted for June 17 and June 18, 2023, using the maximum and minimum temperature prediction model, and the results are shown in Table 4.

Table 4: Projected maximum and minimum temperatures

Dates	Minimum temperature(℃)	Maximum temperature(℃)
June 17, 2023	22	28

The resulting maximum and minimum temperatures for the two days were input into the sine-cosine function temperature prediction model to obtain the 24-hour temperature variation curves for these two days, as shown in Figs. 3, 4.

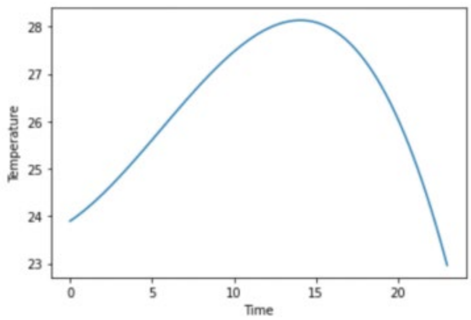

Figure 3: 24-hour temperature profiles for June 17, 2023

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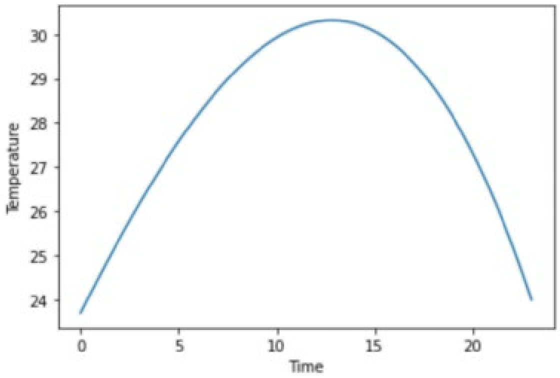

Figure 4: 24-hour temperature change profile for June 18, 2023

From Figure 3, and Figure 4, it can be seen that the fitted image has a continuous law of change, which can be derived from the temperature of each time period, and it is convenient for the prediction of the post-sequence model.

5.3 Predictive modeling of integrated changes in residual chlorine concentrations

The change in water temperature was calculated from the predicted air temperature changes in Figure 3, Figure 4, as shown in Table 5.

Table 5: Water temperatures at various moments on June 18, 2023

Times	Temperature (℃)
9:00	27
10:00	27
11:00	27
12:00	28
13:00	28
14:00	28
15:00	27
16:00	26
17:00	26

Table 6: Reaction rate parameter k values

Times	Numerical (k)
9:00	0.45452552
10:00	0.45452552
11:00	0.45452552
12:00	0.50337839
13:00	0.50337839
14:00	0.50337839
15:00	0.45452552
16:00	0.40094979
17:00	0.40094979

By substituting the residual chlorine concentration and water temperature data into the relational equation, the value of the reaction rate parameter k at each moment can be found, as shown in Table 6.

The threshold point for early dosing was calculated by using the Longe-Kuta method to solve the numerical solution of the differential equation by iteration in order to obtain the pattern of the residual chlorine concentration with the change of each index. Adjustments were made to the original dosing moments as shown in Figure 5.

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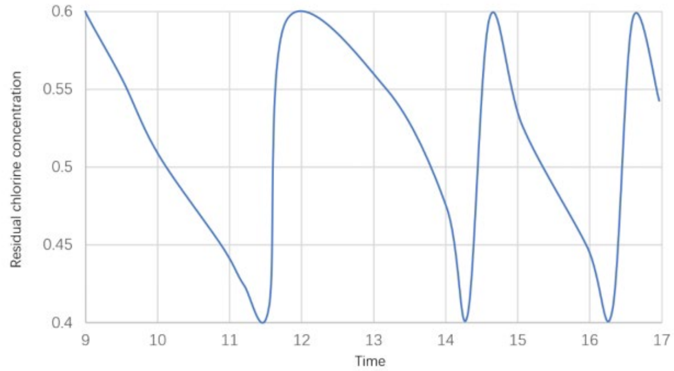

Figure 5: Residual Chlorine Change Curve Considering Dosing Time Adjustment

The decision weight values obtained according to the Topsis method are shown in Figure 6.

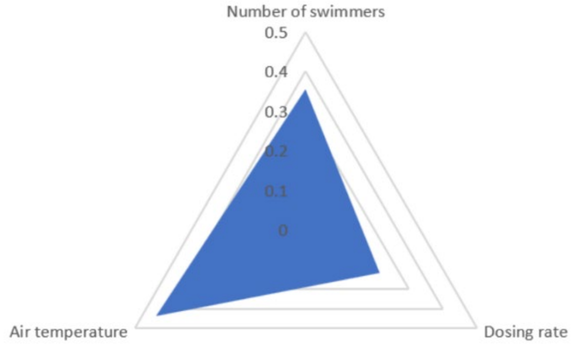

Figure 6: Weights of factors

It can be seen that the number of swimmers and temperature factors have a greater influence on the residual chlorine concentration, and should be an important consideration for swimming pool operators. The decision to control the flow of people can be realized by specifying the swimming time for which swimmers pay for a single session or by opening the swimming pool at different times of the day.

6. Error analysis

In the ARIMA time series model, since many discrete values such as climate, terrain, etc. are ignored, and since it is difficult to speculate accurately about the weather, the predicted results will be in error from the actual situation.

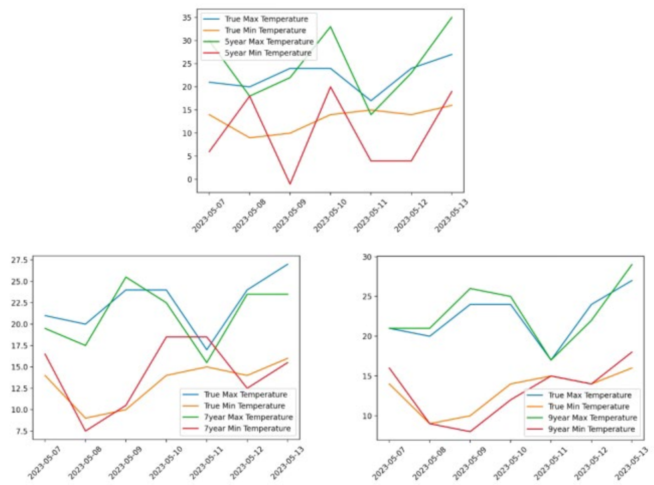

Figure 7: Error analysis of training model with different amounts of data

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In this paper, the amount of data used for training is very large, in order to prove that the model has good accuracy, the following experiments were done in the Shaoxing area from May 7, 2023, to May 13, 2023 temperature as an example, respectively, 5 years, 7 years, 9 years of temperature data trained model predicted results compared with the actual results, the comparison results are shown in Figure 7.

As can be seen from Figure 7, the larger the amount of data used, the temperature error predicted by the trained model is approximately smaller, the model in this paper was trained with 9 years of temperature data, and the experiments show that the error with the reality is within 2 ℃, which has a good prediction ability.

7. Conclusions

In this paper, we have successfully predicted the chlorine residual concentration in swimming pools over time by developing a mathematical model and using weather prediction methods and explored the factors affecting chlorine residual concentration and control strategies. Although the weather data used in the model only include maximum and minimum temperatures, the predictive ability of the model has reached a good level. In order to further improve the model performance, more effective data can be included and a more efficient time-series model can be adopted by means such as feature engineering to improve the accuracy and prediction ability of the model. In addition, the model has a wide range of applications, not only for the control and management of residual chlorine concentration in swimming pools, but also for the control of components in reservoirs as well as in various types of water treatment projects. The research in this paper provides a new idea and method for pool managers and reservoir managers to better manage and control water quality to protect public health and environmental safety.

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