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Reliability Analysis of Shield Tunnel Deformation Stability Based on Uniform Design-Response Surface Method

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Abstract: *Deformation stability of the surrounding rock is essential to ensure the safety of tunnel construction. Due to the multitude of factors affecting the deformation of the surrounding rock, the mechanical properties of the surrounding rock and the structural forces of the lining have significant randomness and uncertainty, and the functional function of the deformation stability for tunnel surrounding rock presents significantly nonlinear or implicit characteristics. In this paper, the influence of geotechnical parameters on the surrounding rock deformation of shield tunnel was addressed, and a homogeneous test scheme was designed to carry out interaction tests on each influencing factor to study the settlement of the tunnel vault under the interaction of different level values. Considering the correlation between the parameter variables, the response surface functions between the maximum value of vault settlement and different parameter variables were fitted using partial least squares to obtain the explicit functional functions of tunnel envelope deformation stability. To verify the feasibility of this paper, the reliability index and failure probability of tunnel envelope deformation stability were calculated by Monte Carlo simulation based on a shield tunnel project in a typical clay stratum in Shanghai area, and the sensitivity of different variation levels of parameters on the reliability was analyzed. The results show that the method is simple and feasible, and solves the difficulty of solving the implicit functional function in the reliability analysis process, and the calculation accuracy can meet the engineering requirements and has certain practicality.*

Keywords: *Shield Tunnel; Surrounding Rock Deformation; Reliability; Uniform Test; Response Surface; Partial Least Squares*

1. Introduction

The surrounding rock deformation is one of the most obvious and concentrated reflections of the tunnel mechanical behavior, mastering the process of surrounding rock deformation also grasps the stable state of the tunnel[1].The mechanical response of the geotechnical formation caused by shield tunnel construction is affected by numerous factors, among which the material parameters of the geotechnical mass, such as the surrounding rock density γ, elastic modulus E, Poisson's ratio μ, cohesion c, and angle of internal friction φ are essential factors affecting the mechanical response of the formation [2]. Due to the multitude of influencing factors, the deformation and stability of the tunnel envelope has been one of the difficult research areas in the field of tunnel engineering.

The spatial variability of soil parameters is a major source of uncertainty in the field of tunnelling, which has major implications for the design of tunnel stability and safety [3].The traditional structural safety design method has large limitations when the random variables are complex or the functions cannot be expressed explicitly. The reliability calculation method based on mathematical statistical theory treats the main factors affecting load effect and structural resistance as random variables, which can evaluate the safety in tunnel engineering more comprehensively and provide a powerful scientific basis for the pre-decision, construction process and post-maintenance of the project[4]. The establishment of a reasonable functional function of structural reliability is the core and key of stability analysis using reliability theory [5].Compared with above-ground structures, tunneling projects are characterized by complex stratigraphic conditions and uncertainties in the mechanical parameters and instability damage modes of the surrounding rock. The functional functions mostly show implicit or highly nonlinear characteristics[6], which makes it difficult to build an explicit expression of the functional functions theoretically, which leads to constraints in the application of reliability analysis in tunnel engineering. In addition, spatial variability of soil parameters exists due to the influence of geological effects. Previous studies on parameter sensitivity mainly focused on the influence of the mean value of random variable

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parameters on the mechanical response of the structure, ignoring the influence of soil parameter variability, the evaluation results of which are difficult to provide accurate references for reliability analysis studies that consider parameter variability.

Therefore, based on the theory of reliability calculation, this paper combines the uniform design method, response surface method and numerical simulation method. The influence of parameter correlation on the regression model is eliminated by fitting the test data with partial least squares method. An explicit response surface functional function for the stability of the surrounding rock deformation considering the parameter correlation is established. The sensitivity ranking of different parameters is clarified by analyzing the influence of parameter variability on reliability under different variation levels. The content of the study provides a new approach to reliability assessment and sensitivity analysis, and the results will offer an important reference basis for shield tunnel construction deformation stability analysis.

2. Reliability analysis of tunnel deformation stability

2.1 Uniformity test scheme design

A decisive step in a successful experimental design is to identify valid test points within the overall data sample. The ultimate goal of conducting an experimental design is to obtain the fullest and most representative information in the system using as few tests as possible. The uniform design was jointly proposed by Chinese researcher Kai-Tai Fang and mathematician Yuan Wang in 1978, and it is an applicable method for multi-factor and multi-level experimental design created on the basis of orthogonal design[7].As a statistical design method, the uniform design method considers the "uniform dispersion" of the test points within the overall data sample, and the test points taken are based on the principles of number theory, which ensure that the test points are very uniformly scattered in the integration range, but also guarantee that the test points are sufficiently close to the various values of the product function to facilitate computer statistical modeling, with good robustness and flexibility. The test points can reduce the number of tests needed to design the experiment due to their representativeness [8].

The problem of stability and reliability of tunnel surrounding rock deformation itself is a highly nonlinear problem under the influence of multiple factors, and the functional relationship between surrounding rock deformation and different geotechnical parameters cannot be expressed explicitly. Therefore, the numerical test scheme can be designed reasonably and efficiently by the uniform test method, thereby fitting the functional model. According to the relevant literature[9], five factors of the geotechnical soil in the tunnel construction process were selected for the uniform test design: the Young's modulus E, Poisson's ratio μ; the cohesion c, the angle of internal friction φ; and the geotechnical weight γ.

Each factor was divided into 10 level numbers and a homogeneous design table U^*_{10} (10^8) was used for the homogeneous test. The uniform design table and using table of U^*_{10} (10^8) are shown in Table 1 and Table 2.

*Table 1 U*10 (108) Uniform test design*

	1	2	3	4	5	6	7	8
1	1	2	3	4	5	7	9	10
2	2	4	6	8	10	3	7	9
3	3	6	9	1	4	10	5	8
4	4	8	1	5	9	6	3	7
5	5	10	4	9	3	2	1	6
6	6	1	7	2	8	9	10	5
7	7	3	10	6	2	5	8	4
8	8	5	2	10	7	1	6	3
9	9	7	5	3	1	8	4	2
10	10	9	8	7	6	4	2	1

ISSN 2522-3488 Vol. 9, Issue 2: 84-90, DOI: 10.25236/IJNDES.2025.090214

*Table 2 U*10 (108) Usage Table*

Number of Factors S	Column number						Uniformity deviation D
2	1	6					0.1125
3	1	5	6				0.1681
4	1	3	4	5			0.2236
5	1	3	4	5	7		0.2414
6	1	2	3	5	6	8	0.2994

Since there are five influencing factors, columns 1, 3, 4, 5, and 7 in Table 1 were chosen to arrange the test, and the deviation D = 0.2414 for these five columns.

2.2 Construction of functional function for tunnel deformation stability

From the perspective of surrounding rock deformation, the reliability limit state equation can be expressed as

$$Z = R - S = u_{\text{limt}} - u_{\max} \tag{1}$$

Where Z is the structure function function, R is the structure generalized resistance function, S is the structure generalized load function, u_{limt} is the ultimate displacement of the surrounding rock, and $u_{\max}$ is the maximum displacement value of the surrounding rock.

The main difficulty in conducting the study of the stability and reliability of the surrounding rock deformation is that it is impractical to obtain the analytical solution of the surrounding rock displacement, so it is impossible to establish the explicit functional function of the surrounding rock deformation stability.

The presence of correlations in geotechnical parameters has been recognized, and such correlations have a significant impact on the results of tunnel stability analysis. The partial least squares regression model is an emerging method for multivariate statistical data analysis, which was first proposed by S.Wold and C.Albano et al. in 1983[10].The partial least squares enables the integrated application of multiple data analysis methods due to the organic combination of multiple linear regression analysis, principal component analysis of variables, and typical correlation analysis between variables, which brings great convenience to multivariate data analysis by simultaneously achieving regression modeling, data structure simplification, and correlation analysis between two sets of variables under one algorithm. The basic principle is as follows.

Setting the dependent variable as y and the independent variables as x (x_1, x_2, x_3, x_4, x_5), the statistical relationship between y and x is as follows.

(1) Extract the principal component q_1 in the system of independent variables x, q_1 is a linear combination of x_1, x_2, x_3, x_4, x_5, implementing the regression analysis of y on x and q_1;

(2) Sequentially extract the principal component q_2 in the independent variable system x, and implement the regression analysis of y on x and q_2 until until m principal components are extracted and the regression equation reaches a satisfactory accuracy (m<5);

(3) Implementing the regression analysis of the dependent variable y on the principal components q_1, q_2, ..., q_m, while the principal component q_i is a linear combination of x_1, x_2, x_3, x_4, x_5, i.e., the dependent variable y can be expressed as the regression equation of the original variables x_1, x_2, x_3, x_4, x_5.

The partial least squares regression method has a unique effect on the regression analysis considering parameter correlation. In this paper, based on uniform test data, the partial least squares regression is used to establish an explicit function between the maximum displacement value of the tunnel crown surrounding rock and the geotechnical parameters, namely:

$$u_{\max}^{\text{crown}} = u(E, \gamma, c, \mu, \varphi) \tag{2}$$

Based on the elasto-plastic theory, Li et al[11] derived the critical settlement formula for the top of a circular tunnel based on the analytical solution of the displacement of the surrounding rock of the circular chamber, taking the shear plastic limit as the critical condition:

$$u_{\text{limt}} = \delta_{top} = \frac{r_0 \sin\varphi(\gamma H + c \cdot \text{ctg}\varphi)}{2G} \tag{3}$$

ISSN 2522-3488 Vol. 9, Issue 2: 84-90, DOI: 10.25236/IJNDES.2025.090214

Where r_0 is the diameter of the excavation; H is the burial depth of the tunnel; γ is the average capacity of the surrounding rock; c is the cohesion of the surrounding rock; φ is the friction angle of the surrounding rock; G is the shear modulus of the surrounding rock.

The expression for the functional function of deformation stability of the surrounding rock (limit state equation) follows:

$$Z = \frac{r_0 \sin\varphi(\gamma H + c \cdot \cot\varphi)}{2G} - u(E, \gamma, c, \mu, \varphi) \tag{4}$$

3. Case analysis and discussion

3.1 Project overview and parameters

Shanghai Metro Line 15, Yongde Road Station ~ Yuanjiang Road Station, the upstream line is 2648.400 m long and the downstream line is 2651.682 m long. 9.75 m~17.2 m buried depth of tunnel. 13.5 m buried depth of a calculated section, the surrounding rock level is V. The tunnel is a circular tunnel with 6.6 m outer diameter and 5.9 m inner diameter, consisting of 350 mm thick, 1.2 m wide ring-shaped precast reinforced concrete pipe sheet, through-joint assembly, strength grade C55. In this paper, the main mechanical parameters of the geotechnical body and concrete pipe sheet are shown in Table 3.

Table 3 Mechanical parameters

Materials	Parameters	Base value	Value range
Surrounding Rock	γ(kN·m^{-3})	17.5	16.6~18.4
	c(kPa)	24.5	11~38
	φ(°)	21	12~30
	μ	0.375	0.33~0.42
	E(MPa)	32.5	28~37
Lining	γ(kN·m^{-3})	25	-
	μ	0.2	-
	E(GPa)	35.5	-
	h(mm)	350	-

3.2 Numerical calculation model

FLAC3D numerical simulation software was used to numerically simulate the construction process of the two-line shield tunnel. The shield tunnel construction problem is simplified to a two-dimensional plane strain model for simulation with model size 60 m×36 m (width×height), tunnel diameter D = 6.6 m, axial burial depth H = 16.8 m, and grid size no larger than 0.75 m. All boundaries are normal constrained except for the free boundary at the ground surface. The geotechnical soil is an ideal elastic-plastic body with Drucker-Prager yield criterion, and the lining structure is simulated with beam elements. Considering that the excavation of the tunnel makes the soil unit at the bottom of the cavern in the unloading state, and the soil shows obvious loading and unloading characteristics, this paper simplifies the problem by dividing the model into two layers, and the subsoil Young's modulus is assigned to three times the original modulus value, which can simply consider the characteristics of the soil unloading stiffness is greater than the loading stiffness, and the obtained internal force of the liner and ground deformation are more in line with the reality. The numerical analysis model is shown in Fig.1.

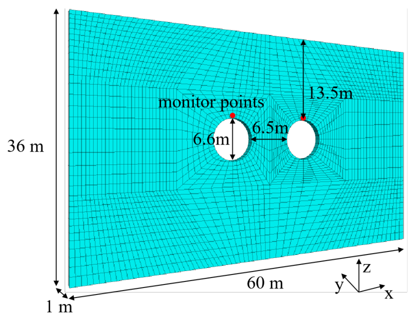

Fig.1 Computational model

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The study focuses on the influence of soil properties on tunnel stability. To simplify the computational procedure, the basic assumptions include.

(1) The shield tunnel excavation and support process is simulated by means of stress release, and the excavation transient stress release rate is selected to be 50% .

(2) Construction parameters of the tunnel boring machine, such as working face support pressure, grouting pressure and thrust force, are not considered in the numerical model.

(3) Factors such as the spacing between the two tunnels and the burial depth are not considered.

3.3 Tunnel deformation stability functional function construction

According to the selected 5 dependent variables, namely Young's modulus E, Poisson's ratio μ, cohesion c, internal friction angle φ and density γ, as the random variables affecting the functional function of the structure, the crown settlement was taken as the response value. Based on the uniform design, the data combination of the test points was designed with reference to the 5-factor 10-level uniform design table at equal intervals within the range of values in Table 3, and the uniform test design scheme and calculation results are shown in Table 4 for the right tunnel as an example.

Table 4 Experimental protocol and results

i	γ (kN·m^{-3})	C (kPa)	φ(°)	μ	E (MPa)	u_{max} (mm)
1	16.6	17	18	0.37	36	-1.516
2	16.8	26	26	0.42	34	-3.041
3	17.0	35	12	0.36	32	-1.808
4	17.2	11	20	0.41	30	-0.261
5	17.4	20	28	0.35	28	-1.764
6	17.6	29	14	0.40	37	-0.836
7	17.8	38	22	0.34	35	-2.266
8	18.0	14	30	0.39	33	-0.895
9	18.2	23	16	0.33	31	-2.711
10	18.4	32	24	0.38	29	-0.864

A pure quadratic polynomial without cross terms was chosen to fit the response surface function between the tunnel vault settlement and the parameter variables. Considering the correlation between the parameters and integrating the interaction of the factors, the explicit expression for the maximum value of settlement of the tunnel at the crown of the arch was obtained using partial least squares regression as:

$$\begin{aligned} u_{\max}^{\text{crown}} &= -241.21435 + 18.29241\gamma - 0.22803c + 0.09411\varphi + 526.46317\mu - 1.45914E \\ &- 0.50431\gamma^2 + 0.00385c^2 - 0.00351\varphi^2 - 682.22362\mu^2 + 0.02188E^2 \end{aligned} \tag{5}$$

To test the accuracy of the regression curves, the parameters in Table 4 were brought into Eq.(5), and the errors between the simulated and regression values were calculated for each group of data, respectively, as shown in Table 4.

The root mean square error (RMSE) of the regression system as:

$$RMSE = \sqrt{\frac{1}{10}\sum_{i=1}^{10}\left(y_i - \hat{y}_i\right)^2} = 0.0028$$

From the above, that the regression curves can fit the simulated values properly and the regression results are significant and feasible. Referring to Eq. (4), the reliability function for the stability of the surrounding rock deformation can be obtained as:

$$\begin{aligned} Z &= \frac{r_0 \sin\varphi(\gamma H + c\cot)}{2G} - u(E,\gamma,c,\mu,\varphi) = \frac{r_0 \sin\varphi(\gamma H + c\cot)(1+\mu)}{E} - \\ &(-241.21435 + 18.29241\gamma - 0.22803c + 0.09411\varphi + 526.46317\mu - 1.45914E - \\ &0.50431\gamma^2 + 0.00385c^2 - 0.00351\varphi^2 - 682.22362\mu^2 + 0.02188E^2) \end{aligned} \tag{6}$$

ISSN 2522-3488 Vol. 9, Issue 2: 84-90, DOI: 10.25236/IJNDES.2025.090214

3.4 Reliability analysis

According to the reliability function, the tunnel depth H and diameter r_0 are assumed to be constant, and the geotechnical parameters are random variables obeying normal distribution. According to the field measurements and relevant specifications, the values of the random variable parameters are shown in Table 5.

Table5 Values of surrounding rock parameters

Parameters	γ(kN·m^{-3})	c(kPa)	φ(°)	μ	E(MPa)
Mean	17.5	24.5	21	0.375	32.5
COV	0.2	0.2	0.2	0.2	0.2

The reliability index of tunnel vault deformation stability β = 3.7903 and failure probability P_f = 7.5240e-05 were calculated using Monte Carlo method. According to the Chinese national standard, Uniform Standard for Structural Reliability Design (GB 50153-2008), the target reliability index is 3.2 for the ductile damage state and 3.7 for the brittle damage state in the case of structural safety class II. The deformation stability reliability index of the tunnel in this case meets the requirements, indicating that the surrounding rock is stable during the tunnel construction.

3.5 Sensitivity analysis considering parameter variability

Sensitivity refers to the gradient relationship between the structural design parameters and the structural response. To obtain a comparative analysis of the sensitivity of each factor, the sensitivity of the parameter in dimensionless form is calculated with reference to Eq (7)

$$S_i = \frac{|\Delta P/P|}{|\Delta X_i/X_i|} = \left|\frac{\Delta P}{\Delta X_i}\right| \cdot \left|\frac{X_i}{P}\right| \tag{7}$$

In order to discuss the sensitivity of soil parameters to the reliability index at different variation levels, three base conditions of variation coefficients are designed, namely, COV = 0.15, 0.2, and 0.25. The variation coefficients of each parameter in the base condition are the same, i.e., $COV_\gamma = COV_c = COV_\varphi = COV_\mu = COV_E$. The coefficient of variation of one parameter is selected and fluctuates within the given range, while the variation characteristics of other parameters are kept unchanged, thus designing the combined working condition of each parameter coefficient of variation. When the base value of coefficient of variation is 0.15, the range of variation is taken from 0.05 to 0.25, when the base value is 0.20, the range of variation is taken from 0.10 to 0.30, and when the base value is 0.25, the range of variation is taken from 0.15 to 0.35. Theoretically, the smaller the fluctuation interval of the parameter coefficient of variation, the closer the calculated value is to the benchmark value, and the more it can reflect the influence of the change of the parameter coefficient of variation on the system in the benchmark state. Considering the efficiency of stochastic calculation, the fluctuation interval of the coefficient of variation in this paper is taken as 0.05.

The sensitivity of geotechnical parameters to the reliability index of arch settlement is shown in Table 6.

Table 6 Sensitivity of parameters

Parameters	Sensitivity		
	COV=0.15	COV=0.2	COV=0.25
γ	1.315	1.1165	0.947
c	0.6455	0.2595	0.1825
φ	1.1135	0.776	0.3125
μ	1.2835	0.9655	0.7305
E	13.2095	8.3305	7.558

The sensitivity of the reliability index to the geotechnical parameters at the same coefficient of variation baseline value is in the following order: $E > \gamma > \mu > \varphi > c$. The sensitivity of the same random variable parameter to the reliability index gradually decreases as the coefficient of variation increases.

4. Conclusion

The geotechnical parameters are considered as random variables and a numerical simulation scheme

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based on uniform tests is designed. An explicit expression between the maximum settlement of the vault and the parameters of the random variable considering the parameter correlation is obtained using partial least squares.

The reliability function for the stability of the surrounding rock deformation is established, and the reliability index of the stability of the tunnel surrounding rock is calculated by Monte Carlo method with the shield construction of Shanghai Metro Line 15 as the engineering support. The calculated reliability index of vault settlement deformation in the engineering example is 3.7903, which is higher than the target reliability index required in the specification, indicating that the surrounding rock is stable during the tunnel construction period.

Considering the spatial variability of the parameters, the influence of different variation levels of the parameter variables on the reliability was analyzed. The reliability of shield tunnel vault settlement deformation is most sensitive to geotechnical Young's modulus at the same variability level, followed by surrounding rock density and Poisson's ratio. Friction angle and cohesion are not sensitive to the influence of reliability index. There are significant differences in the influence of the variability of parameters on reliability at different variation levels, and the higher the variation level of parameters, the smaller its influence on reliability. Among the three variation levels studied in the article, the sensitivity of the Young's of the geotechnical soil is the largest, which should be focused on in the design analysis and investigation.

By using uniform experimental design and numerical analysis, the explicit expression of the settlement deformation of the surrounding rock at the tunnel vault is obtained by combining the partial least squares method, which solves the difficulty of the reliability solution process due to the implicit expression of the functional function, and provides a new calculation idea and a feasible method for the solution of the reliability of tunnel deformation stability.

Author Biography

Shang-Yue Lin works at China Railway 14th Bureau Group Co., Ltd., Jinan 250101, China. His research mainly involves shield tunnel construction technology and the reliability assessment of surrounding rock deformation.

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