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Application of Covariance Matrices in Financial Portfolio

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ABSTRACT. *Covariance matrix is an important concept in both linear algebra and statistics. It is a square matrix that gives the covariance between element pairs of random vectors. Covariance is similar to but slightly different than the definition of correlation. It does not demonstrate the strength of a linear relationship as correlation (scale of -1 to +1) does, but it allows us to determine the directions of linear relationship between two random variables.*

KEYWORDS: *Covariance matrix; random vectors; random variables*

1. Introduction

The diagonals of all covariance matrices are variances while the off-diagonal elements are covariances of each pair of variables. Any covariance matrix is symmetric with the two portions of off-diagonal elements identical to each other.

2. Properties of Covariance Matrix

- If two variables increase together, the coefficient of covariance is positive.
- If two variables go in different directions, the coefficient of covariance is negative.
- Symmetric property: covariance matrices are symmetric along the diagonal.
- The diagonal entries are all positive and are the variance of each data set.
- Covariance matrix is positive semidefinite. Its eigenvalues are nonnegative.

3. Applications of Covariance Matrix

Covariance matrix is widely used in financial engineering, machine learning, and econometrics. Used in portfolio, covariance matrix can determine the standard deviation of stocks. Therefore, portfolio managers are able to assess the risk associated with the stocks.

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3.1 Portfolio Analysis

- There are x stocks in the portfolio and want to allocate optimal capital of each stock so that the risk is minimized.
- Create portfolios with demonstration of different capital allocations of each stock.
- Calculate the standard deviation of each portfolio and the one with lowest risk is the optimal scenario.

3.2 Steps

1) When there are ‘x’ stocks in the portfolio (A_1, A_2, … A_x), all the stock data can be combined into a single matrix denoted as ‘A’:

$$A_1 = \begin{bmatrix} P11 \\ P21 \\ . \\ . \\ . \\ Px1 \end{bmatrix} \quad A_2 = \begin{bmatrix} P12 \\ P22 \\ . \\ . \\ . \\ Px2 \end{bmatrix} \quad A_3 = \begin{bmatrix} P13 \\ P23 \\ . \\ . \\ . \\ Px3 \end{bmatrix} \quad \ldots\ A_X = \begin{bmatrix} P1x \\ P2x \\ . \\ . \\ . \\ Pxx \end{bmatrix}$$

$$A = \begin{bmatrix} P11 & P12 & .. & .. & .. & P1x \\ P21 & P22 & .. & .. & .. & P2x \\ . & . & & & & . \\ . & . & & & & . \\ . & . & & & & . \\ Px1 & Px2 & .. & .. & .. & Pxx \end{bmatrix}$$

2) The average price of each stock is as follows:

Mean price of A_1 : $M_1 = \frac{(P11 + P21 + P31 + P41 + \ldots + Px1)}{x}$

Mean price of A_2 : $M_2 = \frac{(P12 + P22 + P32 + P42 + \ldots + Px2)}{x}$

Mean price of A_{Ξ} : $M_{\Xi} = \frac{(P1x + P2x + P3x + P4x + \ldots + Pxx)}{x}$

3) Combined all the means into matrix ‘M’, with the length ‘x’:

$$M = [M_1\ M_2\ M_3 \ldots\ldots M_n]$$

4) Demean the price / center the matrix:

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Demean is subtracting the sample mean from each data so the sum of the data or the mean of the set is 0. Demeaning the price helps compare how a stock's movement from its mean in relation to the movement of another stock form its mean.

We subtract the mean price of the stocks in each column, resulting in a new matrix with demeaned prices. Each data represents the difference between stock price and the mean of its column.

$$D = \begin{bmatrix} P11 - M1 & P12 - M2 & .. & .. & .. & P1x - Mx \\ P21 - M1 & P22 - M2 & .. & .. & .. & P2x - Mx \\ . & . & & & & . \\ . & . & & & & . \\ . & . & & & & . \\ Px1 - M1 & Px2 - M2 & .. & .. & .. & Pxx - Mx \end{bmatrix}$$

5) Establish Covariance Matrix:

We then establish the covariance matrix by multiplying the transpose of the demeaned price matrix with the demeaned price matrix and divide by the number of data points 'n':

The equation is as follows: $\mathbf{D^T * D / n =}$

$$\frac{1}{n}\begin{bmatrix} P11 - M1 & P21 - M1 & .. & .. & .. & Px1 - M1 \\ P12 - M2 & P22 - M2 & .. & .. & .. & Px2 - M2 \\ . & . & & & & . \\ . & . & & & & . \\ . & . & & & & . \\ P1x - Mx & P2x - Mx & .. & .. & .. & Pxx - Mx \end{bmatrix} * \begin{bmatrix} P11 - M1 & P12 - M2 & .. & .. & .. & P1x - Mx \\ P21 - M1 & P22 - M2 & .. & .. & .. & P2x - Mx \\ . & . & & & & . \\ . & . & & & & . \\ . & . & & & & . \\ Px1 - M1 & Px2 - M2 & .. & .. & .. & Pxx - Mx \end{bmatrix}$$

$$= \begin{bmatrix} {\sigma_1}^2 & \sigma_{12} & .. & .. & .. & \sigma_{1x} \\ \sigma_{21} & {\sigma_2}^2 & .. & .. & .. & \sigma_{2x} \\ . & . & & & & . \\ . & . & & & & . \\ . & . & & & & . \\ \sigma_{x1} & \sigma_{x2} & .. & .. & .. & {\sigma_x}^2 \end{bmatrix}$$

Remember that every covariance matrix is symmetric, thus $\sigma_{12} = \sigma_{21}$.

6) Variance of Portfolio:

We now need to find the standard deviation of the portfolio. We need to find the weights (percentage capital) allocation for every stock. The total percentage should be 1.

The weight distribution is a matrix denoted as 'W' as follows:

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$$W = \begin{bmatrix} W1 \\ W2 \\ . \\ . \\ . \\ Wx \end{bmatrix}$$

$$W1 + W2 + W3 + \dots Wx = 1$$

Expected portfolio return = M × W

4. Example of Covariance Matrix in Portfolio

There are two stocks (stock A and stock B) in the portfolio, each of them will have several possible values a year from now. Those values are correlated with each other as the whole market fluctuates due to various factors; thus, to some extent these two are tied to the whole market.

1) Construct the matrices:

Stock A has possible future prices:

$$A = \begin{bmatrix} 45 \\ 50 \\ 55 \end{bmatrix}$$

Stock B has possible future prices:

$$B = \begin{bmatrix} 85 \\ 150 \\ 35 \end{bmatrix}$$

Then the combined matrix would be :

$$C = \begin{bmatrix} 45 & 85 \\ 50 & 150 \\ 55 & 35 \end{bmatrix}$$

2) Average price:

Mean price of A = $\frac{(45+50+55)}{3}$=50 dollars

Mean price of B = $\frac{(85+150+35)}{3}$=90 dollars

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3) Combine the means:

M = [50 90]

4) Demean the price / center the matrix:

Subtract 50 from each entry in matrix A and subtract 90 from each entry in matrix B. The resulting matrix is as follows:

$$C' = \begin{bmatrix} -5 & -5 \\ 0 & 60 \\ 5 & -55 \end{bmatrix}$$

5) Establish Covariance Matrix:

$$\frac{1}{n}\begin{bmatrix} -5 & 0 & 5 \\ -5 & 60 & -55 \end{bmatrix}\begin{bmatrix} -5 & -5 \\ 0 & 60 \\ 5 & -55 \end{bmatrix} \quad n = 3$$

$$= \frac{1}{3}\begin{bmatrix} 50 & -250 \\ -250 & 6650 \end{bmatrix}$$

6) Variance of the portfolio:

Percentage capital of each stock:

Stock A = 60% = 0.6

Stock B = 40% = 0.4

Weight matrix 'W' = $\begin{bmatrix} 0.6 \\ 0.4 \end{bmatrix}$

7) Expected portfolio return = M × W = $\begin{bmatrix} 50 & 90 \end{bmatrix} \times \begin{bmatrix} 0.6 \\ 0.4 \end{bmatrix}$ = [30+36] = 66

8) Expected portfolio variance = $w_1^2\sigma_1^2 + w_2^2\sigma_2^2 + 2w_1w_2Cov_{1,2}$

- W_1 = portfolio weight of the first stock = 0.6
- W_2 = portfolio weight of the second stock = 0.4
- σ_1 = standard deviation of the first stock = $\sqrt{\frac{50}{3}} \approx 4.082483$

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- σ_2 = standard deviation of the second stock = $\sqrt{\frac{6650}{3}} \approx 47.081490$
- $Cov_{1,2}$ = covariance of two stocks = $p_{(1,2)}\sigma_1\sigma_2$
- $p_{(1,2)}$ = correlation coefficient
- Correlation coefficient = $p_{(1,2)}$

  $(-5) \times (-5) + (0) \times (60) + (5) \times (-55) = -250$

  $\frac{-250}{4.082483 \times 47.081490} \approx -1.300665$

  $\frac{-4.88625}{3-1} \approx -0.650332$

- $Cov_{1,2} = -0.650332 \times 4.082483 \times 47.081490 \approx -125$

  Expected portfolio variance = $w_1^2\sigma_1^2 + w_2^2\sigma_2^2 + 2w_1w_2Cov_{1,2}$
  $= 0.6^2 \times 4.082483^2 + 0.4^2 \times 47.081490^2 + 2 \times 0.6 \times 0.4 \times -125 \approx 353.466672$

5. Analysis of Data

Portfolio variance is the indicator of measurement of risk in a portfolio. With this value, the portfolio managers can see how the stocks or assets fluctuate over time. The expected portfolio variance is 353.466672, which is a large value, indicating the large risk of investing these two stocks with the weight of 60% and 40%. In order to reduce the risk, the portfolio manager should either change the percentage of capital to invest in each stock, or find an alternative stock to invest in.

The covariance that indicates the relationship of stock A and stock B is -125. Since the covariance is negative, the two stocks have inverse relationships. If stock A increases, stock B decreases and vice versa. Applied into economics, these two products or services will be substitutes for each other.

6. Summary

Covariance matrices, including variance and standard deviation, are often mentioned in both statistics and linear algebra. Covariance matrices are used when variables are based on similar scales. To compare and evaluate the relationship of more datasets and values, we can use specific programs to ensure accuracy and avoid inconvenient calculations by hand. We can see the applications of linear algebra, even math, everywhere in our daily life: ranging from shopping in the grocery stores to calculating the GDP of the country. Linear algebra, as a specific and essential branch in math, can be used to solve linear and nonlinear equations and are used in most compute-intensive tasks.

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