ISSN 2616-5902 Vol. 7, Issue 9: 73-78, DOI: 10.25236/AJBM.2025.070911

The MF-DFA Algorithm as a Tool for Testing Market Efficiency

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Abstract: *This article examines the MF-DFA (Multifractal Detrended Fluctuation Analysis) algorithm developed by J. Kantelhardt, a widely used tool for detecting informational inefficiency in the stock market. Its ability to identify subsets of data with varying degrees of correlation has linked it to the concept of market efficiency, where correlated returns imply inefficiency. However, as demonstrated in this study, the algorithm was originally designed for general purposes and does not account for the unique characteristics of financial data. One of the key features of financial data is volatility clustering, which, by itself, does not violate market efficiency. The article shows that volatility clustering generates a significant portion of the multifractal spectrum, often interpreted as a sign of inefficiency. Thus, applying the MF-DFA algorithm to financial data fails to distinguish between (a) the multifractal effect arising from varying degrees of correlation in price returns, and (b) the multifractal effect induced by clustered volatility. Consequently, the multifractal effect detected via MF-DFA analysis cannot serve as a definitive criterion for assessing whether a market is efficient or inefficient in the weak-form sense.*

Keywords: *Financial Multifractality, MF-DFA, Stock Market, Efficient Market Hypothesis (EMH)*

1. Introduction

The notion of informational efficiency in financial markets implies that, at any given moment, market prices fully reflect all available information accessible to investors [1]. If this holds true, predicting price increments becomes impossible, and the corresponding statistical data exhibit no time-dependent correlations. The emergence of this hypothesis in the 1960s [2] spurred the development of statistical methods aimed at either validating or refuting it.

The primary mathematical model aligned with this hypothesis was the random walk process [3]. Consequently, testing market efficiency essentially involved testing the random walk hypothesis. Initially, simple statistical methods were employed for verification (e.g., autocorrelation tests or runs tests). However, the analytical toolkit gradually became more sophisticated. Fractal methods, for instance, were introduced to study the dimensionality of time series (such as prices) to detect long-term memory in price increments.

Within fractal analysis, the so-called Hurst exponent (H) is calculated, ranging from 0 to 1. If the data exhibit no correlations — neither short- nor long-term — this coefficient equals 0.5. During the 1980s and 1990s, empirical estimates of H often significantly exceeded 0.5 (frequently around 0.7), indicating strong long-term correlations and persistent trends in market data. Today, however, most developed markets exhibit Hurst exponents very close to 0.5, which is widely interpreted as evidence of no memory effects — and thus, weak-form market efficiency.

Nevertheless, methodological advancements have since taken the study of financial data's statistical properties to a new level. Pioneered by Benoit Mandelbrot and his followers, the multifractal market hypothesis [4] emerged. This paradigm posits that the statistical properties of financial time series (e.g., variance) are not uniformly distributed but instead localized in specific segments. The degree of this statistical "inhomogeneity" is quantified by the multifractal spectrum of dimensions.

Under this framework, a financial time series is no longer viewed as a homogeneous statistical entity but rather as a complex assembly of distinct statistical sub-patterns, each governed by different dynamics. For example:

a) Large price fluctuations may follow one statistical law (e.g., exhibiting anti-persistence, $H < 0.5$, with frequent trend reversals);

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b) Small price fluctuations, conversely, might display persistent behavior (H > 0.5, current trend continuation).

The elegance with which Mandelbrot drew parallels between physical and economic phenomena, along with his pioneering use of multifractal methods in financial market analysis, had an unintended consequence. Many researchers came to believe that any statistical method examining multifractal properties of time series could be automatically applied to financial data without reservation.

Among the currently most popular tools for studying multifractal properties in stock markets is the MF-DFA (Multifractal Detrended Fluctuation Analysis) algorithm, developed by J. Kantelhardt and colleagues in 2002 [5]. Notably, this method has been widely adopted to test the Efficient Market Hypothesis (EMH) in numerous studies, including [6]-[13]. In such research, the standard methodology involves:

a) Calculating the multifractal spectrum for the original data series;

b) Computing the spectrum for shuffled/surrogate data (see Fig. 1);

c) Comparing both spectra

If a significant difference emerges between the spectra (i.e., region A occupies a substantial portion of the spectral range [α_{min}; α_{max}]), researchers typically conclude that the data exhibit multifractality, implying market inefficiency due to detected long-term memory effects.

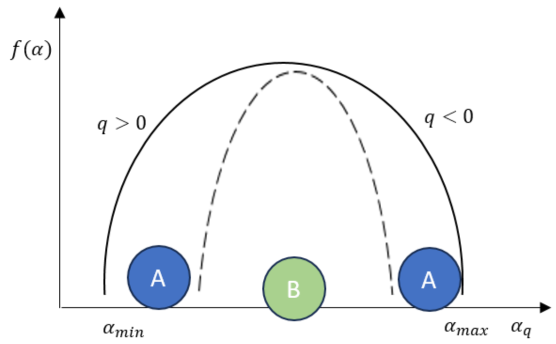

Figure 1: Multifractal spectrum for original (solid line) and shuffled (dashed line) data. Description: f(α) – multifractal spectrum function; α – Hölder exponent; q – scaling parameter (q > 0: analyzes high-variance regions; q < 0: examines low-variance regions). Source: Author's own elaboration.

However, a critical issue persists: most studies pay scant attention to the actual economic interpretation of multifractality. Given that the MF-DFA algorithm wasn't originally designed for financial data, its uncritical application often leads to questionable conclusions. As demonstrated below, results from the standard MF-DFA implementation cannot reliably support claims about weak-form market inefficiency.

2. The Economic Meaning of Multifractality

Before examining the MF-DFA algorithm in the context of the Efficient Market Hypothesis (EMH), it is essential to clarify the economic interpretation of financial multifractality. The foundation for this concept was laid by Benoit Mandelbrot, who introduced the notion of multifractal (or trading) time [4]. This pivotal idea is crucial for analyzing financial time series, such as price quotations. The core premise is that market time does not flow uniformly like physical time. While physical time progresses at a constant rate, trading time is highly non-uniform. Market participants (traders/investors) react to news flows, which themselves are irregularly distributed. For example, some periods feature sparse news, spread over long physical time intervals. On the other hand, other periods experience bursts of high-impact news, compressed into short physical time spans, triggering immediate market reactions.

This time inhomogeneity explains why asset returns (including log returns) almost never follow a normal distribution. Instead, financial return distributions exhibit: (a) heavy tails (excess kurtosis) and (b) high peaks (leptokurtosis). These properties reflect alternating phases of "slow time" (low volatility, minor price changes) and "fast time" (high volatility, large price swings). Such phases can often be distinguished by trading volume patterns.

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Mandelbrot was the first to link trading time inhomogeneity to heavy-tailed return distributions. He proposed his own price fluctuation model with non-uniform (or "trading") time [4], terming it as multifractal because it simultaneously accounts for: (a) self-similarity of statistical properties, and (b) their intrinsic heterogeneity. This statistical heterogeneity was indicative of long-range dependence, but was strictly confined to volatility patterns. As Mandelbrot noted, volatility clusters in certain regions — periods of large price fluctuations alternate with calmer intervals, exhibiting long memory and persistence [14]. His collaborators further clarified: "The multifractal model displays long dependence in the absolute value of price increments, while price increments themselves can be uncorrelated" [4, p. 2].

Thus, the MMAR (*Multifractal Model of Asset Returns*) model proposed by Mandelbrot is essentially a scalable analogue of ARCH-type models (which model conditional volatility), as Mandelbrot himself emphasized [4, p. 26]. Crucially, Mandelbrot's use of "multifractality" refers specifically to the time/volatility relationship: for monofractal (homogeneous) data volatility scales uniformly across all time intervals, whereas for multifractal (heterogeneous) data it scales unevenly — sometimes rapidly, sometimes slowly. This non-uniform scaling of volatility (divergent "scaling exponents") generates multifractal effects. At this point, it becomes evident that the observed multifractal effect is intrinsically linked to volatility clustering. Importantly, this phenomenon is distinct from long-term memory in price increments, as later research stresses: "increments of financial time series are well known to be uncorrelated (for large enough time lags), while their amplitude ('local volatilities') exhibits power-law correlations" [15, p. 10].

We have previously described the model proposed by Mandelbrot and his students. However, currently the majority of works dealing with the concept of a multifractal market and investigating multifractal properties of financial time series employ not Mandelbrot's MMAR model, but rather the MF-DFA algorithm developed by J. Kantelhardt for analyzing multifractal properties of time series [5]. The mathematical tools used by both Kantelhardt and Mandelbrot's followers (the Hurst exponent, Hölder exponent, multifractal spectrum, etc.) are identical. Yet paradoxically, Kantelhardt's paper contains no references to the works of Mandelbrot or any of his students. This explains why within the MF-DFA algorithm, multifractality carries a somewhat different meaning than in Mandelbrot's multifractal time paradigm.

As we have seen, Mandelbrot's school interprets multifractality of time series as time inhomogeneity (particularly of trading time) and the associated inhomogeneity in the rate of volatility growth during scaling. Thus, in the MMAR model, multifractality is used as a way to model clustered volatility when describing price fluctuations.

At the same time, Kantelhardt's analysis employs the concept of multifractality differently - namely as a property of a time series to contain subsets, some of which demonstrate persistent behavior while others show anti-persistent behavior. Importantly, Kantelhardt interprets persistence not as stability (clustering) of volatility, but precisely as stability of changes in the time series. He interprets persistence as "persistent and anti-persistent *increments*, where a positive increment is likely to be followed by another positive or negative increment, respectively" [16, p.8]. Accordingly, multifractal properties emerge as a consequence of different correlation dependencies linking such increments: "multifractal scaling is observed if the scaling behaviour of small and large fluctuations is different. For example, extreme events might be more or less correlated than typical events" [16, p.11].

The difference between Mandelbrot's and Kantelhardt's approaches to multifractality can be demonstrated with the example of a heavy-tailed fractal random walk. From the perspective of Mandelbrot's followers, such a process would be multifractal (since heavy tails are manifestations of time inhomogeneity, or multifractality). At the same time, from Kantelhardt's viewpoint, this same process would be monofractal, as all increments of this time series are correlated according to a single logic.

3. The MF-DFA Algorithm as a Tool for Analyzing Market Efficiency

As previously noted, MF-DFA analysis can detect multifractal effects arising from the presence of subsets in price increment time series characterized by diverse correlation dependencies. When such statistical heterogeneity exists in the data, the MF-DFA algorithm will indicate multifractal effects. Understood in this way, multifractality suggests market inefficiency: "A higher degree of multifractality in market price returns indicates more significant inefficiency in that market" [17, p.2].

However, such heterogeneity is not the only source of multifractality in the data. Kantelhardt and his

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followers have repeatedly emphasized that heavy tails in the original data distribution make a substantial contribution to the multifractal effect (manifested in the width of the multifractal spectrum) [16, p.11]. These two sources of multifractality in financial data have been mentioned in almost every article using the MF-DFA algorithm to analyze market efficiency.

At the same time, as we have shown, Mandelbrot's MMAR model clearly demonstrates the existence of another effect - multifractality induced by volatility clustering. This naturally raises the question: *can volatility clustering contribute to the multifractal effect detected by the MF-DFA algorithm*? To test this hypothesis, we generated a series of 100,000 normally distributed returns r_t following a GARCH(1,1) process, modeled by the equations:

$$\sigma_t^2 = \omega + \alpha r_{t-1}^2 + \beta \sigma_{t-1}^2 \quad (1)$$

$$r_t = \mu + \sigma_t \varepsilon_t, \text{где } \varepsilon_t \sim N(0,1) \quad (2)$$

In our example, the parameters were selected as follows: $\omega = 0.1, \alpha = 0.09, \beta = 0.9, \mu = 0$.

For the original return series, we constructed a multifractal spectrum plot following standard methodologies used in similar studies (Fig. 2). The lower right portion of the graph shows the multifractal spectrum for both the original and shuffled data. A wider spectrum indicates a more pronounced multifractal effect.

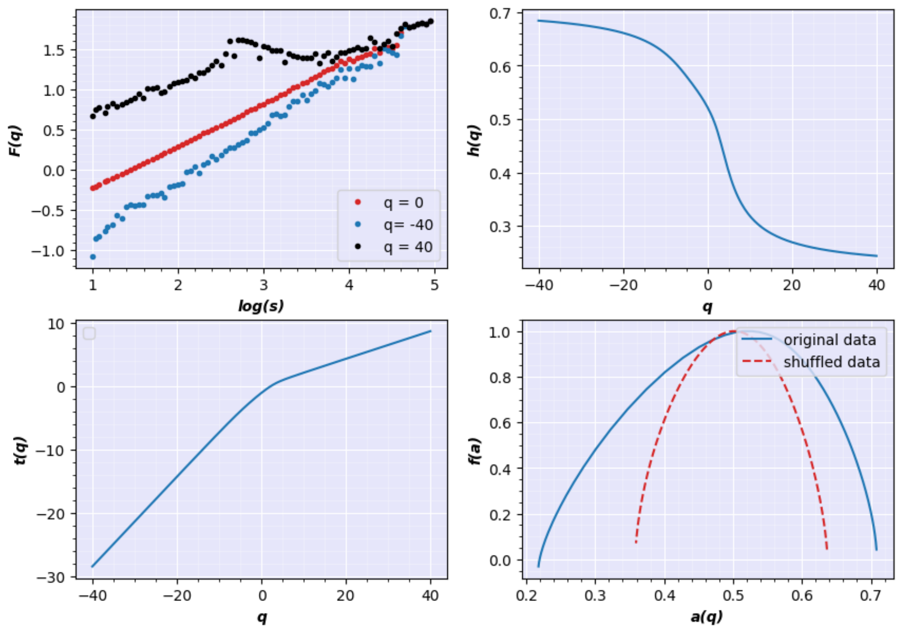

Figure 2: Multifractal spectrum for both the original and shuffled data. Source: Author's own elaboration.

The presented figure clearly demonstrates that the original data exhibits a relatively broad multifractal spectrum with a width of approximately 0.5. However, when the data are shuffled, this width significantly narrows to about 0.3. Critically, neither dataset contains "true" multifractality associated with differences in return correlations), as such correlations are intentionally excluded during data generation. This confirms that the observed spectrum narrowing stems exclusively from eliminated volatility clustering. Thus, volatility clustering alone can generate significant multifractal effects under MF-DFA.

Since real financial data inherently exhibits volatility clustering (as demonstrated by Mandelbrot, among others), applying the MF-DFA algorithm to stock market data will systematically detect multifractality from this source. We shall refer to this phenomenon as "cluster multifractality." To distinguish such statistical artifacts from the "true" multifractality arising from correlation differences in price increments (which we shall term "increment multifractality"), preliminary statistical procedures are required that remove volatility memory while preserving other types of memory in the data.

However, since classical MF-DFA analysis cannot distinguish between these two types of memory (memory in returns and volatility memory), the algorithm's results cannot provide definitive evidence of market inefficiency. This is relevant because, according to the standard definition of weak-form market efficiency, two conditions must be met: 1) price increments must be unpredictable, and 2) prices must

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incorporate all available statistical information. While "increment multifractality" undoubtedly violates both principles (being based on long-term memory in price movements) and thus indicates inefficiency, "cluster multifractality" stems from volatility clustering - a phenomenon that does not directly contradict weak-form EMH requirements.

Market volatility spikes may simply reflect intensified flows of significant news, with each news item being instantly incorporated into prices. In such cases, prices may still fully reflect all available statistical (and other) information at every moment, maintaining market efficiency. Therefore, the presence of cluster multifractality - evident in most multifractal spectra of stock market behavior – does not constitute valid grounds for rejecting the EMH. The algorithm's inability to differentiate between these fundamentally distinct sources of multifractality severely limits its usefulness as a tool for assessing market efficiency/inefficiency.

4. Conclusion

The algorithm proposed by Kantelhardt was not originally designed for analyzing financial data. However, its ability to detect long-range dependence effects led researchers to adopt MF-DFA as a tool for testing the Efficient Market Hypothesis (EMH). This approach — using MF-DFA to assess market efficiency — has gained widespread traction in academic literature. Yet, its uncritical application to financial data poses significant challenges, as it conflates two distinct phenomena:

1) Multifractal volatility (or *cluster multifractality*), arising from volatility clustering;

2) Multifractal returns (or *true multifractality*), which we associate with market inefficiency.

This reveals the fundamental limitations of the MF-DFA algorithm as a tool for assessing market efficiency. If volatility clustering is absent, MF-DFA reliably identifies multifractality (i.e., heterogeneity in scaling behavior) in data with long-memory increments. Kantelhardt himself demonstrated that applying MF-DFA to standard fractional Brownian motion yields *no multifractal effect* [5, p. 93]. For real financial data, which exhibit *both* long-range correlations in returns *and* volatility clustering (as demonstrated by Mandelbrot), MF-DFA produces a broad multifractal spectrum *even when price increments are uncorrelated*. This means the algorithm misinterprets cluster multifractality as true multifractality. The multifractality detected by MF-DFA in such cases merely reflects volatility clustering — a phenomenon *compatible* with EMH, as it does not imply predictability of price increments (Fama's core criterion for inefficiency).

Thus, the key findings of our study can be summarized as follows:

a) Multifractality ≠ Market Inefficiency: volatility clustering (a multifractal trait) does not imply predictability of price *direction*;

b) MF-DFA Pitfall: standard multifractal analysis (MF-DFA) cannot distinguish between:

○ Long-term memory in increments (violating EMH)

○ Long-term memory in volatility (compatible with EMH);

c) This distinction is critical for interpreting multifractal spectra in market efficiency studies.

Therefore, to identify the true multifractality in financial data that could indicate market inefficiency, we must first eliminate the clustering effect (i.e., remove cluster-induced multifractality), preserving only the component generated by long-term correlations in price increments. The development of such a methodology — specifically, the refinement of the MF-DFA algorithm — should become the focus of future research.

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