Practical Research on the Professional Development of Senior High School Mathematics Teachers from the Perspective of Difficulty Coefficient Analysis

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Abstract: *Driven by the reform of senior high school education methods and core competency-oriented policies, integrating scientific difficulty coefficient setting for mathematics test items with teachers' professional development is key to breaking the bottleneck in improving mathematics education quality. Currently, college entrance examination mathematics difficulty coefficients are highly subjective and lack systematic theoretical support, while senior high school mathematics teachers face dilemmas such as outdated curricula, disconnected teaching and research, and single evaluation mechanisms. This study constructs a "Comprehensive Difficulty Coefficient Model for Mathematics Test Items" with 10 dimensions and 3 levels, explores its internal relationship with teachers' professional development using literature research, questionnaires, interviews, and empirical methods, and proposes a "four-in-one" professional development strategy: curriculum system optimization, teaching ability improvement, research ability enhancement, and evaluation system improvement. A two-year empirical test shows the strategy significantly improves teachers' difficulty control ability, teaching quality, and students' mathematical core competencies. It provides a new theoretical perspective and practical path for teachers' professional development and empirical references for policymakers to promote educational equity.*

Keywords: *difficulty coefficient; senior high school mathematics; teachers' professional development*

1. Introduction

With the issuance of policies such as *Opinions on Deepening the Reform of Education and Teaching to Comprehensively Improve the Quality of Compulsory Education*, senior high school mathematics education has entered a transition stage centered on core competency cultivation[1]. As a key link connecting curriculum standards, teaching practice, and academic evaluation, reasonable difficulty coefficient setting directly affects students' learning motivation and teachers' teaching decisions[2]. However, three practical dilemmas persist: subjective coefficient setting disconnected from students' cognitive levels[3], teachers' cognitive biases toward difficulty dimensions hindering teaching strategy adjustment[4], and insufficient integration of difficulty coefficient analysis with teachers' professional development[5].

Senior high school mathematics teachers are pivotal to core competency cultivation, but they face challenges including uneven educational resources, fragmented training, and disjointed teaching and research[6]. This study constructs a scientific difficulty coefficient model, explores its impact on teachers' professional development, and proposes targeted strategies—filling research gaps and providing a new path for improving mathematics education quality.

From a theoretical perspective, the model enriches mathematics education evaluation theory and broadens teachers' professional development research horizons[7]. Practically, it helps teachers optimize teaching strategies and supports policymakers in promoting educational equity[8]. Policy-wise, it aligns with national educational reform goals, providing empirical support for policy implementation[9].

This study follows the logic: propose problems → deconstruct problems → investigate problems → solve problems → expand problems. Research methods include literature research, questionnaire surveys (300 teachers from 15 provinces), interviews (20 teachers), empirical research (20 classes from 4 schools), and multivariate statistical analysis using SPSS 26.0 and AMOS 24.0[10].

2. Literature Review

The difficulty coefficient, first proposed by E. L. Thorndike, measures test item difficulty. Foreign research has formed mature systems: L. J. Cronbach proposed an item response theory-based calculation method, and J. Hattie found reasonable difficulty setting has an effect size of 0.42 on learning outcomes.

Domestic research started in the 1980s: Wang Xiaoling proposed a score rate-based calculation method[11], and Zhang Dianzhou identified item type[12], knowledge content, and thinking requirements as key difficulty factors. Recent core competency-oriented research includes Shi Ningzhong's difficulty design principles[13] and Cao Yiming's multi-dimensional difficulty model. However, existing research lacks comprehensive dimensions, ignores teaching strategy connections, and disconnects from teachers' professional development.

The concept of teachers' professional development emerged in the U.S. in the 1960s. Foreign theories include A. M. Lieberman's "professional development school" model and E. Hoyle's stage theory. Domestic research focuses on connotation (Tu Rongbao[14]), influencing factors (policies, training), and strategies (school-based training). Limitations include insufficient attention to research and difficulty control abilities, and lack of quantitative support.

Research on the connection between difficulty coefficients and teachers' professional development is scarce. Foreign studies (D. L. Ball, J. Cohen) and domestic studies (cognitive status, teaching strategy impact) lack systematic models and large-scale empirical testing[15]. This study fills this gap.

3. Theoretical Basis and Research Framework

3.1 Definition of Core Concepts

1) Comprehensive Difficulty Coefficient of Mathematics Test Items: A multi-dimensional quantitative index calculated via weighted sum of 10 dimensions (context background, parameter classification, etc.), reflecting test requirements for knowledge, ability, and literacy.

2) Teachers' Professional Development: Continuous growth in professional knowledge, teaching ability (difficulty control, etc.), research ability, and professional concepts.

3) Mathematical Core Competencies: Key abilities including mathematical abstraction, logical reasoning, mathematical modeling, intuitive imagination, mathematical operation, and data analysis.

3.2 Theoretical Basis

Constructivist Learning Theory: Emphasizes matching test difficulty with students' cognitive levels.

Teacher Professional Development Stage Theory: Guides differentiated strategy design.

Educational Evaluation Theory: Supports teaching diagnosis via difficulty coefficient analysis.

Core Competency Cultivation Theory: Ensures difficulty setting and teaching align with competency goals.

3.3 Research Framework

The framework includes four parts:

1) Difficulty Coefficient Model: 10 dimensions × 3 levels, weighted sum calculation (weights: reasoning ability 0.18, cognitive level 0.14, etc.).

2) Teachers' Professional Development Elements: Professional knowledge, teaching ability, research ability, professional concepts.

3) Four-in-One Strategy: Curriculum optimization, teaching ability improvement, research enhancement, evaluation improvement.

4) Practical Effect Evaluation: Assesses teachers' development, teaching quality, and students' competencies.

4. Construction and Verification of the Comprehensive Difficulty Coefficient Model

4.1 Basis for Model Construction

Policy Basis: Aligns with *General Senior High School Mathematics Curriculum Standard (2017 Edition, Revised in 2020)*.

Theoretical Basis: Integrates mathematics education evaluation and core competency theories.

Practical Basis: Analyzes teaching practice and college entrance examination papers, incorporating expert opinions.

4.2 Dimension Division and Connotation

The comprehensive difficulty coefficient model for mathematics test items comprises ten core dimensions, each divided into three progressive levels, collectively reflecting the multi-faceted requirements of test items for students' mathematical abilities.

The Context Background Dimension classifies items by their application scenarios: pure mathematics (A1) focuses on direct knowledge application; subject integration (A2) links math to daily life/production, requiring mathematization of practical contexts; scientific inquiry (A3) involves interdisciplinary scientific scenarios, demanding extraction of mathematical elements from cross-disciplinary information.

The Parameter Classification Dimension is based on parameter characteristics: no parameters (B1) relies on static data; fixed parameters (B2) involve constant unknowns without discussion; variable parameters (B3) require comprehensive analysis of parameter ranges and scenarios.

The Operation Level Dimension reflects computational complexity: basic operations (C1) include numerical calculations and simple non-symbolic operations; intermediate operations (C2) involve symbol derivation and formula transformation; advanced operations (C3) integrate complex logical deduction, such as proofs and regression modeling.

The Reasoning Ability Dimension measures logical requirements: direct reasoning (D1) involves 1-3 steps of straightforward deduction; chain reasoning (D2) forms sequential logical links; complex reasoning (D3) requires multi-branch or circular logic with reverse verification.

The Knowledge Content Dimension covers knowledge scope: single-point mastery (E1) centers on one knowledge unit; two-point integration (E2) combines two cross-unit knowledge concept；multi-point integration (E3) involves three or more cross-chapter/interdisciplinary knowledge modules.

The Thinking Direction Dimension describes problem-solving paths: forward thinking (F1) follows conventional logical sequences; reverse thinking (F2) involves reverse application or transformation of knowledge; multi-directional thinking (F3) encourages multi-angle analysis and flexible method selection.

The Cognitive Level Dimension reflects depth of knowledge application: comprehensive understanding (G1) tests basic concept mastery; application of knowledge (G2) requires transferring abstract knowledge to practical problems; in-depth analysis (G3) involves screening implicit conditions and constructing models in complex environments.

The Reading Volume Discrimination Dimension is based on text information load: short reading (H1, ≤50 characters) allows quick information extraction; medium reading (H2, 50-100 characters) requires careful screening of key conditions; long reading (H3, >100 characters) demands distinguishing redundant and implicit information.

The Condition Base Dimension depends on the number of effective conditions: single condition (I1) enables direct application; dual conditions (I2) require collaborative integration; multiple conditions (I3) involve sorting and optimizing complex interrelated conditions.

The Innovation Empowerment Dimension reflects item novelty: conventional level (J1) uses fixed problem-solving models; moderate innovation (J2) includes ill-structured items with novel methods; in-depth innovation (J3) introduces new definitions or models beyond textbooks, testing rapid knowledge transfer.

These ten dimensions interact to comprehensively evaluate students' mathematical knowledge, logical thinking, practical application, and innovative abilities, providing a systematic framework for

assessing test item difficulty.

4.3 Weight Determination and Model Verification

- Weight Determination: 15 experts (professors, researchers, senior teachers) conducted three rounds of anonymous scoring, with weights normalized and verified.

- Model Verification: Content validity (10 experts confirmed comprehensiveness) and empirical validity (correlation coefficient 0.87 with official difficulty coefficients, p<0.01).

5. Correlation Analysis between Difficulty Coefficient and Teachers' Professional Development

5.1 Questionnaire and Interview Results

- Questionnaire Results (278 valid responses, 92.7% recovery rate):

  o Only 32.4% of teachers accurately understand difficulty coefficients; 45.7% have preliminary knowledge; 21.9% have little understanding.

  o Teachers excel in traditional dimensions (operation level, knowledge content) but struggle with modern dimensions (context background, innovation empowerment).

  o 78.8% need difficulty coefficient training; 67.3% want competency-oriented teaching design training.

- Interview Results: Teachers face disconnection between difficulty analysis and practice, lack scientific difficulty design methods, disjointed teaching and research, and mismatched training.

5.2 Correlation between Dimensions and Professional Development

1) Context Background: Relates to mathematical modeling/situational teaching ability (A1: pure knowledge teaching; A2: life problem mathematization; A3: interdisciplinary integration).

2) Parameter Classification: Links to variable thinking/logical reasoning teaching (B1: basic operations; B2: parameter-containing operations; B3: parameter discussion).

3) Operation Level: Directly connects to operation teaching ability (C1: basic skills; C2: symbol derivation; C3: complex logical deduction).

4) Reasoning Ability: Relates to logical reasoning cultivation (D1: direct reasoning; D2: chain reasoning; D3: complex reasoning).

5) Knowledge Content: Links to knowledge integration teaching (E1: single-unit teaching; E2: cross-unit integration; E3: multi-unit/interdisciplinary integration).

6) Thinking Direction: Relates to thinking quality cultivation (F1: forward thinking; F2: reverse thinking; F3: multi-directional thinking).

7) Cognitive Level: Connects to cognitive guidance (G1: basic knowledge teaching; G2: knowledge application; G3: complex situation analysis).

8) Reading Volume Discrimination: Relates to information extraction teaching (H1: simple extraction; H2: key information screening; H3: implicit condition mining).

9) Condition Base: Links to condition analysis teaching (I1: single condition application; I2: dual-condition correlation; I3: multi-condition integration).

10) Innovation Empowerment: Relates to innovative teaching (J1: conventional；J2: ill-structured；J3: new context).

6. Teachers' Professional Development Strategies

6.1 Curriculum System Optimization

- Core Modules: "Difficulty coefficient theory + core competency cultivation + teaching practice

application".

- Elective Courses: Differentiated courses (basic difficulty strategies for junior high; advanced strategies for senior high; research methods for weak researchers).
- Dynamic Update: Incorporate latest teaching cases and proposition trends.

6.2 Teaching Ability Improvement

- Difficulty Control Training: Lectures, case discussions, simulated teaching.
- Teaching Design Optimization: Align with difficulty dimensions and student cognition.
- Classroom Implementation Enhancement: Classroom observation, microteaching, reflection.
- Competency Cultivation: Link difficulty dimensions to core competencies (e.g., parameter classification → logical reasoning).

6.3 Research Ability Enhancement

- Research Awareness Cultivation: Lectures and experience sharing.
- Method Training: Workshops on questionnaire surveys, empirical research, and statistical tools.
- Topic Guidance: Special projects on difficulty coefficients and teaching/competency cultivation.
- Result Transformation: Share and promote research outcomes.

6.4 Evaluation System Improvement

- Index System: Four first-level indicators (difficulty control, teaching, research, professional concepts) with sub-indicators.
- Diversified Methods: Combine quantitative/qualitative, process/summative, and self/others' evaluation.
- Feedback Application: Link results to performance and personalized development plans.

7. Empirical Research Results

7.1 Research Design

- Objects: 20 classes (10 experimental, 10 control) with comparable teachers and students.
- Cycle: 2 years (preparation: 2023.9-12; implementation: 2024.1-2025.1; summary: 2025.2-6).
- Data Collection: Teachers' development, teaching quality, and students' competencies.

7.2 Key Results

- Teachers' Professional Development:
  - Difficulty control ability: Experimental class (92.3% identification accuracy, 89.7% judgment accuracy, 88.5 points adjustment ability) > control class (76.5%, 72.3%, 75.2 points).
  - Teaching ability: Experimental class (87.6 design, 89.2 implementation, 86.8 competency cultivation) > control class (74.3, 75.6, 73.5).
  - Research ability: 18 projects/32 papers (experimental) vs. 6 projects/11 papers (control).
- Teaching Quality:
  - Classroom evaluation: 88.7 (experimental) vs. 76.4 (control).
  - Student scores: 89.5 average, 32.6% excellent rate (experimental) vs. 78.3, 18.5%

(control).

- Students' Core Competencies: All six competencies scored significantly higher in the experimental class (84.3-88.6 points) than the control class (72.5-76.4 points).

8. Conclusions and Prospects

8.1 Conclusions

First, this study successfully constructs a comprehensive difficulty coefficient model for mathematics test items, which consists of ten core dimensions and three hierarchical levels per dimension. Through rigorous content validity verification by mathematics education experts and empirical validity testing against official difficulty coefficients of college entrance examination papers (with a correlation coefficient of 0.87, $p<0.01$), the model demonstrates good reliability and validity, enabling comprehensive and scientific reflection of the difficulty characteristics of mathematics test items.

Second, the research reveals the intrinsic correlation between each dimension of the difficulty coefficient and the professional development of senior high school mathematics teachers. Specifically, dimensions such as reasoning ability, cognitive level, and operation level have the closest connection with teachers' teaching ability, directly influencing their ability to design teaching activities and guide students in problem-solving; dimensions including innovation empowerment and context background are closely related to teachers' teaching innovation ability and scientific research ability, promoting teachers to break conventional teaching models and conduct in-depth educational research; while knowledge content and thinking direction dimensions exert an important impact on the optimization of teachers' professional knowledge structure and the renewal of professional concepts.

Third, based on the above correlation analysis, this study proposes a "four-in-one" professional development strategy for senior high school mathematics teachers, covering curriculum system optimization, teaching ability improvement, research ability enhancement, and evaluation system improvement. This strategy is highly targeted, as it directly addresses the practical dilemmas faced by teachers in difficulty coefficient application and professional growth, and features strong operability by providing clear implementation paths and specific methods for each component.

Fourth, the two-year empirical research confirms that the proposed "four-in-one" strategy achieves significant practical effects. It not only effectively improves teachers' professional development level—including enhancing their difficulty control ability, teaching design ability, and scientific research achievement output—but also significantly promotes the improvement of classroom teaching quality and the all-round development of students' mathematical core competencies, fully verifying the effectiveness and practical value of the strategy.

8.2 Limitations and Prospects

Despite the valuable research findings, this study still has certain limitations. In terms of research objects, the experimental sample is limited to 20 classes from 4 senior high schools, and the sample size needs to be further expanded to enhance the representativeness and promotion value of the research results. In terms of research cycle, the two-year tracking period may not fully reflect the long-term impact of the professional development strategy on teachers and students, requiring extended follow-up observations. In terms of application scope, the research mainly focuses on senior high school mathematics teachers, and its applicability to junior high school mathematics teachers and other education stage educators remains to be further verified.

Looking forward, future research could be carried out in the following aspects. First, the sample size should be expanded by recruiting teachers from diverse regions, school types, and academic stages to carry out large-scale empirical research, thereby enhancing the generalizability of the research findings. Second, the research cycle ought to be extended to conduct long-term tracking of participating teachers' professional growth and the sustainable development of students' competencies, with a focus on exploring the long-term effect mechanism of the proposed strategy. Third, the application scope of the research outcomes should be broadened: the professional development strategy should be adjusted and optimized in accordance with the characteristics of mathematics teaching at stages such as junior high school, so as to facilitate the widespread adoption of the research findings. Fourth, efforts should be made to keep pace with advances in educational technology; artificial intelligence and big data technologies should be integrated to develop intelligent analytical tools for difficulty coefficients and personalized

platforms for teachers' professional development, which can provide more precise and efficient support for the professional development of mathematics teachers.

Acknowledgements

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