ISSN 2522-6398 Vol. 7, Issue 8: 95-101, DOI: 10.25236/FER.2024.070816

Deepening Mathematical Thinking: A Study on the Examination of Higher-order Thinking Skills in the 2023 GCE New Mathematics Curriculum Paper

Wanling Liu[1,a,*], Tao Chen[2,b]

[1]*School of Mathematics and Computer Science, Shaanxi University of Technology, Hanzhong, China*
[2]*Professor from Shaanxi University of Technology, Hanzhong, China*
[a]*2218553527@qq.com,* [b]*24523106@qq.com*
[*]*Corresponding author*

Abstract: *In the context of the current education reform, higher-order thinking skills have become an important quality for students to adapt to social change. This paper explores the role of the question papers in evaluating and developing higher-order thinking skills by analyzing the new standard question papers of Mathematics in the 2023 HKALE. The study found that the two sets of question papers covered all levels of thinking skills, but there was a potential imbalance, particularly in the apparent lack of assessment of creative thinking skills. The study concludes with suggestions for optimizing the cognitive level assessment, innovative problem design, and multi-scenario design, which will serve as a reference for optimizing the GCE examination papers and the teaching of mathematics in senior secondary school.*

Keywords: *New Curriculum Mathematics paper; higher-order thinking skills; developmental advice; mathematical thinking*

1. Introduction

Higher-order thinking skills refer to students' ability to perform higher cognitive activities such as analyzing, evaluating, and innovating, specifically including creativity, critical thinking, and problem-solving skills[1]. In the 21st century, higher-order thinking skills have become essential for students. In the 21st century, higher-order thinking skills have become essential core competencies for students to adapt to the rapidly evolving and increasingly complex challenges of society. This not only equips students with the ability to cope with a wide range of challenges but also develops in them the excellent qualities to solve problems and make informed decisions in complex environments.

While examination papers have always been regarded as an important indicator for assessing students' ability or quality, the GCE examination papers are an important tool for assessing the higher-order thinking ability of senior secondary school students. An in-depth analysis of the GCE Mathematics examination paper to explore its performance in examining students' higher-order thinking ability in mathematics is of great significance in understanding the quality of education and teaching, optimizing the design of the GCE examination paper, and improving the quality of talent training.

At present, research on the Mathematics GCE examination papers focuses mostly on the theoretical structure, degree of difficulty, core literacy, and analysis of the consistency of the examination questions with the curriculum standards[2]-[5] and there is relatively little discussion on the effective examination of students' higher-order thinking skills through test paper analyses. Given this, this paper selects the 2023 New Curriculum Standard Mathematics I and II papers as the research object, aiming at exploring the current situation and problems of examining higher-order thinking in the mathematics papers under the New Curriculum Standard model through in-depth analyses of the test papers and putting forward corresponding cultivation suggestions, with a view to providing meaningful references for the optimization of the design of the future college entrance examination papers and for the teaching of mathematics in high schools.

2. Relevant Concepts

Mathematical thinking refers to the ways of thinking and methods used in solving mathematical problems[6]. Mathematical thinking is the way of thinking and methods used in solving mathematical

ISSN 2522-6398 Vol. 7, Issue 8: 95-101, DOI: 10.25236/FER.2024.070816

problems. It has been mentioned that profundity, rigour, divergence, flexibility, criticality, and creativity are important qualities of mathematical thinking.[7] . However, with the continuous development of society, it is not possible to develop mathematical thinking solely on its own. However, with the continuous development of society, the simple understanding and application of mathematics can no longer meet the requirements of the quality of mathematical thinking, and to achieve the development of the quality of students' mathematical thinking, it is necessary to explore the deeper mathematical higher-order thinking. Most educational research has identified four key elements in the definition of higher-order thinking in mathematics: mathematical critical thinking, mathematical creative thinking, mathematical problem-solving ability, and mathematical metacognitive ability[8]. Mathematical critical thinking is the foundation, which is characterized by analytical, systematic, confident, and inquisitive thinking; mathematical creative thinking is the source of motivation, which is characterized by fluency, novelty, and adaptability; mathematical problem-solving is the key support, which encompasses the three processes of representation, inference, and reflection; and mathematical metacognitive ability is the safeguard, which encompasses the three dimensions of knowledge, experience, and monitoring. These four elements together constitute the structure of higher-order thinking in mathematics[9].

Bloom's Taxonomy of Objectives theory was proposed by Benjamin Bloom, a famous educator in the United States; in 2001 Anderson (L.W.), Krathwohl (D.R.), and others modified the cognitive objectives classification system, in which the cognitive process dimensions, from low to high, include memory, comprehension, application, analyzing, evaluating and creating at six levels[10]. The theory is widely recognized in the educational community as the most effective way to categorize cognitive goals. It is widely recognized in the educational community that the theory provides a clear framework for understanding and developing higher-order thinking skills, especially at the levels of analysis, synthesis, and creation, which directly correspond to the key elements of higher-order thinking. In this paper, the research object is the real mathematics questions of the college entrance examination, which is oriented to high school students with high thinking levels, and since the applicability of mathematics is supposed to be the problem-solving of mathematical thinking from abstraction to concrete reality, and the problem-solving process is the key element of mathematical higher-order thinking, the last four levels correspond to mathematical higher-order thinking, and the first two levels correspond to mathematical lower-order thinking. Mathematical metacognitive ability is mainly manifested in students' planning, regulation, and monitoring of the whole learning process, and the test paper cannot directly react to the metacognitive performance, which is not studied in this paper for the time being.

Based on the connotative elements of mathematical higher-order thinking and applying Bloom's theory of target classification, this paper will analyze the new high school examination paper in terms of its hierarchical division and examination of thinking. So the four key elements of mathematical higher-order thinking and the connotations of the six cognitive levels are compared and analyzed, and a low-to-high correlation is obtained, regarding the description of the behavioral verbs in Ma Shuo Bing's article to derive the evaluation indexes of the analyzed examination papers as follows Table 1[11].Table 1. There is an approximate one-to-one correspondence between higher-order thinking in mathematics and Bloom's cognitive level.

Table 1: Evaluation table of mathematical thinking behaviors under Bloom's classification of objectives

cognitive level	Level Connotation	level of thinking	mathematical thinking	Evaluation of behavioral indicators
memorization	Recall information, facts, concepts	lower-order thinking	lower-order thinking	Ability to understand and list basic mathematical concepts, formulas, and rules and apply them accurately in problem-solving without having to consult information
understandings	Understanding, interpreting, and summarizing the meaning of information	lower-order thinking	lower-order thinking	Be able to understand, explain, and express the conditions, requirements, and relationships of a mathematical problem and be able to summarize key information about the problem
put to use	Knowledge application, problem-solving	higher order thinking	Problem-solving skills	To be able to explore and solve real-life problems, and to propose and apply mathematical knowledge flexibly to solve them.
analyzed	Information decomposition, identification of structures	higher order thinking	critical thinking	Ability to summarize and organize problem information, break down complex problems into manageable parts, and accurately analyze key factors
evaluations	Information assessment, judgmental value	higher order thinking	critical thinking	Be able to Assess and review the problem-solving process, Judge the strengths and weaknesses of problem-solving methods, reflect and make suggestions for improvement
creation	Creating new ideas, products, and solutions	higher order thinking	creative thinking	Ability to abstract, discover, and generate innovative solutions and build unique problem-solving paths

ISSN 2522-6398 Vol. 7, Issue 8: 95-101, DOI: 10.25236/FER.2024.070816

3. Analysis of examination papers

3.1 Example of an Analysis of a Higher Order Thinking Paper in Mathematics

The following is a selection of sample questions from the 2023 New Academic Standard Higher Education Examination, based on Bloom's Cognitive Level Characteristics, to analyze the behavioral performance of integrating mathematical thinking, and to discuss in depth the examination of mathematical thinking in the Higher Education Examination questions.

Example (2023 Gao Xue Real Exam Questions New Standard I Volume 12) The following object, which can be placed by the whole into the prism length of 1 (Unit: m) The following objects can be put into a square container (the thickness of the wall is ignored) with a prism length of (in.)

A. The diameter of the 0.99 m of the sphere

B. All tetrahedra with prisms of length 1.4 m tetrahedron

C. The diameter of the base is 0.01 m , the 1.8 m cylinder of height

D. The diameter of the base is 1.2 m The diameter of the base of the cylinder is 0.01 cylinder of height

Analysis: this topic question and answer belongs to the multiple choice question type, and the four options in turn to increase the difficulty of analysis, the topic set itself need to be analyzed and determine whether each option is consistent with the meaning of the question. This topic involves many three-dimensional figures in space, to determine the different sizes of the figure can be placed inside a fixed container is the center of the problem, only students need to analyze the meaning of the question is mainly to examine the nature of the figure inside the square, you need to grasp the key nature of the different figures to a simple calculation to determine whether the whole can be put into the square container. Therefore, the question mainly examines mathematical critical thinking, the cognitive level reaches the level of analysis and effectively exercises the students' mathematical higher-order thinking skills.

3.2 Statistics and Analysis of Results of Question Paper Analyses

The next step is to determine the number of cells that can be used according to the Table 1 of the evaluation behaviors indicators to analyze the statistics of the 2023 NSS Mathematics questions. In this paper, each question is regarded as an independent entity, e.g., "17.1" means the first question of question 17; the requirements of the question and the performance of the thinking process in solving the question are attributed to different cognitive thinking categories. To reduce the error caused by subjective judgement, when encountering analytical and comprehensive questions, which involve different levels of thinking, the thinking types are classified according to the requirements of the problem center. Based on the analyses, the following tables 2 and 3 can be obtained.

Table 2 Distribution of questions in the NSS I paper based on cognitive level and mathematical thinking

arrangement of ideas / mathematical thinking	memorization	understandings	put to use	analyzed	evaluations	creation
Lower-order thinking in math	1, 2, 3, 8	4, 5, 6, 14, 15, 17.1, 17.2				
Problem-solving skills			22.1, 10, 13, 21.1, 21.3			
critical thinking				7, 9, 11, 12, 16, 18.1, 18.2, 19.1, 20.1, 20.2, 21.2	19.2, 22.2	
creative thinking						

Table 3: Distribution of questions in the NSS II paper based on cognitive levels and mathematical thinking

arrangement of ideas / mathematical thinking	memorization	understandings	put to use	analyzed	evaluations	creation
Lower order thinking in math	1, 13	3, 4, 5, 6, 7, 14, 21.1				
Problem-solving skills			12, 19.1			
critical thinking				2, 8, 9, 10, 11, 16, 17.1, 17.2, 18.1, 19.2, 20.2, 22.2	15, 18.2, 20.1, 21.2, 22.1	
creative thinking						

ISSN 2522-6398 Vol. 7, Issue 8: 95-101, DOI: 10.25236/FER.2024.070816

From the distribution table, it can be seen that the distribution of topics in Paper I and Paper II is roughly the same, with most of the topics focusing on comprehension, application, and analysis, indicating that both lower-order and higher-order mathematical thinking are the key targets of the college entrance examination. Paper I and II have the most amount of critical thinking questions set on analysis and evaluation, and the more amount of lower order mathematical thinking questions set on memorization and comprehension, but both are very deficient in the amount of creative thinking examined at the highest level.

In order to qualitatively analyze the level of thinking in the new standard question papers, on the basis of Tables 2 and 3, the statistical data of the different levels of mathematical thinking were obtained in Figure 1. On the basis of Table 2 and Table 3, statistics on the examination scores of different levels of mathematical thinking were obtained in Figure 2, and in order to analyze the examination of different levels of thinking in a more detailed way, the scores of each question were subdivided using the types of questions (single choice, multiple choice, fill-in-the-blanks, computation, and demonstration) in Figure 2, and for the sake of graphical simplicity, the mathematical thinking in Table 1 was numbered from low to high, respectively. "A", "B" "C", "D" "E ", "F" six levels and six cognitive levels correspond to each other.

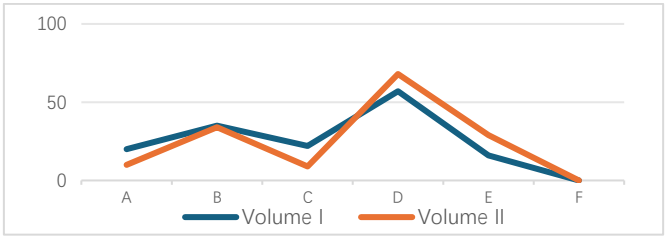

Figure 1: 2023 New Syllabus Paper Mathematical Thinking Marks Distribution Folding Plot

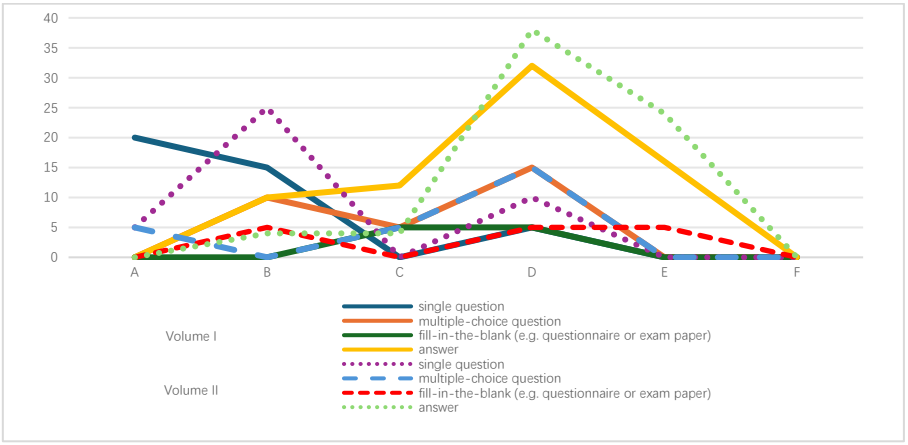

Figure 2: 2023 New Syllabus Question Paper Mathematical Thinking in Question Type Mark Distribution Folding Plot

According to the statistics and analysis of the results of the NSS papers, it can be seen from Fig.1: both sets of papers involved the examination of various levels of thinking on the whole, mainly focusing on the comprehension level of lower-order thinking and the analytical level of higher-order thinking; it is obvious that Volume I, which examines the degree of lower-order thinking, is slightly higher than Volume II, and Volume II, which examines the degree of higher-order thinking, is slightly higher than Volume I; it can be seen that the overall investigation of thinking in the two papers shows an imbalanced state, especially the investigation of students' ability to solve practical problems is very weak. It can be seen that the two sets of papers showed an overall imbalance in the examination of thinking, especially the examination of students' ability to solve practical mathematical problems was very weak. It can also be seen that there is an overall imbalance in the examination of thinking in the two sets of papers, especially in the examination of students' ability to solve practical mathematical problems in a creative way, which may be due to the fact that creative thinking, as the highest level of thinking, has its own difficulty and challenge, and the creative questions will make the examination paper more difficult, which is easy to go beyond the tolerance range of the students. As can be seen from Figure 2, the two sets of question papers have different degrees of examination of mathematical thinking in terms of question types. In Paper I, the distribution of scores of single-choice and multiple-choice questions is higher than

ISSN 2522-6398 Vol. 7, Issue 8: 95-101, DOI: 10.25236/FER.2024.070816

that of higher-order mathematical thinking, and the distribution of scores of fill-in-the-blank and solution questions is higher than that of higher-order mathematical thinking; in Paper II, the distribution of scores of single-choice questions is higher than that of higher-order mathematical thinking, the distribution of scores of multiple-choice and solution questions is higher than that of higher-order mathematical thinking, and the distribution of scores of multiple-choice and solution questions is higher than that of higher-order mathematical thinking. Distribution of scores was higher than the distribution of scores for mathematical higher-order thinking. The distributions of scores for lower-order thinking and fill-in-the-blanks questions were comparable to the distributions of scores for higher-order thinking.

According to the analysis of the above question papers in general, we can get: the new standard I volume and II volume of both the investigation of mathematical higher-order thinking and the assessment of mathematical lower-order thinking; lower-order thinking involves the investigation of the memory and understanding level, the higher-order thinking is mainly the investigation of critical thinking; II volume of both the amount of questions and the score is higher than the I volume, which means that the II volume of the investigation of higher-order thinking than I volume of the strength of the investigation; the investigation of both sets of volumes in creative thinking, whether the number of questions or score is rarely involved in real-life problem-solving ability thinking is also relatively low. The two sets of papers are less involved in the investigation of creative thinking, both in terms of the number of questions and marks, and the investigation of problem-solving ability in real life is also relatively low.

4. Recommendations for the Proposition of the Senior Examination Paper

Based on the analysis of the statistics and results of the NSS papers, to examine students' higher-order thinking skills in mathematics in depth, the following suggestions are made concerning the design of the question papers:

4.1 Optimising the Examination of Cognitive Levels to Develop Higher Order Thinking in Mathematics

In order to qualitatively examine students' thinking ability, the overall design of the GCE examination paper should be qualitatively tilted towards higher-level mathematical thinking. Higher-level cognition is not only an important training goal, but also a profound reflection of lower-level cognition, which should account for a slightly larger proportion of content than lower-level cognition, both in teaching and in examinations. Therefore, the test paper should focus on the qualitative examination of high-level thinking, and the test questions should not be designed in such a way that the memorization and comprehension of the topics should appear several times. The difficulty of the test questions should not be too low or too difficult, and the questions should be designed so that there is no repetition of omissions, and all the questions in the test paper can be organically combined for the design of high-level questions.

4.2 Innovative Problem Design to Develop Creative Thinking in Mathematics

Creative thinking is generally high-level thinking that can only be shown in solving unknown problems, which requires learners to break through thinking stereotypes and explore new ways of solving problems. In the gaokao proposition should be designed to students unknown areas of innovative mathematical problems, unknown areas can be interdisciplinary areas of real problems, requiring students to use what they have learned to design a solution to the problem, to reduce the difficulty can be given to the open question-to-answer, one of the answers can be full marks; can also be unknown mathematical theorems or formulas, guiding the students in the knowledge they have already learned to prove the reasoning of a new problem-solving method. For example, when encountering the volume of irregular three-dimensional geometry in high school, students often think of cutting the irregular polyhedral into common cones, triangles, tetrahedrons, etc., and then use the general volume formula to solve the problem. To help students develop creative thinking, a math question can be designed with the Newton-Simpson formula as the background. In the first question students are guided to prove the calculation of the new volume formula, and then after creative thinking and understanding in the second question they can be guided to use the conclusion of the first question to calculate the volume of a polyhedron that meets the conditions.

4.3 Multi-scenario Design to Develop Mathematical Problem-Solving Skills

The gaokao proposition is not only to examine students' understanding and memorization of

ISSN 2522-6398 Vol. 7, Issue 8: 95-101, DOI: 10.25236/FER.2024.070816

knowledge points but also to pay more attention to examining students' ability to apply what they have learned to solve practical problems. On the one hand, the design of test paper scenarios can be based on mathematical culture, which is a scenario material that can't be ignored both in the presentation of test questions and the introduction of teaching. Mathematical culture as the background of the design of mathematical problems, can effectively enhance students' understanding of the developmental nature of mathematics, the breadth of mathematical knowledge comes from different periods of scholars of mathematical problems based on the continued exploration of the predecessor, mathematical culture for the scenario of the proposition, can effectively expand the mathematical thinking.

On the other hand, the propositions can be based on practical application scenarios in which the problem-solving skills of mathematics will be greatly cultivated. Mathematics is an instrumental subject, which has a great impetus in physics, chemistry, computers, and other fields of science. Interdisciplinary scenarios to set propositions can help students apply their mathematical knowledge and skills to real life and other fields of study and can promote the deepening of mathematical thinking. For example, in the field of physics, by designing mathematical problems related to mechanics and electromagnetism, students can be helped to understand the importance of mathematics in explaining natural phenomena and laws and to enhance their awareness of mathematical applications. In addition, mathematical questions related to daily life and social practice can be designed in conjunction with problematic situations in real life.

5. Conclusion

This study provides an in-depth analysis of the 2023 New Curriculum Mathematics paper with a view to assessing and exploring the role of these papers in examining and developing students' higher-order thinking skills. The study found that although the two sets of papers covered all levels of thinking skills, there were significant deficiencies in assessing creative thinking skills, suggesting that the current test design needs to be further optimized. Based on Bloom's goal classification theory, this study puts forward three targeted suggestions, which not only provide a reference for the optimization of college entrance examination papers, but also provide guidance for the practice of high school mathematics teaching, which is helpful to improve the depth and breadth of students' mathematical thinking, and then cultivate their ability to solve complex problems and make wise decisions. Through the analysis and recommendations of this study, we hope that the future Gaokao mathematics papers can more effectively assess students' higher-order thinking ability and promote the development of students' deep thinking and innovation ability in mathematics learning. This not only helps to improve students' mathematical literacy, but also lays the foundation for their future success in a rapidly changing society.

References

[1] Zhang Xiaoping. Cultivation of higher-order thinking in high school mathematics teaching should focus on the "three forces"[J]. Mathematics World (Upper), 2023(07):3-5.
[2] Fu Wandi, Chen Yong. A comparative study of Chinese and American college entrance examination papers in mathematics based on SOLO theory[J]. Research on Science Examination, 2023,30(05):2-8.
[3] JIN YANPING, PAN XIN, CHEN BIFEN. Comparative Analysis of Comprehensive Difficulty of Mathematics Higher Examination Questions in 2023[J]. Secondary School Teaching and Research (Mathematics), 2023,(11):39-44.
[4] HE Xue, ZHOU Liren,HUANG Xiaomei. Analysis of the consistency between the national college entrance examination papers in mathematics and the curriculum standard in the past three years--Taking the new college entrance examination I,II papers in 2021-2023 as an example[J]. Research on Science Examination, 2024, 31(03):7-12.
[5] LIU Zhifeng, LI Ke, ZHANG Yuhuan. Comparative analysis of 14 sets of gaokao mathematics examination papers in 2020 and 2021 under the perspective of core literacy[J]. Secondary Mathematics Teaching Reference, 2022,(13):62-66.
[6] Shan Changsong. Strategies for Cultivating High School Students' Mathematical Arithmetic Ability under Core Literacy[J]. Secondary school curriculum counselling,2024(03):102-104.
[7] LIU Zaiping, LIU Zuoxi, LU Xinbing. Examining Mathematical Thinking and Guiding Effective Teaching--An Evaluation of the National B Paper of Mathematics of the 2023 College Entrance Examination and Implications[J]. Research on Classroom Teaching in Primary and Secondary Schools, 2023, (12):64-68.
[8] Zhou Ying, Lin Yi. Structural modelling and questionnaire development of higher-order thinking in

ISSN 2522-6398 Vol. 7, Issue 8: 95-101, DOI: 10.25236/FER.2024.070816

mathematics for junior high school students[J]. Mathematics Bulletin, 2021, 60(02):16-24.
[9] Ning Yimin. The relationship between deep learning and higher-order thinking in mathematics: a questionnaire development and survey study [D]. Guangxi: Guangxi Normal University, 2023, 000754.
[10] Zhang Xianlei, Zhao Rumu. Analysis of textbook exercises based on Bloom's classification of educational objectives--Taking high school physics compulsory one as an example[J]. Hunan Secondary School Physics, 2021, 36(06):91-94.
[11] Ma, Shuobing. A study on the refinement of the assessment framework of core literacy in mathematics and its application[J]. New Curriculum Introduction,2023,(07):20-23.