

# A Study of Ecosystems Based on the Sex Transition of Lampreys

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**Abstract:** In this paper, we focus on the effect of the sex ratio of lampreys on the ecosystem and establish a population dynamics ecosystem model. The problem is solved using the following models: Model I - Logistic growth model for population, Model II - Lotka-Volterra model considering gender, Model III - population symbiosis model, and Model IV - population model with degenerate stage structure. For Problem 1, on the basis of Model I, this paper first establishes the Lotka-Volterra model with Model II, excluding the denaturation stage. Considering that the sex ratio of lampreys is influenced by the external environment and the growth rate of the larval stage, this paper includes information about lampreys being capable of undergoing sex conversion based on Model II. Additionally, it constructs a population model IV with a denaturation stage structure. Next, the results of Model II and Model IV are compared. It is concluded that in situations with limited resources, sex conversion can help maintain a stable population. Problem 2 is actually an extension of problem 1. According to the findings of problem 1, this paper concludes that the benefit of sex ratio conversion is its ability to quickly stabilize the population size. Comparing the sex ratio data before and after the 500-year gender transition, it can be found that the sex ratio transition will lead to serious imbalance, and the sex ratio imbalance may lead to the extinction of species. For Problem 3, which analyzes stability in the larger ecosystem of Problem 1, we compared the phase diagrams before and after the sex switch. We found that the number of lamprey baits stabilized after the sex switch was performed. It is concluded that the conversion of sex ratios is beneficial to the stability of the ecosystem. For problem 4, this paper first constructs Model I, which is a logistic growth model for the population.

**Keywords:** Lamprey; Sex ratio conversion; Population Dynamics Model; Stability of Ecosystem

## 1. Introduction

### 1.1 Problem Background

The sex ratio of animals in the animal kingdom often influences the population size of their species. Most animals are divided into male and female, particularly to adapt to changes in sex ratios that depend on external environmental factors [1]. The consortium aims to gain a better understanding of how changing sex ratios affect sea lampreys. Firstly, since sea lampreys are predators, fluctuations in their sex ratio can impact their predatory behavior and their position in the food chain. Changes in the sex ratio of sea lampreys can impact their reproductive rates and, consequently, the stability of the ecosystem. Finally, changes in the population and sex ratios of sea lampreys may impact other coexisting organisms, including parasites (Figure 1).



Figure 1: Sea lamprey

Sea lamprey: The distribution is concentrated in freshwater and Marine ecosystems around the North Atlantic Ocean [2].

### 1.2 Restatement of the Problem

Given the background information and constraints identified in the problem statement, we need to address the following problems:

In this study, a mathematical model was established to analyze the potential effects of the sex ratio of lampreys on the ecosystem.

This study takes into account changes in the sex ratio of lampreys and analyzes the advantages and disadvantages of lampreys in the ecosystem as shown in the model.

Based on the prediction results of the model, the specific impact mechanism of lampreys on the ecosystem was discussed in this study.

This study uses the constructed model to analyze the potential benefits to other organisms in the ecosystem, such as parasites, when the sex ratio of lamprey populations changes.

### 1.3 Our Work

Our study focuses on examining the dynamics of sex ratios in lampreys and, consequently, the potential implications for the larger ecosystem. We will conduct a comprehensive analysis of the advantages and disadvantages of this sex ratio change for lamprey populations. Additionally, we will investigate the potential effects of this change on ecosystem stability and its ecological implications for other organisms. Our work mainly includes the following:

We construct four models: Model I - Logistic growth model for population, Model II - Lotka-Volterra model considering gender, Model III - population symbiosis model, and Model IV - population model with degenerate stage structure.

Firstly, a model was constructed to compare the two scenarios, one with a denaturation stage and one without, in order to analyze the relationship between sex ratio conversion and population size.

Secondly, we construct models using lamprey population density, birthrate, death rate, capacity, and maximum environmental capacity. We then summarize the advantages and disadvantages of the ecosystem based on the simulation results.

Furthermore, we analyzed the effect of changes in the sex ratio of populations on ecosystem stability.

Finally, we developed a population symbiosis model, illustrating how alterations in sex ratios can impact other organisms.

In summary, the entire modeling process is illustrated in the following figure 2.

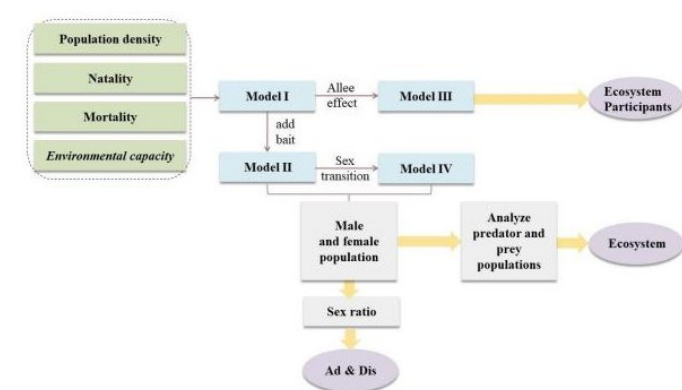


Figure 2: Our Work

## 2. Assumptions and Justifications

To simplify the problem, we make the following basic assumptions, each of which is properly justified.

Assumption 1: Changes in the sex ratio are predictable

Justification: Although the sex ratio conversion of sea lampreys does not necessarily adhere to the law of sex ratio change, the behavior of the group, according to the law of large numbers, will eliminate the presence of unpredictable chance factors. Therefore, by considering the population effect, we can more accurately simulate the sex conversion of sea lampreys.

Assumption 2: There is no need to consider the Allee effect.

Justification: The sea lamprey population is sufficiently large, allowing individuals to mate without restriction based on reproductive needs, and without apparent reproductive competition. Hence, the model can disregard the Allee effect, which is the phenomenon where population density below a certain threshold leads to a decrease in reproductive ability.

Assumption 3: Predator populations have the same predatory ability.

Justification: In the Lotka-Volterra model incorporating sex, we assume that the predatory abilities of male and female sea lampreys are equal. There were no significant physiological or behavioral differences in predation between female and male sea lampreys. If predation affects both sexes similarly, simplifying the model can reduce complexity and facilitate analysis.

Assumption 4: Consistency of action coefficients in the symbiosis model.

Justification: Since obtaining specific information about the inhibitory effect of each individual may be difficult, using a consistent effect coefficient makes the model easier to analyze. This is because it assumes that the inhibitory effect of each individual on others has the same coefficient, simplifying the mathematical expression and solution process and improving the model's operability.

### 3. Notations

The key mathematical notations used in this paper are listed in Table 1.

*Table 1: Notations used in this paper*

Symbol	Description
S	Sex ratio.
b	Birth ratio
m	Mortality
K	Carrying capacity
P	Population density
c	Hunting ability
r	Nature rate of growth

### 4. Population Dynamics Model

Since the population and the environment together form the ecosystem, the analysis of how thesea lamprey affects the ecosystem should start by examining the population, tribe, and the environment. Firstly, the food chain of lampreys is depicted in Figure 3, illustrating their roles as predators, competitors, and parasite in the ecosystem. This includes predation by other animals, competition with aquatic life, and obtaining nutrients by parasitizing the bodies of fish. A complex food chain forms, revealing the interrelationships between different organisms.

When analyzing Figure 4 within abroad ecosystem, the larger ecological environment reflects the diversity of the population and communities. Therefore, we need to consider that there is a relatively complete food chain in thesea lamprey population, indicating a hierarchical relationship between different species. The sex ratio of an animal has an impact on the population's growth rate, which in turn affects the entire ecosystem, including the food chain and the environment.

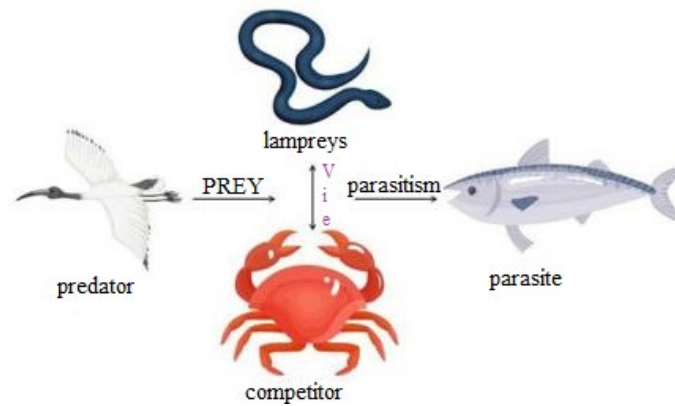


Figure 3: Food chain of lampreys

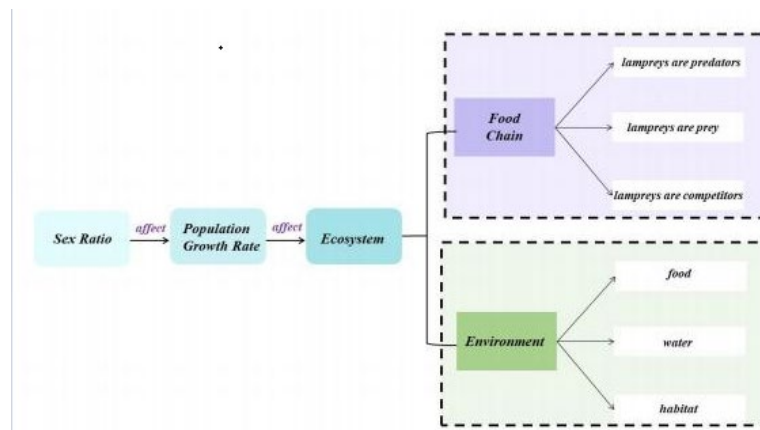


Figure 4: An entire ecosystem

#### 4.1 Logistic Model of Population Growth

When the population size of sea lampreys is sufficient, there is no competition forming among sea lampreys, so the Allee effect does not need to be considered in the mathematical model. We can obtain the equation as below:

$$\begin{cases} \frac{dN_m}{dt} = b_m N_f \left(1 - \frac{N_m + N_f}{K_L}\right) - m_m N_m \\ \frac{dN_f}{dt} = b_f N_f \left(1 - \frac{N_m + N_f}{K_L}\right) - m_f N_f \end{cases} \quad (1)$$

Where  $N_m$  represents the density of male sea lamprey;  $N_f$  is the density of female sea lamprey;  $b$  is the birthrate of the prey population;  $b_m$  is the birthrate of male sea lamprey;  $b_f$  is the birthrate of female sea lamprey;  $m_m$  represents the mortality of male sea lamprey;  $m_f$  represents female sea lamprey mortality;  $K_L$  represents the environmental capacity of sea lamprey.

#### 4.2 Lotka-Volterra Model Considering Sex

$$\begin{aligned} \left( \frac{dP}{dt} = rP \left(1 - \frac{P}{K_P}\right) - (c_m N_m + c_f N_f)P \right. \\ \left. \frac{dN_m}{dt} = b_m N_m \left(1 - \frac{N_m + N_f}{K_L}\right) - m_m N_m + k c_m P N_m \right. \\ \left. \frac{dN_f}{dt} = b_f N_f \left(1 - \frac{N_m + N_f}{K_L}\right) - m_f N_f + k c_f P N_f \right. \end{aligned} \quad (2)$$

Where  $P$  represents the density of prey population;  $r$  is the natural growth rate of prey population;  $c_m$  is the predatory ability of male sea lamprey;  $c_f$  is the predatory ability of female sea lamprey;  $k$

represents the conversion coefficient of predator population to predator population  $K_p$  represents the environmental capacity of prey population.

Assuming that the sex ratio of thesea lamprey population at time  $t$  is  $S(t)$ , then we can get that:

$$S'(t) = \frac{N'_m(t)N_f(t) - N_m(t)N'_f(t)}{N_f^2(t)} \quad (3)$$

When the predatory ability of the male sea lamprey is consistent with that of the female sea lamprey, in other words,  $c_m = c_f = c$ , we can obtain that  $S(t)$  satisfy the equation:

$$S'(t) = \left(1 - \frac{N_m + N_f}{K_L}\right) (b_m - b_f S(t)) + (m_f - m_m) S(t) \quad (4)$$

Therefore, when both sexes of the predator population have the same predatory ability, the density of the prey population does not affect the sex ratio of the predator population.

When the predatory ability of the male sea lamprey is not consistent with that of the female sea lamprey, that is to say,  $c_1, c_2$ , then  $S(t)$  should satisfy the following equation:

The sex ratio variation in the sea lamprey population depends not only on the population itself, but also on the prey population and the differences in predation ability between the two sexes of the sea lamprey population.

#### 4.3 Species Coexistence Model

For any two individuals, their effect on inhibiting population growth is equal. Referring to Logistic equation, we can get the following model:

Where  $r_x$  is the natural growth rate of the  $x$  population;  $X$  is the density of the  $x$  population;  $K_x$  represents the environmental capacity of the  $x$  population;  $\beta_1$  is the coefficients of the effect of sea lamprey population on the  $x$  population;  $\beta_2$  represents the coefficients of the effect of the  $x$  population on sea lamprey population.

#### 4.4 Population Model with a Degenerate Stage Structure

Considering that the sex ratio of sea lampreys can vary depending on the external environment, the determination of sea lampreys' sex as male or female relies on their growth rate during the larval stage, and the growth rate of these larvae is affected by the food supply, we introduced the juvenile population density of sea lamprey based on (2), then we can get the following equation:

Where  $N_b$  is the juvenile population density of sea lamprey;  $m_b$  represents the mortality of juvenile sea lamprey;  $k_2$  is the conversion coefficient of the prey population into the predator population;  $a$  is the conversion coefficient of juvenile sea lamprey population to male adult population;  $\beta$  represents the conversion coefficient of juvenile sea lamprey population to female adult population.

The parameters  $a$  and  $\beta$  are related to the density of the prey population  $x$ . When food resources are abundant, the juvenile sea lamprey population is more likely to transition into a male adult population, with parameter  $a$  being larger than parameter  $\beta$ . On the contrary, when food resources are scarce, the parameter  $a$  is smaller than the parameter  $\beta$ . Based on this analysis, the paper establishes the relationship between the parameters  $a$  and  $\beta$  and the density  $x$  of the prey population, yielding the following equation:

### 5. Result

#### 5.1 The Impact of Changing Sex Ratio on Ecosystems

In order to better compare with the population with the structure of denaturation stage, this paper simulated and compared the models in sections 4.2 and 4.4 to study and analyze the impact of the changing sex ratio of sea lamprey in section 5.1 on the ecosystem.

The simulation indices was  $b = 0.6; d = 0.1; k = 0.2; c_1 = 0.01; c_2 = 0.02; a_1 = 0.2; a_2 = 0.2; m_1 = 0.2; m_2 = 0.4; K_1 = 800; K_2 = 1000; x(0) = 100; y_1(0) = 30; y_2(0) = 20; y_3(0) = 3/2$ .

It is evident that in the simulation process, when the initial prey density is relatively low, the population without sex conversion results in a lower prey population and a decrease in the number of predators during the subsequent survival process, both of which exhibit some fluctuations. In contrast, sex-switching sea lamprey populations were able to stabilize their reproduction more rapidly in the face of resource scarcity.

The data showed that the scarcity of initial prey density did not have a positive effect on the non-sex-converted population. Instead, the trend of decreasing prey and predator numbers indicates a more fragile adaptation to the environment. However, the sea lamprey population that can switch sexes appears to be more adaptive, able to adjust and cope with resource scarcity more quickly. As a result, the population can reach a relatively stable reproductive state in a short period of time.

This observation suggests that gender switching may confer certain short-term survival advantages in resource-limited settings, which may be linked to the ability of the gender switching mechanism to utilize limited resources more efficiently. Therefore, individuals with the ability to switch sex may be more likely to ensure the survival and reproduction of their populations during periods of scarce resources.

### 5.2 Strengths and Weakness of Lamprey Population

From the results of 5.1, it can be seen that lampreys can sustain a large population.

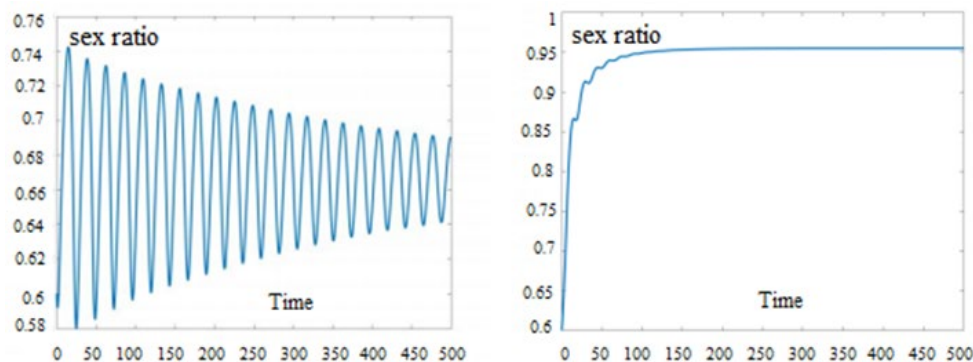


Figure 5: Phase diagram without(left)/with (right)gender transition

As you can see from Figure 5(left):

At the beginning, the sex ratio of lampreys without sex conversion fluctuated widely, ranging from about 0.15. The sex ratio then fluctuates less and less as the years goes on. This is because the population is gradually stabilizing. When the population has lived for 500 years, the fluctuation of the sex ratio of the population decreases to 0.04. Finally, the sex ratio of lampreys reached 0.69, indicating that there was not much difference in the number of males and females.

As can be seen from Figure 5(right):

During the first 50 years, the sex ratio of the transitioned lampreys increased rapidly until it stabilized at about 0.95 in the 100th year. This suggests that switching the sex ratio would result in a serious imbalance between male and female lampreys. In contrast to Figure 5(left), the sex ratio of lampreys undergoing sex switching has little fluctuation, indicating that the sex ratio switching is conducive to the stability of the population

From the observation of Figures 5, In the long term, an unbalanced sex ratio could put species at risk of accelerated extinction. In resource-limited environments, lampreys exhibit greater flexibility in coping with resource scarcity through sex switching, and as a result, are more likely to experience an unbalanced sex ratio. Due to its limited survival and reproductive strategy, such a sex ratio imbalance can have several negative effects on the species. It may lead to a decrease in reproductive opportunities, thereby limiting the potential for population growth.

From an ecological perspective, an imbalanced sex ratio could jeopardize the population's long-term survival. If the sex ratio is maintained in an extreme state for a prolonged period, it may lead to the loss of genes, a reduction in genetic diversity, and ultimately accelerate the extinction process of species. Thus, maintaining a moderate sex ratio is essential for the survival and reproduction of species, particularly in environments facing stresses such as resource scarcity.

### 5.3 Effects of Changes in Population Sex Ratio of Lamprey on Ecosystem Stability

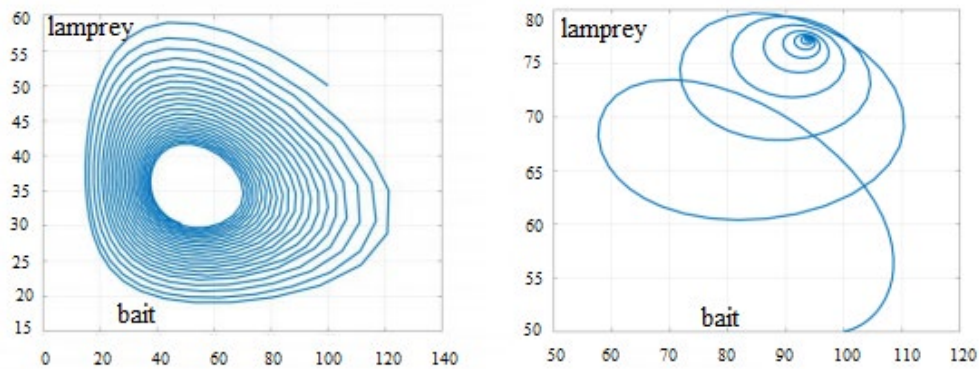


Figure 6: Phase diagram without/with

As can be seen from Figure 6 (left) :

Before the sex transition, the number of bait and the number of lampreys population fluctuated, and there was no regularity between the number of bait and the number of lampreys population.

As can be seen from Figure 6 (right) :

After sex switching, the number of bait and lampreys stabilized at 100 and 75, respectively, indicating that sex switching can maintain population stability.

This indicates that sex ratio switching contributes to maintaining the stability of lamprey and prey populations, thereby promoting ecosystem balance and stability. This stability is essential for maintaining ecological balance and the interrelationships among individual species.

### 5.4 Benefits for Other Ecosystem Participants

By manipulating the sex ratio of sea lampreys, it may be possible to prevent the chain reaction triggered by food scarcity, thereby slowing down or preventing ecosystem collapse. When the food supply is insufficient, the sea lamprey may adjust its sex ratio, which affects its reproductive strategy and makes the population more suitable for the current resource situation. Such adjustments may involve alterations in reproductive inputs, population densities, and relationships upstream and downstream of the food chain, leading to a state of equilibrium that prevents excessive resource pressures from causing catastrophic effects on the entire ecosystem.

Lampreys exhibit significant variations in sex ratios across different environments. The sex ratios of adult lampreys stocked as larvae in the backwater and stream environments were skewed towards males, with a male to female ratio of 3.8 to 1 in the backwater and 2.3 to 1 in the stream. The sex ratio of unlabeled adult lampreys captured from the same trap in the same year was 1.4 males to 1 female.

## 6. Sensitivity Analysis

In Section 4.2, two parameters are introduced to evaluate the parameters of the Lotka-Volterra model: male and female lampreys' predation capacity.

When the predation ability of male lampreys is the same as that of female lampreys, that is,  $c_m = c_f = c$ , then  $S(t)$  can satisfy the equation:

$$s'(t) = \left(1 - \frac{N_m + N_f}{K_L}\right) (b_m - b_f s(t)) + (m_f - m_m) s(t) \quad (5)$$

The reason why the sexes of predator populations are thought to have the same predatory ability is because there are no significant physiological or behavioral differences between female and male sea lampreys in terms of predation, in order to reduce the complexity of the model.

The results show that there are fluctuations in the parameters of the sex ratio of both sexes as the parameter decreases by 5% per step. However, the fluctuation of the parameters gradually decreases, and at 0.01 for  $c_1$  and  $c_2$ , it gradually stabilizes. This is reasonable and explicable. The trend of curve



change obtained by sensitivity test was consistent with the actual situation

## 7. Model Evaluation

### 7.1 Strengths

Our model has the following strengths:

- The main advantage is its great extensibility and the inclusion of all factors in a single, powerful framework. For example, by calculation and using the algorithm shown in 4.3, the population symbiosis model can be applied not only to this species, lamprey, but also to other species such as clownfish;
- The population model with degenerate stage structure is scientific and reasonable. In addition, we have creatively considered the introduction of larval stages that can be applied to different practical situations.
- We extensively studied animal sex ratio switching strategies, taking ecosystem stability into account, to ensure that our model closely reflects reality.
- Our model is based on sound biological principles.
- We did a good job of visualization, introducing schematics of lampreys, population simulations, lampreys sex ratios, Phase diagram without(left)/with (right)gender transition
- The effectiveness of the model under different parameters can be proved by sensitivity analysis. Therefore, the model can be applied to other organisms.

### 7.2 Weaknesses and possible improvements

We make some approximations in the process of solving the model, which results in small errors. Our model makes some assumptions, which may differ slightly from the actual situation.

## 8. Conclusions

Because lampreys population is large enough, the Allee effect is ignored in this paper. On the basis of model 1, the mutual inhibition among populations was studied, and the symbiosis model of model 3 was developed. Through the analysis of population simulation map and sex ratio map, it is concluded that the sex ratio of lampreys will cause environmental changes, and then affect other organisms. In addition, since the models used in the above four problems already incorporate the concepts of competition and symbiosis between populations, we can also consider the sex of lampreys and analyze the stages of denaturation. Our model is very adaptable. It can be used not only on lampreys, but also on other organisms.

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