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# MOFA/D-URAM for Solving the Air and Missile Defense Problem Based on Uncertainty Theory

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***Abstract:*** *For the uncertain factors in the fire allocation process of air and missile defense problem, the uncertainty theory is used to deal with the uncertain factors in the problem, and an uncertain multi-objective dynamic weapon target assignment model is proposed. In order to deal with the above model, a multi-objective evolutionary algorithm based on decomposition is proposed, which adds the displacement mechanism of firefly algorithm and uniformly randomly adaptive weights mechanism. Then, the simulation results show that the proposed algorithm has good convergence and distribution uniformity for solving multi-objective optimization problem. Lastly, using the algorithm to solve the above model, the results verify the rationality of the model.*

***Keywords:*** *Fire allocation; Adjustment mechanism; Multi-objective optimization; Uncertainty theory*

## 1. Introduction

Generally, the fire allocation problem in air and missile defense is a kind of weapon-target assignment problem (WTA), whose main purpose is to find an optimal assignment scheme of weapons and targets to minimize the probability of a missile destroying a protected asset under some constraints. The original study on WTA can stretch back to the last century, which mainly focus on the simple mathematical model and relevant algorithm. Previously, since limitations of computer technology, many scholars usually make some hypotheses to simplify the WTA problem and proposed some precise algorithms for solving small scale WTA, like Branch and Bound method and enumeration method [1]. In later studies, many scholars have improved the mathematical model of WTA to make it more close to real situation. There are two distinct categories of the WTA: the Static WTA (SWTA) and the Dynamic WTA (DWTA). The original SWTA is modeled by Manne[1], which defines a scenario that a known number of incoming targets are detected and a finite number of weapons, with known probabilities of destroying the targets, are available for a single exchange. In the SWTA, no subsequent engagements are considered since time is not a dimension considered in the problem. By contrast, the DWTA model takes the effect of time into account [2].The DWTA replicates the SWTA in its first stage, but includes a second stage where the number and the status information of targets and weapons are known only to a probability distribution, then the solution to the DWTA informs the defense on how to allocate the weapons in the first stage and how many to reserve for the second stage in order to minimize the probability of destruction. Thus, DWTA is a shoot-look-shoot process can be seen as a variant of SWTA. Meanwhile, with the scale of targets and weapons increases, the computational complexity of WTA grows exponentially as well, and it leads to a NP-hard problem [10] and the precise algorithm [3,4,5] is no longer applicative for solving WTA. Recently, with the advent of a variety of heuristic algorithms and intelligent algorithms, the above difficulties were solved, and many of them have been successfully applied to solving large-scale WTA problems, like ant colony algorithm[6], genetic algorithm[7], clonal selection algorithm[8], and particle swarm optimization algorithm[9].

Although the above WTA model is constantly improving, many uncertain factors are still ignored in the process of modeling construction, and they are still build based on deterministic environment. In the real battlefield, the situation is rapidly changing and full of many uncertainties, and many parameters cannot be obtained accurately. For example, the release of various interferences, sudden maneuvering flight of the targets and the change of attack intention, which can not be measured accurately by sensors and can not be ignored when the commander choose the fire allocation scheme. Some scholars used probability theory or fuzzy set theory to deal with these uncertain factors[11]. But in the real battlefield,

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there are usually not enough samples to properly estimate their probabilities distribution, and it requires the experts or commanders to estimate the belief degree that each event will happened based on their experience. Literature[12] point out that human beings tend to overestimate unlikely events, and persisting in using probability theory or fuzzy set theory to deal with the belief estimation may leads to false conclusions[18].The Uncertainty Theory was established by Liu in 2009[18], and it is a mathematical theory that satisfies the normality, monotonicity, self-duality, and countable subadditivity axioms which can deal with such uncertain problem. Liu point out that Uncertainty Theory is a mathematical system that parallel to probability theory, but the results of some problems derived from Uncertainty Theory is superior to that derived from probability theory, especially the problem involving subjective experience[13]. At present, this theory has been widely used in risk analysis, logistics, UAV path planning and other fields [14][15][16].

When solving the WTA problem, the commander usually takes into account not only the total damage efficiency of all targets, but also the operational cost and the total expected surviving value of protected assets and so on. Especially in the dynamic WTA model, only considering the maximization of the total damage efficiency may results in the waste of ammunition, which may lead to insufficient ammunition for subsequent incoming targets. Therefore, the WTA model of Air and Missile Defense usually contains more than one target function, and it leads to a Multi-objective optimization problem [17]. There is usually no unique optimal solution for multi-objective optimization problems, but a set of multiple equal non-dominant solutions which are usually called pareto optimal solution, and these provide the commander with a series of alternatives. Additionally, not all the pareto optimal solutions for multi-objective optimization problems are uniformly and continuously distributed in solution space, which brings difficulties to the design of multi-objective optimization algorithm [19]. In the past, some scholars proposed to decompose a multi-objective optimization process into multiple single-objective optimization processes, such as the weighted method and the Tchebycheff approach [20], which improved the distribution uniformity of pareto front to some extent.

Considering the uncertain factors in the battlefield of Air and Missile Defense, firstly the basic concepts of Uncertainty Theory are introduced in Section 2, and a DWTA model based on the uncertainty theory is proposed in Section 3, which treats the target threat degree and the probability of each asset being destroyed as uncertain variables. Then, the model's equivalent model is deduced by this theorem. In order to solve the model effectively, an algorithm with firefly algorithm displacement mechanism and uniformly randomly adaptive weights mechanism is proposed in Section 4, and it's performance in convergence and distribution uniformity is improved, which can be testify in the simulation results. Finally, the algorithm is applied to solving the above model to verify that the model is feasible.

## 2. Preliminaries

In this section, we introduce some foundational concepts and properties of uncertainty theory, which are used throughout this paper.

**Definition (1)** Let $\Gamma$ be a nonempty set， and $\mathcal{L}$ is a $\sigma$-algebra over $\Gamma$ ,Each element $\Lambda$ in $\mathcal{L}$ is called an event. A set function $\mathcal{M}$ from $\mathcal{L}$ to [0,1] is called an uncertain measure if it satisfies the following axioms.

**Axiom (1)** (Normality Axiom) $\mathcal{M}\{\Gamma\}=1$ for the universal set $\Gamma$ .

**Axiom (2)** (Duality Axiom) $\mathcal{M}\{\Lambda\}+\mathcal{M}\{\Lambda^c\}=1$ for any event $\Lambda \in \mathcal{L}$ .

**Axiom (3)** (Subadditivity Axiom)For every countable sequence of events $\Lambda_1, \Lambda_2, \ldots \in \mathcal{L}$ ,we have

$$\mathcal{M}\left\{\bigcup_{i=1}^{\infty}\Lambda_i\right\} \le \sum_{i=1}^{\infty}\mathcal{M}\{\Lambda_i\},$$

**Axiom (4)** (Product Axiom)Let $(\Gamma_i, \mathcal{L}_i, \mathcal{M}_i)$ be uncertainty space for $i=1,2,\ldots$ $i=1,2,\ldots$ .The product uncertain measure $\mathcal{M}$ is an uncertain measure satisfying

$$\mathcal{M}\left\{\prod_{i=1}^{\infty}\Lambda_i\right\} = \bigwedge_{i=1}^{\infty}\mathcal{M}\{\Lambda_i\}$$

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Additionally, Liu define an uncertain measure of product space $(\prod_{k=1}^{n} \Lambda_k, \prod_{k=1}^{n} \mathcal{L}_k)$ in [15].

**Definition (2)** An uncertain variable is a measurable function $\xi$ from an uncertainty space $(\Gamma, \mathcal{L}, \mathcal{M})$ to the set of real numbers, i.e., for any Borel set $B$ of real numbers, the set

$$\{\xi \in B\} = \{\gamma \in \Gamma \mid \xi(\gamma) \in B\} \in \mathcal{L}$$

**Definition (3)** The uncertain variables $\xi_1, \xi_1, ..., \xi_n$ are said to be independent if

$$\mathcal{M}\left\{\bigcap_{i=1}^{\mathrm{n}} (\xi_i \in B_i)\right\} = \bigwedge_{i=1}^{\mathrm{n}} \mathcal{M}\{(\xi_i \in B_i)\}$$

for any Borel sets $B_1, B_2, ..., B_n$ of real numbers.

**Theorem (1)** The uncertain variables $\xi_1, \xi_2, ..., \xi_n$ are said to be independent if and only if

$$\mathcal{M}\left\{\bigcup_{i=1}^{\mathrm{n}} (\xi_i \in B_i)\right\} = \bigvee_{i=1}^{\mathrm{n}} \mathcal{M}\{(\xi_i \in B_i)\}$$

for any Borel sets $B_1, B_2, ..., B_n$ of real numbers.

**Theorem (2)** Let $\xi_1, \xi_1, ..., \xi_n$ be uncertain variables, and $f$ is a real-valued measurable function. Then $f(\xi_1, \xi_1, ..., \xi_n)$ is an uncertain variable.

**Definition (4)** The uncertainty distribution $\Phi$ of an uncertain variable $\xi$ is defined by

$$\Phi(x) = \mathcal{M}\{\xi \le x\}$$

for any real number.

**Definition (5)** Let $\xi$ be an uncertain variable with regular uncertainty distribution $\Phi$. Then the inverse function $\Phi^{-1}$ is called the inverse uncertainty distribution of $\xi$.

**Definition (6)** Let $\xi$ be an uncertain variable with regular uncertainty distribution $\Phi$ .If $\Phi(x)$ is strictly increasing in the set $\{x \mid 0 < \Phi(x) < 1\}$ ,we call $\xi$ obey to regular distribution.

**Definition (7)** An uncertain variable $\xi$ is called zigzag if it has a zigzag uncertainty distribution

$$\Phi(x) = \begin{cases} 0 & \text{if} \quad \mathrm{x} < \mathrm{a} \\ (\mathrm{x}-\mathrm{a})/2(\mathrm{b}-\mathrm{a}) & \text{if} \quad \mathrm{a} \le \mathrm{x} \le \mathrm{b} \\ (\mathrm{x}+\mathrm{c}-2\mathrm{b})/2(\mathrm{c}-\mathrm{b}) & \text{if} \quad \mathrm{b} \le \mathrm{x} \le \mathrm{c} \\ 1 & \text{if} \quad \mathrm{x} > \mathrm{c} \end{cases}$$

denoted by $\xi \sim \mathrm{z}(\mathrm{a}, \mathrm{b}, \mathrm{c})$ where a, b, c are real numbers with a<b<c.

**Definition (8)** Let $\xi$ be an uncertain variable. Then the expected value of $\xi$ is defined by

$$\mathrm{E}[\xi] = \int_0^{+\infty} \mathcal{M}\{\xi \ge x\} \mathrm{d}x - \int_{-\infty}^{0} \mathcal{M}\{\xi < x\} \mathrm{d}x$$

provided that at least one of the two integrals is finite.

**Theorem (3)** Let $\xi$ be an uncertain variable with regular uncertainty distribution $\Phi$. If the expected value exists, then

$$\mathrm{E}[\xi] = \int_0^1 \Phi^{-1}(\alpha) \mathrm{d}\alpha$$

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**Theorem (4)** Let $\xi_1, \xi_2, ..., \xi_n$ be independent uncertain variables with regular uncertainty distributions $\Phi_1, \Phi_1, ... , \Phi_n$ respectively. If the function $f(x_1, x_2, ..., x_n)$ is a measurable function which is strictly increasing with respect to $x_1, x_2, ..., x_m$ and strictly decreasing with respect to $x_{m+1}, x_{m+2}, ..., x_n$ ,then $\xi = f(\xi_1, \xi_2, ..., \xi_n)$ is an uncertain variable with inverse uncertainty distribution

$$\Psi^{-1}(\alpha) = f(\Phi_1^{-1}(\alpha), ..., \Phi_m^{-1}(\alpha), \Phi_m^{-1}(1-\alpha), ..., \Phi_n^{-1}(1-\alpha))$$

**Remark (1)** See the details of the above definitions and theorems in reference[15].

The above content are some basic definitions and theorems of Uncertainty Theory, which is used in the following process of modeling. For detailed derivation and proof, please refer to the references [18].

## 3. DWTA Formulation

Considering the uncertain factors in the real battlefield, the target threat degree and the destroyed probability of each asset can not be obtained precisely, and we need experts to give the belief degree of relevant parameters. Thus, they can be treats as uncertainty variables, and a DWTA model based on uncertainty theory is propose.

### *3.1. Problem Describtion*

Firepower allocation in air and missile defense warfare play an important role in the cooperative combat. In this paper, a DWTA is adopted to discuss the problem, and the firepower assignment process is decomposed into multiple time stages, which is more conducive to considering some parameters that may change over time. Give the following scenario: In an air defense area, the radar detects a batch of incoming targets at a time, and the attacking intentions of each target are known (The attack asset of each target is known). The ground has a certain number of defensive weapons, and each weapon at each stage can only shoot one target (if a weapon can shoot multiple targets at each stage, it can be regarded as multiple weapons units). The commander has a maximum of $S$ stages in which weapons can hit the targets before these targets break through the defenses. The value of S is determined by the performance indexes of the targets and the weapons [19]. The parameters and variables in the DWTA model are listed in Table1.

*Table 1: Declaration*

| The meaning of variables: |
|---|
| $K(t)$ : the number of assets at stage $t$ ; $t = 1, 2, ..., S; K(1) = K$ |
| $T(t)$ : the number of targets at stage $t$ ; $t = 1, 2, ..., S; T(1) = T$ |
| $W(t)$ : the number of weapons at stage $t$ ; $t = 1, 2, ..., S; W(1) = W$ |
| $\boldsymbol{V} = [v_k]_{1\times K}$ : the vector of asset value ( $v_k$ denotes the value of asset $k$ ) |
| $\boldsymbol{W} = [w_j]_{1\times T}$ :the vector of threat value ( $w_j$ denotes the threat value of target $j$ ) |
| $\boldsymbol{Q} = [q_{jk}]_{T\times K}$ : the target lethality matrix( $q_{jk}$ denotes the probability that target $j$ destroys asset $k$ , $j = 1, 2, ...T$ , $k = 1, 2, ...K$ ) |
| $\boldsymbol{P}^t = [p_{ij}(t)]_{W\times T}$ :the kill-probability matrix( $p_{ij}(t)$ denotes the probability that weapon $i$ destroys target $j$ when assigned to it at stage $t$ , $i = 1, 2, ...W$ , $j = 1, 2, ...T$ , $t = 1, 2, ...S$ ) |
| $m_j$ :the maximal number of weapons that can be assigned to target $j$ at a stage, $j = 1, 2, ...T$ |
| $N_i$ : the maximal number of missiles that weapon $i$ can launch |
| $C_i$ : the value that each missile of weapon $i$ costs |
| $\boldsymbol{F}^t = [f_{ij}(t)]_{W\times T}$ : the engagement feasibility matrix at stage $t$ ( $f_{ij}(t) = 0$ if weapon $i$ cannot shoot target $j$ ,and $f_{ij}(t) = 1$ otherwise) |
| $x_{ij}(h)$ : the number of weapons $i$ that assigned to target $j$ at stage $h$ |

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### *3.2. Objective function*

In the given scenario above, we consider following multiple indicators as the objective function. The mathematical formula of them is as follows.

$$\max J_1(X^t) = \sum_{k=1}^{K} v_k \prod_{j=1}^{T(t)} \left[ 1 - q_{jk} \prod_{h=t}^{S} \prod_{i=1}^{W} (1 - p_{ij}(h))^{x_{ij}(h)} \right] \tag{1}$$

$$\max J_2(X^t) = \sum_{j=1}^{T} w_j \left[ 1 - \prod_{h=t}^{S} \prod_{i=1}^{W} (1 - p_{ij}(h))^{x_{ij}(h)} \right] \tag{2}$$

$$\min J_3(X^t) = \sum_{i=1}^{W} \left( C_i \sum_{j=1}^{T} x_{ij} \right) \tag{3}$$

where $J_1(\bullet)$ stands for the total expected surviving value of protected assets， $J_2(\bullet)$ stands for the expected effectiveness of destroying incoming targets， $J_3(\bullet)$ stands for the operation cost; $X^t = [X_t, X_{t+1}, ..., X_S]$ with $X_k = [x_{ij}(k)]_{W\times T}$ is the decision matrix at stage $k$ ( $x_{ij}(k)$ denotes the number of missiles that weapon $i$ allocates to target $j$ at stage $k$ ).

Notice that $X^t$ just represent the global decision matrix at stage $t$. When it gets to the $t+1$ stage, the status parameters of targets and weapons need to be updated. For example, if some targets were destroyed in stage $t$, then it is necessary to generate a new decision matrix $X^{t+1} = [X_{t+1}, X_{t+2}, ..., X_S]$ with updated targets state in stage $t+1$. If no state parameter changes in stage $t+1$, The distribution of weapons and targets is still carried out according to the $X^t$ decision matrix. In this way, commanders can adjust the assignment scheme according to the changes of actual battlefield conditions.

### *3.3. Constraint*

The DWTA needs to satisfy the following constraints.

$$\sum_{j=1}^{T} \delta(x_{ij}(t)) \le 1, \forall t \in \{1, 2, ..., S\}, \forall i \in \{1, 2, ..., W\} \tag{4}$$

$$\sum_{i=1}^{W} x_{ij}(t) \le m_j, \forall t \in \{1, 2, ..., S\}, \forall j \in \{1, 2, ..., T\} \tag{5}$$

$$\sum_{t=1}^{S} \sum_{j=1}^{T} x_{ij}(t) \le N_i, \forall i \in \{1, 2, ..., W\} \tag{6}$$

$$\delta(x_{ij}(t)) \le f_{ij}(t), \forall t \in \{1, 2, ..., S\},$$

$$\forall i \in \{1, 2, ..., W\}, \forall j \in \{1, 2, ..., T\} \tag{7}$$

where $\delta(\alpha) = \begin{cases} 1, & \alpha > 0 \\ 0, & \alpha \le 0 \end{cases}$.

Constraint (4) means that each weapon can only be assigned to one target at each stage. If a weapon can assign missiles to multiple targets at one stage, it can be regarded as multiple weapons units; constraint (5) limits the maximum number of missiles that weapons can launch to a target at each stage, and it is aimed to prevent too many missiles from being assigned to one target and resulting in resource shortages in subsequent stage; constraint (6) limits the maximum number of missiles that each weapon can assign, and it is determined by missile stock of the weapon; constraint (7) reflect the influence of time window on the engagement feasibility of weapons ( $f_{ij}(t) = 1$ denotes that is feasible; otherwise is infeasible), and if a weapon is to be assigned to a target, it must satisfy this constraint. Noted that, above

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constraints increase the complexity of solving DWTA and the make it harder to design an algorithm to generate a feasible solution.

### *3.4. Improved DWTA based on uncertainty theory*

In the above model, the threat value of target is determined by many factors of targets like the type, the flight speed, the track angle and the maneuverability, etc. Thus, the threat value of target $w_j\,(j=1,2,...,T)$ are treated as uncertain variables defined in the uncertain space $(\Gamma, \mathcal{L}, \mathcal{M})$. The uncertain distribution function of each uncertain variable is obtained by the experience of experts using uncertain statistical method [25]. It is denoted by

$$\xi_j \sim \Phi_j \quad (j=1,2...,T)$$

Meanwhile, the probability $q_{jk}$ are determined by targets performance which can not be obtained accurately, and they can be treated as uncertain variables, too. The uncertain distribution functions of them can be obtained by using the same method as above. It is denoted by

$$\eta_{jk} \sim \Psi_{jk} \quad (j=1,2...,T, k=1,2...,K)$$

Then, the above model in (1)(2)(3) can be rewritten as follows

$$\max J_1(X^t) = \sum_{k=1}^{K} v_k \prod_{j=1}^{T(t)} \left[ 1 - \eta_{jk} \prod_{h=t}^{S} \prod_{i=1}^{W} (1 - p_{ij}(h))^{x_{ij}(h)} \right] \tag{8}$$

$$\max J_2(X^t, \xi)) = \sum_{j=1}^{T} \xi_j \left[ 1 - \prod_{h=t}^{S} \prod_{i=1}^{W} (1 - p_{ij}(h))^{x_{ij}(h)} \right] \tag{9}$$

$$\min J_3(X^t) = \sum_{i=1}^{W} \left( C_i \sum_{j=1}^{T} \sum_{h=1}^{S} x_{ij}(h) \right) \tag{10}$$

In the above model, the performance of each target is independent of each other, so the uncertain variables $\xi_1, \xi_2, ..., \xi_T$ and $\eta_{11}, \eta_{12}, ..., \eta_{TK}$ are also independent of each other. Generally, the uncertain distribution of each uncertain variable obtained by uncertain statistical method are regular. In equation (8) and (9) , if the $X^t$ is a known value, $J_1$ and $J_2$ become the function of $\eta_{11}, \eta_{12}, ..., \eta_{TK}$ and the function of $\xi_1, \xi_2, ..., \xi_T$ ,respectively, denoted as $J_1(\eta_{11}, \eta_{12}, ..., \eta_{TK})$ and $J_2(w_1, w_2, ..., w_T)$. Furthermore, both $J_1(\bullet)$ and $J_2(\bullet)$ are measurable function, then $J_1$ and $J_2$ are also uncertain variables(Theorem 2.2) whose uncertain distributions are denoted as $\Upsilon_1$ and $\Upsilon_2$, respectively.

Different problems have different meanings of valuation and need appropriate principle to define the valuation principle. In equation (8) and (9), the objective functions $J_1$ and $J_2$ can be converted to determinate form by using the Expected-Value principle, denoted by ($E[J(\bullet)]$). Additionally, $J_1$ is a measurable function which is strictly decreasing with respect to $\eta_{11}, \eta_{12}, ..., \eta_{TK}$, and $J_2$ is a measurable function which is strictly increasing with respect to $\xi_1, \xi_2, ..., \xi_T$. Combined with the theorem2.3 and theorem2.4, it can be proved that the inverse function of uncertain distribution $\Upsilon_1^{-1}$ and $\Upsilon_2^{-1}$ satisfy

$$\Upsilon_1^{-1}(\alpha) = J_1(\Psi_{11}^{-1}(\alpha), \Psi_{12}^{-1}(\alpha), ..., \Psi_{TK}^{-1}(\alpha))$$

$$\Upsilon_2^{-1}(\alpha) = J_2(\Phi_1^{-1}(\alpha), \Phi_2^{-1}(\alpha), ..., \Phi_T^{-1}(\alpha))$$

where $\Phi_1^{-1}(\alpha), \Phi_2^{-1}(\alpha), ..., \Phi_T^{-1}(\alpha)$ and $\Psi_{11}^{-1}(\alpha), \Psi_{12}^{-1}(\alpha), \quad ..., \Psi_{TK}^{-1}(\alpha)$ are the inverse uncertainty distribution of $\xi_1, \xi_2, ..., \xi_T$ and $\eta_{11}, \eta_{12}, ..., \eta_{TK}$ respectively. Then the equation (8) (9) can be rewritten as follows

$$\max E[J_1(X^t,\boldsymbol{\xi})]=\int_0^1\sum_{k=1}^{K}v_k\prod_{j=1}^{T(t)}\left[1-\Psi_{jk}^{-1}(1-\alpha)\prod_{h=t}^{S}\prod_{i=1}^{W}(1-p_{ij}(h))^{x_{ij}(h)}\right]\mathrm{d}\alpha \tag{11}$$

$$\max E[J_2(X^t,\xi)]=\int_0^1\sum_{j=1}^{T}\Phi_j^{-1}(\alpha)\left[1-\prod_{h=t}^{S}\prod_{i=1}^{W}(1-p_{ij}(h))^{x_{ij}(h)}\right]\mathrm{d}\alpha \tag{12}$$

Finally, the DWTA based on uncertainty theory (UDWTA) can be summarized as equations (10)(11)(12) with the constraints (4)-(7). As it can be seen from the UDWTA, the three objective functions are positively correlated, that is, the larger the total expected surviving value or the expected effectiveness of destroying targets means more resource consumption. In other word, (10) and (11) are conflict with (12).

## 4. MOEA/D-URAM for solving UDWTA

Considering the uncertain, multi-objective and combinatorial nature in UDWTA, in order to improve the quality and spread of the solutions, an improved multi-objective evolutionary algorithm is proposed for multi-objective optimization problem in this section.

In this section, through adding the optimization displacement mechanism of firefly algorithm and uniformly randomly adjustment mechanism into the traditional multi-objective optimization evolutionary algorithm based on decomposition (MOEA/D), the convergence and distribution uniformity of the algorithm are improved effectively.

### *4.1. Tchebycheff Approach*

A multi-objective optimization problem with M-dimensional decision vectors and N objective functions is described as follows (minimization problem here)

$$\begin{aligned}\text{Minimize}\quad & \boldsymbol{y}=f(\boldsymbol{x})=(f_1(\boldsymbol{x}),...,f_n(\boldsymbol{x}))\\ & \boldsymbol{x}=(x_1,...,x_m)\in\boldsymbol{X}\end{aligned} \tag{13}$$

where $X$ is the range of feasible solutions. The set of corresponding target function vectors is denoted by $\mathbb{Z}$, $\mathbb{Z}=\{\boldsymbol{z}=f(\boldsymbol{x})\,|\,\boldsymbol{x}\in\boldsymbol{X}\}$ .A decision vector $\boldsymbol{a}\in\boldsymbol{X}$ is said to dominate another decision vector $\boldsymbol{b}\in\boldsymbol{X}$ (denoted as $\boldsymbol{a}<\boldsymbol{b}$) if and only if

$$\begin{aligned}&\forall i\in\{1,...,n\}: f_i(\boldsymbol{a})\le f_i(\boldsymbol{b})\\ \textit{and}\quad &\exists j\in\{1,...,n\}: f_j(\boldsymbol{a})<f_j(\boldsymbol{b})\end{aligned}$$

In order to find the pareto solution of the above model effectively, we first decompose a multi-objective optimization problem into multiple single-objective optimization problems by using the Tchebycheff approach [20]. The multi-objective optimization model is replaced as follows

$$\min\quad g(x|\lambda,f^*)=\sqrt{\sum_{i=1}^{n}\lambda_i(f_i(x)-f_i^*)^2} \tag{14}$$

where $f^*=(f_1^*,f_2^*,...f_n^*)$ is the optimal value vector of the objective functions, and $\lambda_i$ denotes the weight value of the *i*-th objective function which satisfies $\sum_{i=1}^{n}\lambda_i=1$.

**Theorem 4.1** The optimal solution of model (14) must be a pareto-optimal solution of model (13).

The prove of Theorem 4.1 is in reference[18] requires that the optimal value $f_j^*$ of each objective function exists; Additionally, the distance formula with ideal point in (14) can also be replaced by following and theorem 4.1 still holds.

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$$g(x|\lambda, f^*) = \left( \sum_{i=1}^{n} \lambda_i ( f_i(x) - f_i^*)^q \right)^{\frac{1}{q}}$$

$$g(x|\lambda, f^*) = \max_{0 \le i \le n} \left( \lambda_i \left| f_i(x) - f_i^* \right| \right)$$

If a multi-objective optimization problem in (13) is decomposed into multiple single-objective models in (14) with different weight vectors, the optimization algorithm can guide each individual to approach different pareto-optimal solution. Generally, the optimal solutions of subproblems with similar weight vectors are small apart in the solution space. This method could improve the distribution uniformity of output. In addition, compared with the method of adding the weights to the objective function directly, the Tchebycheff approach can better overcome the shortcoming of uneven distribution, which is caused by the non-convex set of true pareto-front[20].

### *4.2. Firely Algorithm*

Firefly algorithm is a new intelligent algorithm proposed by Yang of Cambridge University in 2008[29]. It mainly imitates the luminous behavior of fireflies in nature. By observing the brightness and attractiveness of other individuals in the neighborhood, firefly chooses its own direction of movement, so as to continuously update its position and achieve the ultimate goal optimization location. The relevant definitions of firefly algorithm are as follows.

Different from PSO algorithm, this algorithm has the advantages of local attraction and automatic recombination. This is because the light intensity in this algorithm decreases with the increase of distance. By adjusting the absorption coefficient of light intensity, the attraction between individuals can be local or global. Therefore, firefly algorithm is more suitable for multi-mode global optimization problem and in this paper we use the optimization mechanism of firefly algorithm to generate offspring.

### *4.3. Weight Vector Generator and Uniformly Randomly Adaptive Weight Mechanism*

When using the Tchebycheff approach to decompose the multi-objective optimization problem, most scholars use the constant weight vector. This leads to a problem that when solving complex multi-objective optimization problems (the true pareto front are discontinuous or they not uniformly distributed, etc.), the output does not perform well in the distribution uniformity [20]. A weight vector generation method and a uniformly randomly adjustment mechanism are introduced in this part, which is to improve the performance of the output.

(1) Weight vector generator

Here, N weight vectors are generated by uniform random method [21]. The specific process is as follows

1) Randomly generate 5,000 weight vectors to construct the set $\boldsymbol{\lambda}_1$, and initialize a vector set $\boldsymbol{\lambda}$ that contains all the dimensional unit weight vector include $(1 \quad 0 \quad ... \quad 0)$, $(0 \quad 1 \quad ... \quad 0)$, …, $(0 \quad 0 \quad ... \quad 1)$.

2) Find the weight vector $\boldsymbol{\lambda}_1^k$ in $\boldsymbol{\lambda}_1$ which has the longest distance with the vectors in $\boldsymbol{\lambda}$ v (Euclidean distance), and move $\boldsymbol{\lambda}_1^k$ to $\boldsymbol{\lambda}$.

3) If the number of vectors in $\boldsymbol{\lambda}$ reaches $N$ (population size), stop the operation; Otherwise repeat step 2.

Then, normalize the weight vector in $\boldsymbol{\lambda}$ to obtain the generated weight vector, and the normalized formula is as follows

$$\boldsymbol{\lambda}' = WS(\boldsymbol{\lambda}) = \left( \frac{\frac{1}{\lambda_1}}{\sum_{i=1}^{m} \frac{1}{\lambda_i}} \dots \frac{\frac{1}{\lambda_m}}{\sum_{i=1}^{m} \frac{1}{\lambda_i}} \right) \tag{15}$$

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(2) Uniformly randomly adjustment mechanism

The weight vector of the individual is adjusted based on the sparsity level of the individual. It is proposed to adjust once every 5% of the total number in each iteration, and it is not executed in the last 10% iterations. During each adjustment, the 5% individuals with the lowest sparsity level were removed from the current population. Then, choose the individual from external population (EP which stores the nondominant solutions founded so far) $\boldsymbol{x}^{sp}$ which has the lowest sparsity level to current population, and give it a new weight vector by following equation (17). Finally, add the new individual with new weight vector $\boldsymbol{\lambda}^{sp}$ to current population. Repeat the process until the number of individuals in the population reaches $N$. Individual sparsity level [21] is defined as follows

$$SL(ind^j, pop) = \prod_{i=1}^{m} L_2^{NN_i^j} \tag{16}$$

where $L_2^{NN_i^j}$ is the *j*-th individual Euclidean distance, $ind^j$ ,along with its *i*-th nearest neighbor of the population, $pop$.

The update formula of the weight vector $\boldsymbol{\lambda}^{sp}$ of the new individual $\boldsymbol{x}^{sp}$ is as follows

$$\boldsymbol{\lambda}^{sp} = \left(\frac{\frac{1}{f_1^{sp} - f_1^*}}{\sum_{i=1}^{m} \frac{1}{f_i^{sp} - f_i^*}} \cdots \frac{\frac{1}{f_m^{sp} - f_m^*}}{\sum_{i=1}^{m} \frac{1}{f_i^{sp} - f_i^*}}\right) \tag{17}$$

### *4.4. The Franmework of MOFA/D-URAM*

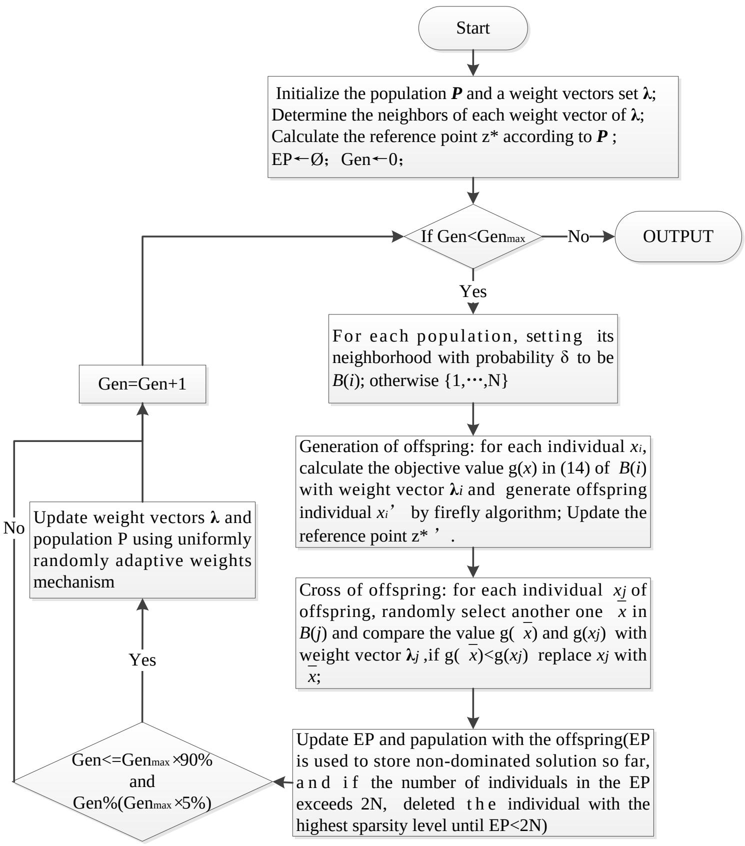


*Figure1: The framework of MOFA/D-URAMFirstly.*

A multi-objective optimization problem is decomposed into multiple single-objective optimization problem by generating a weight vector for each individual. Then, the displacement mechanism of

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firefly algorithm is used to generate new progeny individual, and some individual in the parent population is replaced in a certain proportion according to the respective optimization problem( $\boldsymbol{B}(i)$ is the serial number of the i-th individual's neighbor, whose weight vectors is close to that of i-th individual). Then update the EP which stores the nondominant solutions founded so far(elitism). In addition, the weight vector and individual of the population are adjusted every 5% iteration period.

Through the above discussion, a multi-objective firefly algorithm based on decomposition is proposed(MOFA/D-URAM). This algorithm add the displacement mechanism of firefly algorithm, which has the advantage of the global and local optimization. When the population iterates, uniformly randomly adjusting the individual and weight vector of population could avoid overcrowded population and improve the distribution uniformity of output. Finally, the output uniformly converges to the true pareto front.

The framework of MOFA/D-URAM is shown in Figure.1

## 5. Simulation and Analysis

In this section, two simulation experiments are designed to verify the convergence and uniformity of solutions obtained by the algorithm and the feasibility of UDWTA in section 3.

### *5.1. Performance test*

Here we choose ZDT1~ZDT4 [18] as the test function, then set the variables number $d$ of test functions ZDT1~ZDT3 as 30 and that of the test function ZDT4 is 10. For ZDT1, the pareto-optimal solution is obtained when $g=1$,and the pareto front is represented as $f_2=1-\sqrt{f_1}$ (convex set); for ZDT2, the pareto-optimal solution is obtained when $g=1$ and the pareto front is represented as $f_2=(1-f_1)^2$ (non-convex set); for ZDT3, the pareto-optimal solution is obtained when $g=1$,but its pareto front is discontinuous. The range of $f_1$ is [0,0.852] and $f_2$ is [-0.773,1]; for ZDT4, its pareto-optimal solution and Pareto front are the same as ZDT1, but ZDT4 has many local pareto-optional sets. Here we choose NSGA-II [31] as the comparison algorithm, which is widely applied for solving multi-objective optimization problems.

Set the population number of the two algorithms to 100 and the number of iterations as 300. The EP capacity was set to twice the population size. For MOFA/D-URAW, take the dynamic step length coefficient $\alpha(t)=0.99^t\alpha_0$ with $\alpha_0=0.5$ (it is beneficial to global optimization in the early stage of population iteration and local convergence in the later stage). Each algorithm runs independently for 30 times, and randomly select one in 30 results. The comparison simulation results are shown in Figure.2.

In order to compare the performance of two algorithms, Inverse Generational Distance(IGD) and Spacing Metric (SP) [22] are selected as performance parameters to measure the convergence and uniform distribution of the algorithm respectively.

The definition of IGD is as follows

$$IGD(Q^{t*},Q^t)=\frac{\sum_{v\in Q^{t*}}d(v,Q^t)}{\left|Q^{t*}\right|} \tag{18}$$

where $Q^{t*}$ denotes true pareto front, $Q^t$ denotes the pareto front obtained by the algorithm; $\left|Q^{t*}\right|$ is the collection of $Q^{t*}$ , and here taking the number of elements in the set; $d(v,Q^t)=\min_{u\in Q^t}\left\|F(v)-F(u)\right\|$ denotes the minimum distance from $u\in Q^t$ to $v$.

The definition of SP is as follows

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$$SP = \sqrt{\frac{1}{n_p - 1}\sum_{k=1}^{n_p}(\bar{d} - d_k)^2} \tag{19}$$

$$d_k = \min_{r \neq k}\left(\sum_{\xi=1}^{m}\left| f_\xi(\boldsymbol{x}_k) - f_\xi(\boldsymbol{x}_r)\right|\right), \boldsymbol{x}_k \in Q^t (k = 1, 2..., n_p) \tag{20}$$

$$\bar{d} = \frac{1}{n_p}\sum_{k=1}^{n_p} d_k \tag{21}$$

According to the above definition, the smaller the IGD value is, the better the convergence of the algorithm is; the smaller the SP value is, the better the uniformity of pareto front distribution is. The comparison average of the results of the 30 runs is shown in Table 2 and Table 3.

*Table 2: IGD of two algorithms*

| **Algorithm** | **Test Function** | | | |
|---|---|---|---|---|
| | **ZDT1** | **ZDT2** | **ZDT3** | **ZDT4** |
| MOFA/D-URAM | 0.0083 | 0.0093 | 0.1942 | 1.0947 |
| NSGA-II | 0.0071 | 0.0351 | 0.2114 | 1.6870 |

*Table 3: SP of two algorithms*

| **Algorithm** | **Test Function** | | | |
|---|---|---|---|---|
| | **ZDT1** | **ZDT2** | **ZDT3** | **ZDT4** |
| MOFA/D-URAM | 0.0058 | 0.0081 | 0.0129 | 0.0083 |
| NSGA-II | 0.0109 | 0.0249 | 0.0294 | 0.3374 |

It can be seen from Table 2 and Table 3 and Figure.2 that, under the same population size and iteration times, IGD and SP of MOFA/ D-URAM are generally smaller than those of NSGA-II, which indicate that the algorithm can effectively converge to the true pareto front. Specifically, figure(b) and figure(c) show that the uniformity of pareto front obtained by MOFA/D-URAW is superior than that of NSGA-II; figure(d) shows that the MOFA/D-URAW has a better performance in jumping out of the local Pareto optimal solution and approaching to the global optimal solution. The above simulation results show that the multi-objective optimization algorithm based on decomposition and firefly algorithm displacement mechanism can better guide the population convergence to different Pareto optimal solutions, and the weight adaptive adjustment mechanism is added to avoid population overcrowding.

### *5.2. Solving UDWTA problem with MOFA/D-URAM*

(1) Coding for population

The above UDWTA problem is a discrete model with discrete variable, hence the variables in the problem need to be coded before using the algorithm. Here a matrix is used to encode the assignment scheme as follows

$$X = \begin{pmatrix} X_t \\ \vdots \\ X_S \end{pmatrix} = \begin{pmatrix} x_{11}^t & \cdots & x_{1T}^t \\ \vdots & \ddots & \vdots \\ x_{W1}^t & \cdots & x_{WT}^t \\ x_{11}^{t+1} & \cdots & x_{1T}^{t+1} \\ \vdots & \ddots & \vdots \\ x_{W1}^{S-1} & \cdots & x_{WT}^{S-1} \\ x_{11}^S & \cdots & x_{1T}^S \\ \vdots & \ddots & \vdots \\ x_{W1}^S & \cdots & x_{WT}^S \end{pmatrix}$$

where $x_{ij}^k$ has the same meaning with $x_{ij}(k)$ in section 3.

(2) Adjusting illegal individual

In the iteration of population, some individuals not meet the constraints (4)-(7) will be generated. Therefore, it is necessary to check the position of the individual and adjust the position of the illegal individual. For example, the adjustment mechanism for constraint (5) is as follows.

If the individual does not satisfy the constraint (5), which means

$$\exists t' \in \{1, 2, ..., S\} \quad and \quad \forall j' \in \{1, 2, ..., T\}, \sum_{i=1}^{W} x_{ij'}(t') > m_j$$

*Step1*. Pick a non-zero element $x_{kj'}(t)$ randomly from $x_{ij'}(t'), i \in \{1, 2, ..., W\}$, and do $x_{kj'}(t) = x_{kj'}(t) - 1$;

*Step2*. Check to see if it satisfies $\sum_{i=1}^{W} x_{ij'}(t') \le m_j$. If not satisfied, continue to repeat *Step1*; Otherwise, the adjustment ends.

The adjustment process for constraints (4), (6) and (7) is similar to the above mechanism and will not be repeated here. Finally, output the new individual which satisfies all constraints.

(3) The displacement mechanism of discrete variable

For the discreteness of the problem, the standard displacement mechanism of firefly algorithm needs to be improved. The displacement mechanism in reference[23] is adopted in this section, which is not repeated here.

In the above mechanism, although the decision variables in the UDWTA problem are discrete variables, it has the same properties as the continuous variable. Firstly, the values of each objective function corresponding to adjacent discrete variables are close. Further, the optimal solutions of subproblems with similar weight vectors still are small apart in the solution space. Above properties indicate that MOFA/ D-URAW is still suitable for solving UDWTA model.

(4) Scene description

In order to verify the feasibility of UDWTA, the following air and missile defense combat scenarios are presented. Suppose that at some point, 10 incoming targets $(j = 1, 2..., 10)$ are detected by radar, and here are 5 weapon units $(i = 1, 2..., 5)$ on the ground to protect 3 ground assets $(k = 1, 2, 3)$ by intercepting the targets; the interception process can be decomposed into four stages $(t = 1, 2, 3, 4)$; based on the detected targets information and the resource constraints of the ground weapon units, $m_j$ is all set to 3; $f_{ij}(t)$ is all set to 1. The remaining parameters are shown in Table 4~Table 6.

(5) The results and analysis

The algorithm runs 30 times at different population sizes. The average running time of the algorithm is shown in Table7. It can be seen from Table 6 that the real-time performance of the algorithm satisfies the requirement.

The algorithm outputs a set of pareto-optimal solutions which corresponds to the allocation scheme in Figure.3. Generally, the survival efficiency of assets damage probability of targets increases as the combat cost increases. Here list the solutions with maximal survival efficiency of assets found by the algorithm, and the Table 8 shows which targets each weapon was assigned to at each stage and the number of missiles assigned.

The output of MOFA/D-URAW provides the commander with a set of alternative assignment schemes, and each scheme is relatively optimal. The commander can choose one according to different preferences. Compared with the single objective optimization algorithm which can only output a single optimal solution, the multi-objective optimization algorithm is more suitable to the actual requirement in the changing battlefield.

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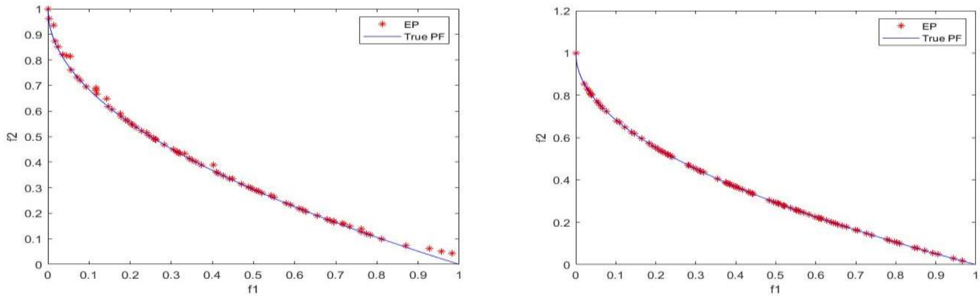


Figure (a) ZTD1

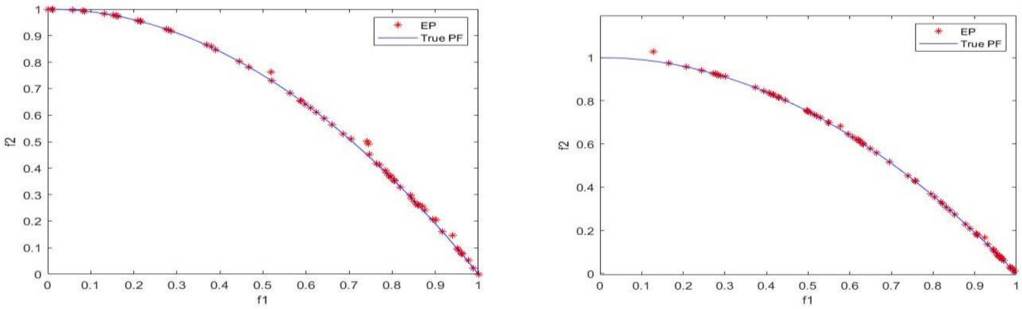


Figure (b) ZTD2

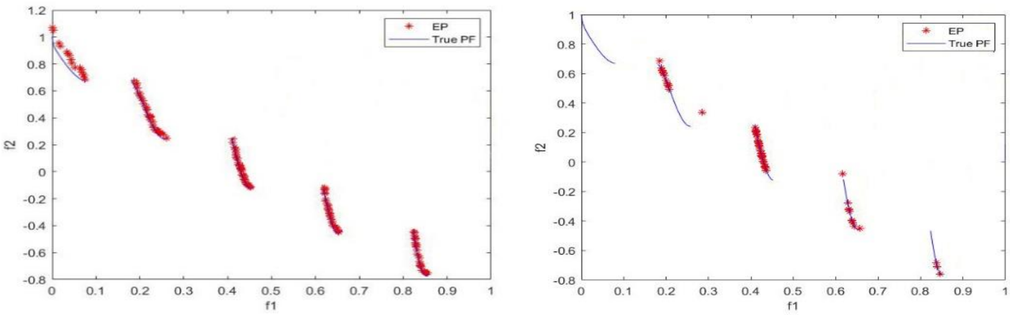


Figure (c) ZTD3

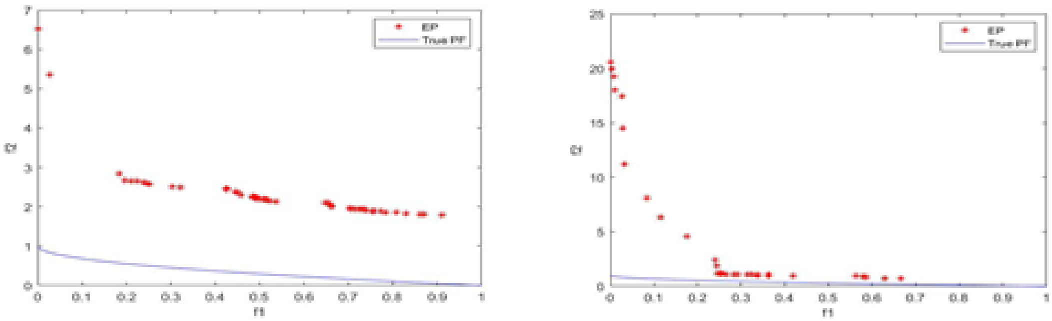


Figure (d) ZTD4

*Figure 2: Pareto front of ZDT1 ~ZDT4 obtained by two algorithms. (MOFA/D-URAM on the left, NSGA-II on the right)*

*Table 4: Average running time*

| The scale of population | Iterations times | | |
|---|---|---|---|
| | 50 | 100 | 200 |
| 50 | 3.3652 | 5.5396 | 10.0772 |

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*Table 5: The damage probability and uncertain distribution of threat value of targets*

| $p_{ij}$ | $T_1$ | $T_2$ | $T_3$ | $T_4$ | $T_5$ | $T_6$ | $T_7$ | $T_8$ | $T_9$ | $T_{10}$ |
|---|---|---|---|---|---|---|---|---|---|---|
| $W_1$ | 0.45 | 0.32 | 0.54 | 0.47 | 0.75 | 0.10 | 0.17 | 0.26 | 0.68 | 0.59 |
| $W_2$ | 0.38 | 0.55 | 0.18 | 0.66 | 0.52 | 0.41 | 0.35 | 0.68 | 0.13 | 0.64 |
| $W_3$ | 0.85 | 0.76 | 0.69 | 0.42 | 0.33 | 0.58 | 0.74 | 0.70 | 0.35 | 0.15 |
| $W_4$ | 0.28 | 0.62 | 0.58 | 0.56 | 0.57 | 0.13 | 0.55 | 0.85 | 0.70 | 0.63 |
| $W_5$ | 0.50 | 0.27 | 0.73 | 0.78 | 0.27 | 0.86 | 0.34 | 0.27 | 0.88 | 0.65 |
| $w$ | $z(0.41,0.43,0.45)$ | $z(0.48,0.51,0.54)$ | $z(0.33,0.35,0.37)$ | $z(0.87,0.89,0.91)$ | $z(0.90,0.92,0.94)$ | $z(0.41,0.43,0.45)$ | $z(0.65,0.67,0.69)$ | $z(0.73,0.75,0.77)$ | $z(0.52,0.54,0.56)$ | $z(0.31,0.33,0.35)$ |

*Table 6:The damage probability and the value of assets*

| | $T_1$ | $T_2$ | $T_3$ | $T_4$ | $T_5$ | $T_6$ | $T_7$ | $T_8$ | $T_9$ | $T_{10}$ | Value of the asset |
|---|---|---|---|---|---|---|---|---|---|---|---|
| $K_1$ | 0 | 0.58 | 0.76 | 0 | 0 | 0.26 | 0 | 0.34 | 0 | 0 | 0.75 |
| $K_2$ | 0.64 | 0 | 0 | 0 | 0.25 | 0.31 | 0.58 | 0.28 | 0 | 0.85 | 0.6 |
| $K_3$ | 0.36 | 0.69 | 0 | 0.65 | 0.73 | 0.14 | 0 | 0 | 0.52 | 0 | 0.85 |

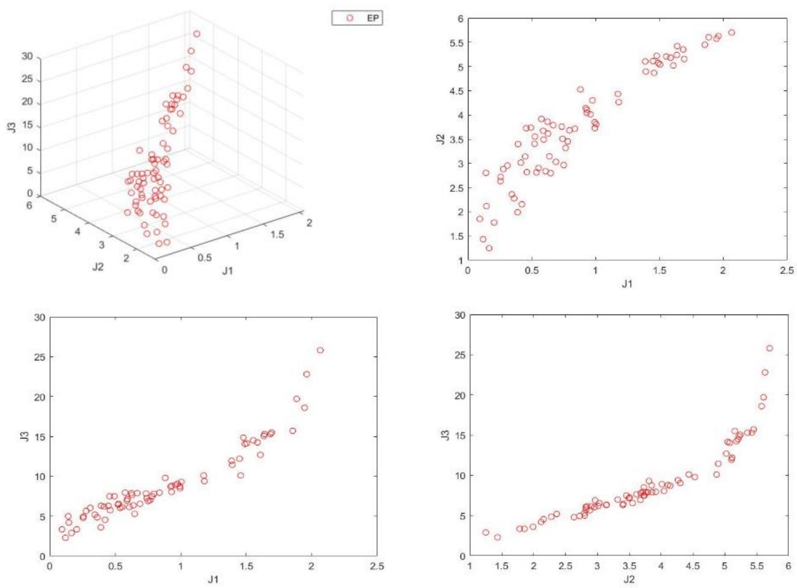


*Figure 3: Pareto-optimal solutions of MOEA/D-URAM for solving the UDWTA.*

*Table 7: Average running time*

| The scale of population | Iterations times | | |
|---|---|---|---|
| | 50 | 100 | 200 |
| 50 | 3.3652 | 5.5396 | 10.0772 |

*Table 8: Solutions with maximal survival efficiency of assets*

| Weapon | Target/the number of missiles | | | |
|---|---|---|---|---|
| | Stage1 | Stage2 | Stage3 | Stage4 |
| W1 | 9/1 | 2/3 | 9/2 | 5/2 |
| W2 | 2/2 | 4/2 | 0 | 1/3 |
| W3 | 7/3 | 2/2 | 3/1 | 4/2 |
| W4 | 8/3 | 9/3 | 3/1 | 4/1 |
| W5 | 2/1 | 10/3 | 4/3 | 6/3 |

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## 6. Conclution

In this paper, the uncertain factors in the dynamic WTA problem is discussed based on uncertainty theory and the UDWTA is proposed. Aiming at solving the multi-objective problem, the MOFA/D-URAW algorithm is proposed by adding the firefly algorithm displacement mechanism and uniformly randomly adjustment mechanism. Then, it is verified by the test function that the convergence and uniform distribution of the algorithm are improved. Finally, the algorithm is applied to UDWTA model to verify its feasibility.

## References

*[1] Manne, A. S . "A Target Assignment Problem." Operations Research 5.3(1957).*
*[2] R.H.Day, Allocating. "weapons to target complexes by means of nonlinear programming," Operation Research,1966,14:992–1013.*
*[3] Lu, Y., and D. Z. Chen . "A new exact algorithm for the Weapon-Target Assignment problem. " Omega 98(2021).*
*[4] Ni, M. , et al. "A Lagrange Relaxation Method for Solving Weapon-Target Assignment Problem." Mathematical Problems in Engineering,2011,(2011-11-24) 2011.PT.4(2011):264-265.*
*[5] Cao, M. , and W. Fang . "Distributed MMAS for weapon target assignment based on Spark framework." Journal of Intelligent and Fuzzy Systems 35.3(2018):1-12.*
*[6] Chang, T. Q., et al. "Terminating control of ant colony algorithm for armored unit dynamic weapon-target assignment." Systems Engineering and Electronics 37.2(2015):343-347.*
*[7] Wang, R. H., and C. Wang . "Variable Value Control Technology of Genetic Algorithm for WTA of Ground Target Attacking." Acta Armamentarii (2016).*
*[8] Hongtao, L., and K. Fengju . "Adaptive chaos parallel clonal selection algorithm for objective optimization in WTA application." Optik - International Journal for Light and Electron Optics 127.6(2016):3459-3465.*
*[9] Fan, C. L., et al. "Weapon-target allocation optimization algorithm based on IDPSO." Systems Engineering and Electronics 37.2(2015):336-342.*
*[10] Lloyd, S. P. , and H. S. Witsenhausen . "Weapons Allocation is NP-Complete." (1986).*
*[11] Gao, X. G. , et al. "Bayesian approach to learn Bayesian networks using data and constraints." International Conference on Pattern Recognition IEEE, 2017.*
*[12] Yao, Y., et al. MADM of Threat Assessment with Attempt of Target. Springer Berlin Heidelberg, 2012.*
*[13] Liu, B.. "Uncertain Urn Problems and Ellsberg Experiment." Soft Computing (2018).*
*[14] Wang, J. , et al. "Uncertain Team Orienteering Problem With Time Windows Based on Uncertainty Theory." IEEE Access PP.99(2019):1-1.*
*[15] Zg, A , et al. "Measuring trust in social networks based on linear uncertainty theory." Information Sciences 508(2020):154-172.*
*[16] Wang, et al. "Uncertain multiobjective traveling salesman problem.".*
*[17] Zhang, Y., et al. "Improved Decomposition-Based Evolutionary Algorithm for Multi-objective Optimization Model of Dynamic Weapon-target Assignment." Acta Armamentarii (2015).*
*[18] Baoding Liu. Uncertainty Theory(5nd ed).Tsinghua University Department of Mathematical Sciences,2017.*
*[19] Reyes-Sierra, M. , and C. C. A. Coello . "Multi-Objective Particle Swarm Optimizers: A Survey of the State-of-the-Art." International Journal of Computational Intelligence Research 2.3(2006):287-308.*
*[20] Zhang, Q. , and H. Li . "MOEA/D: A multiobjective evolutionary algorithm based on decomposition. " IEEE International Conference on Advanced Learning Technologies IEEE, 2005.*
*[21] Qi, Y. , et al. "MOEA/D with Adaptive Weight Adjustment." Evolutionary Computation 22.2(2014):231-264.*
*[22] Schott, J. R. "Fault Tolerant Design Using Single and Multi-Criteria Genetic Algorithms." Master's Thesis, Massachusetts Institute of Technology 37.1(1995):1–13.*
*[23] Tang, X. , et al. "A Discrete State Transition Algorithm for Generalized Traveling Salesman Problem." Springer International Publishing (2015).*
*[24] Deb, K. , et al. "A fast and elitist multiobjective genetic algorithm: NSGA-II." IEEE Transactions on Evolutionary Computation 6.2(2002):182-197.*
*[25] Wang, X. , Z. Gao , and H. Guo . "Delphi Method for Estimating Uncertainty Distributions." International journal on information 15.2(2012).*
*[26] Karasakal, O. "Air defense missile-target allocation models for a naval task group." Computers &*

ISSN 2616-5775 Vol. 5, Issue 3: 1-16, DOI: 10.25236/AJCIS.2022.050301

*Operations Research 35.6(2008):1759-1770.*

*[27] Xu, H., Q. Xing, and Z. Tian. "MOQPSO-D/S for Air and Missile Defense WTA Problem under Uncertainty." Mathematical Problems in Engineering,2017,(2017-12-14) 2017.pt.12(2017):1-13.*

*[28] Konak, A., D. W. Coit , and A. E. Smith . "Multi-objective optimization using genetic algorithms: A tutorial." Reliability Engineering & System Safety 91.9(2006):992-1007.*

*[29] Yang, X. S. . "Firefly Algorithms for Multimodal Optimization." International Symposium on Stochastic Algorithms Springer, Berlin, Heidelberg, 2009.*

*[30] Chang, J. , et al. "Multi-period portfolio selection with mental accounts and realistic constraints based on uncertainty theory." Journal of Computational and Applied Mathematics 377(2020):112892.*