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Dynamic Simulation and Optimization Analysis of the Bench Dragon Based on Differential Whale Optimization Algorithm

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Abstract: *This study is centered on the intricate mathematical problems within the traditional "dragon dance" model. To overcome this, the research leverages dynamic optimization and real - time feedback adjustment. Techniques such as the Archimedean spiral help in accurately determining the positions of the dragon's components over time. The Separating Axis Theorem is used for collision analysis, while optimization algorithms like the Differential Whale Optimization Algorithm (DWOA) enhance the overall model. Specifically, the research optimizes the timing by calculating the position - velocity equations at different times, improves the pitch for smooth rotational transitions, and refines the trajectory by calculating the arc lengths of the turning path. These efforts provide both theoretical and technical support for the design and real - time simulation of dragon dances, promoting a more in - depth exploration of bench dragon motion and the integration of traditional culture with modern research methods.*

Keywords: *Bench Dragon, Collision Detection, Traditional Culture, Differential Whale Optimization Algorithm (DWOA)*

1. Introduction

The bench dragon, as an important part of Chinese rural culture and excellent traditional Chinese culture, play a positive impact in inheriting excellent traditional Chinese culture[1]. Traditional ethnic sports serve as a concentrated embodiment of Chinese village customs, values, cultural traditions, and lifestyle interests. They provide an effective means of aligning with the rural revitalization strategy while preserving and promoting China's rich cultural heritage [2].

The mathematical mysteries behind the bench dragon have gradually become a research hotspot, with significant research value. In the field of motion trajectory planning, the Archimedean spiral and related iterative models are widely used to determine the positions of the dragon's head, body, and tail at different moments [3]. In terms of collision analysis, the Separating Axis Theorem [4] is employed, combined with a modified Whale Optimization Algorithm [5] and Genetic Algorithm [6], to optimize the bench positions and pitch, and to determine the collision termination moment of the bench dragon. In speed calculation and control, Euclidean distance is used to compute the speed [7], and cubic spline interpolation [8] is applied for precise analysis and control of the dragon's head speed. This ensures that the speeds of all the handles meet the required specifications. These mathematical applications have significantly enhanced the optimization and design capabilities for the dragon dance performance [9].

The use of mathematical modeling to simulate and optimize the dynamic motion and interaction of the bench dragon demonstrates how powerful computational techniques can be in the real-time performance and design of complex systems. The combination of path planning algorithms, collision detection methods, and control techniques is crucial for achieving the desired outcomes in both theoretical and practical applications, especially when dealing with intricate movements such as those involved in dragon dances. These methods not only improve the accuracy and efficiency of the design process but also enhance the aesthetic and functional qualities of the performance itself.

The current research on the bench dragon model faces many limitations. Due to the complexity of modeling the bench dragon, existing studies often simplify it, which leads to discrepancies between the model and real-world scenarios, resulting in a lack of accuracy. Moreover, the model optimization process lacks dynamic optimization and real-time feedback adjustment mechanisms, preventing the

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model from making real-time adjustments and challenging its accuracy and applicability.

This study introduces a novel approach to solving the mathematical complexities of the bench dragon model. It presents a method for determining the position and velocity equations of the dragon dance team over time, optimizing the pitch to ensure smooth transitions between clockwise and counterclockwise movements. Additionally, the research develops a geometric model for calculating the arc lengths of an S-shaped turning path, where two tangent arcs of different radii are formed, enhancing the precision of trajectory planning in the dragon dance.

2. Research Focus Elaboration

2.1 Research Problem Definition

The traditional Chinese dragon-shaped bench, originating from China, exhibits a complex motion composed of linear, curvilinear, angular, and oscillatory movements. It encapsulates intricate physical models and mathematical principles. By analyzing its trajectory, motion equations, and turning dynamics, one can gain deeper insights into the evolution of compound motion within complex mechanical environments. This provides a valuable empirical model for refining the theoretical framework of mechanical dynamics and kinematics.

Beyond enhancing the analysis of multi-dimensional motion coupling—an area often limited in traditional kinematics—this study offers theoretical support for advanced fields such as modern robotic motion planning and bionic design. By addressing existing technical bottlenecks, it contributes to the development of more efficient and flexible motion control systems. Furthermore, it bridges abstract mathematical theory with vibrant folk cultural practices, fostering an interdisciplinary integration that drives both theoretical innovation and cultural appreciation.

2.2 Motion parameterization

Determination of the starting point of the entry: The entry trajectory of the dragon dance team is a helical line with a pitch of 55 cm and equal intervals. Moreover, the entry point of the dragon head is at position A on the 16th circle of the x-axis.$x_0 = 55 \times 16 = 880cm$， $y_0 = 0$， So the initial position is (880,0)

Effective bench length calculation: the speed and position of the handle are calculated when the speed and position are calculated, so the effective distance of the bench is the total length minus the overlap distance, that is, the effective distance length head = 341-55=386cm, and the effective distance length body=220-55=165cm. Therefore, we need to delete the overlap when iteratively calculating the handle position and speed of each dragon body

Faucet position:

Given that the motion trajectory is an isometric helix and the speed of the faucet is 1m/s, this paper established a joint equation of Archimedes helix and motion equation to solve the position of the faucet as follows:

$$r(\theta) = a + b\theta \tag{1}$$

$$L(\theta) = b\int_0^\theta \sqrt{1+\theta^2}\ d\theta \tag{2}$$

$$L\big(\theta(t)\big) = v \cdot t \tag{3}$$

Were θ denotes the head along the helix clockwise rotation Angle, a denotes the distance from the starting point to the origin, here $a = 880$. And b represents the distance between two adjacent threads, here $b = 55$.

In this paper, the dragon head is separated separately, and the derivation based on geometric model and the constraint of physical model are carried out. First of all, the path of the head per second is a spiral, which is approximately calculated as an arc. The central Angle of the faucet in the helix can be calculated by the formula of the central Angle of the arc. And according to the characteristics of the helix, the length of the initial outlet diameter can be derived. Then using the iterative idea, the updating formula of the central Angle and the polar diameter can be obtained as follows:

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$$S = v \cdot t, \theta_1 = \frac{S}{r_1} \tag{4}$$

$$r_2 = r_1 - \frac{b \cdot \left(\frac{S}{r_1}\right)}{2\pi}, \theta_2 = \frac{S}{r_2} \tag{5}$$

$$r_{t+1} = r_t - \frac{b \cdot \left(\frac{S}{r_t}\right)}{2\pi}, \quad \theta_{t+1} = \frac{S}{r_{t+1}} \tag{6}$$

The polar diameter and center Angle are updated as follows shown in Figure 1:

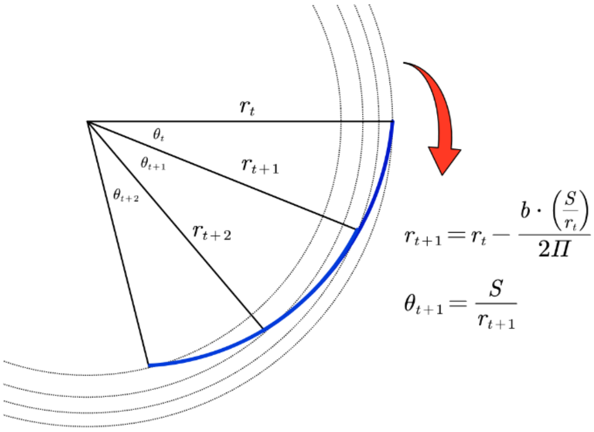

Figure 1 Update of polar diameter and center Angle

Where v is the speed of the faucet, here is 1m/s, S is the arc length traveled by 1s. When the faucet passes 1s, the corresponding center Angle is θ_1, and the initial pole diameter of the faucet pan is entered, that is, the radius of the approximate arc of the first isometric thread line r_1. θ_{i+1}It's the corresponding central Angle when the faucet passes through $(\mathrm{i}+1)$s.

Transformation of rectangular coordinate system:

Now the polar coordinate information of the head's position has been obtained. At this time, we are using the formula to convert the polar coordinates into cartesian coordinates, as follows:

$$x_{t+1} = r_{t+1} \cdot \cos(\theta_{t+1}) \tag{7}$$

$$y_{t+1} = r_{t+1} \cdot \sin(\theta_{t+1}) \tag{8}$$

3. Methodology

3.1 Multi-objective particle swarm optimization

Step1: Initialize

The number of the particle swarm is initialized as P, each particle corresponds to a solution vector x_i containing the position information of the dragon segment, and the velocity v_i of each particle is initialized [10]. In addition, the global non-dominated solution set G is initialized to store the Pareto optimal solution found so far.

Step2: Fitness function

The fitness value of each particle is calculated by two objective functions $f_1(\mathrm{x})$ and$f_2(\mathrm{x})$

Calculate the area of the dragon polygon by the coordinates of the dragon body segment

$$A = \frac{1}{2}\left|\sum_{i=1}^{n}(x_i y_{i+1} - x_{i+1} y_i)\right| \tag{9}$$

where (x_i, y_i) are the coordinates of the ith segment.

Collision detection: For each pair of adjacent segment sums i and i+1, calculate their Euclidean

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distances $d_{i,i+1}$:

$$d_{i,i+1} = \sqrt{(x_i - x_{i+1})^2 + (y_i - y_{i+1})^2} \tag{10}$$

Step3: Update particle velocity and position

The velocity v_i and position of each particle x_i are updated as follows:

$$\mathrm{v}_i^{(t+1)} = \omega \mathrm{v}_i^{(t)} + c_1 r_1 (\mathrm{p}_i - \mathrm{x}_i^{(t)}) + c_2 r_2 (\mathrm{g}_i - \mathrm{x}_i^{(t)}) \tag{11}$$

$$\mathrm{x}_i^{(t+1)} = \mathrm{x}_i^{(t)} + \mathrm{v}_i^{(t+1)} \tag{12}$$

where ω is the inertial weight, which controls the particle's ability to retain its current velocity. c_1 and c_2 are learning factors that control the speed at which particles move toward their own historical and global best solutions. r_1 and r_2 are random numbers used to increase the randomness of the search. p_i is the historical optimal position of the particle and g_i is the current global optimal position.

Step4: Update the non-dominant solution set

After each iteration, the solutions of all particles are used to update the non-dominant solution set G. If a solution of a particle dominates a solution in G, that solution is replaced. The domination relation is defined as follows: for the solution x_i and x_i ,is said x_i to dominate x_j if $f_1(\mathrm{x}_i) \le f_1(\mathrm{x}_j)$ and $f_2(\mathrm{x}_i) \le f_2(\mathrm{x}_j)$ and at least one inequality is strictly true.

Step5: Iteration stop condition

1) Reach the maximum number of iterations.

2) The non-dominated solution set does not change in multiple iterations.

3.2 Adopt genetic algorithm to solve pitch

Step1. Initialize the population:

In the initial population P_0 each individual represents a possible pitch b_i and its initial value $[b_{\min}, b_{\max}]$is randomly generated in the range. set the range of pitch is [0.1,0.5] and the initial population size is $N = 50$.

$$P_0 = \{b_1, b_2, \dots, b_{50}\} \tag{13}$$

Step2. Fitness function:

The fitness of each pitch is calculated by simulating the spiral path $f(b_i)$, and the goal of the simulation result is to have the tap reach the turning area in total time seconds $t_{\text{total}} = 300$, radius meters $r_{\text{turn}} = 4.5$m.

The fitness function can be expressed as follows.

$$f(b_i) = \begin{cases} r_{\text{final}}, & \text{if } r_{\text{final}} \le 4.5 \text{ m} \\ \infty, & \text{if } r_{\text{final}} > 4.5 \text{ m} \end{cases} \tag{14}$$

where, r_{final} is the final radius of the faucet at the end of the simulation.

Step3. Select the parent:

We select the best performing pitch individual as the parent based on the fitness value. The smaller the fitness value, the closer the pitch is to the optimal value. The probability of selecting each individual based on the inverse of its fitness $f(b_i)$ is:

$$p_i = \frac{\frac{1}{f(b_i)}}{\frac{\sum_{j=1}^{50} 1}{f(b_j)}} \tag{15}$$

Here, the probability p_i that an individual b_i is selected as a parent is determined by the inverse of its fitness value.

Step4. Crossover:

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Two individuals b_{p_1} and b_{p_2} are selected from the parent generation for crossover to generate the offspring. The pitch b_{child} of the offspring is a linear combination of the parent, and the weight is controlled by a random coefficient α.

The crossover formula is as follows.

$$b_{\text{child}} = \alpha \cdot b_{p_1} + (1-\alpha) \cdot b_{p_2} \tag{16}$$

where $\alpha \sim \mathcal{U}(0,1)$is a random number that controls the weight between the two parents.

Step5. Update the population:

The generated offspring and the parents of the previous generation are merged to form a new population P_{new}. The new population contains the optimal parent and the newly generated offspring,

The new population formula is as follows.

$$P_{\text{new}} = \left\{b_{\text{child}_1}, b_{\text{child}_2}, \ldots, b_{\text{child}_k}, b_{\text{parent}_1}, b_{\text{parent}_2}, \ldots\right\} \tag{17}$$

where k is the number of offspring and the remaining part is the retained parent.

step7. Termination condition:

The genetic algorithm continues to iterate until any of the following conditions are met:

- The maximum number of iterations is reached
- Fitness is no longer significantly improved (the difference in fitness values is less than a certain threshold).

The pitch b_{best} of the optimal solution is the individual with the smallest fitness in the final population, which is expressed as follows.

$$b_{\text{best}} = \text{argmin} f(b_i) \tag{18}$$

where i is all individuals in the current population.

3.3 Establishing the Turning Model and Solving for Arc Length

(1)Turning Trajectory Analysis

The turning space is defined as a circle with a diameter of 9 meters, centered at the spiral's reference point. The entry and exit spirals are symmetrical about this center, meaning the entry spiral is rotated 180° clockwise around the center to form the exit spiral.

Within the turning region, the turning trajectory consists of two standard circular arcs that are tangent at the connection point. The radius of the larger arc is twice that of the smaller arc.

Through geometric analysis, the resulting turning trajectory within the turning region closely resembles a Taiji diagram, as shown in the Figure 2.

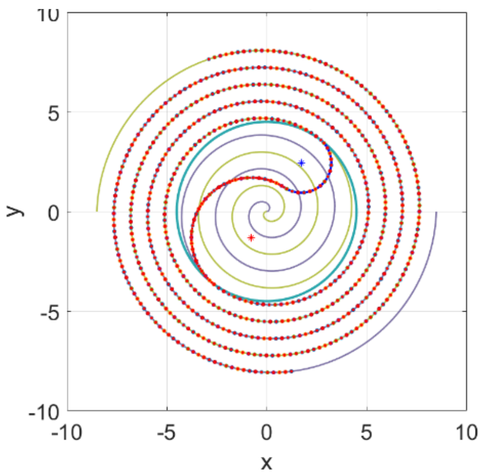

Figure 2 The turning trajectory in the turning area

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(2) Establishment of turning trajectory model

To accurately analyze the U-turn trajectory in the u-turn area, this paper separates the u-turn trajectory in the u-turn area and makes a separate mathematical modeling analysis, as shown in the Figure 3.

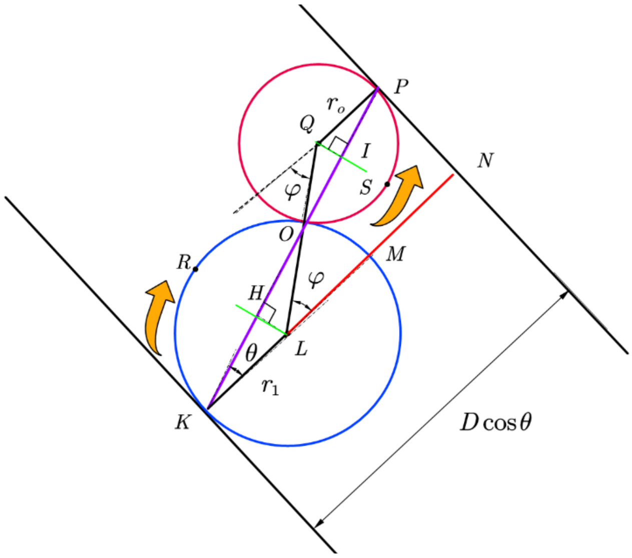

Figure 3 Geometric model of the U-turn trajectory

In the figure, the dragon head starts to turn around from the turning area at point K, where $\widehat{KRO}$is the first circular arc with radiusr_1 when turning around. $\widehat{OSP}$ is the second circular arc of the turn, with radius r_0. That is, when turning around, the dragon dance team walks along the path of the arrow direction and two circular arcs.

First, make the tangent of the circle through point K and point P. The tangent of the dragon dance team's disc entry helix at point K and the tangent of the disc exit helix at point P are also centersymmetric about the origin.

Calculate the arc length and radius:

First, calculate the tangent slope of the intersection point: The tangent slope of the arc is given by the formula:

$$\text{slope} = \frac{k\sin(\theta_{ru}) + R\cos(\theta_{ru})}{k\cos(\theta_{ru}) - R\sin(\theta_{ru})} \tag{19}$$

Where θ_{ru} is the intersection Angle between the incoming spiral and the circle boundary. k and R are the parameters and radius of the helix.

The Angle between the tangent line and the center of the circle: The Angle between the tangent line and the center of the circle $\theta_{\max\_1}$ is given by:

$$\theta_{\max_1} = \arctan\left(-\frac{1}{\text{slope}}\right) + \pi \tag{20}$$

Base Angle of isosceles triangle in geometric relation: According to geometric relation, the base Angle of isosceles triangle between circles C1 and C2 is as follows.

$$\theta_{\text{dengyao}} = \arctan\big(\tan(\theta_{ru})\big) + \pi - \theta_{\max_1} \tag{21}$$

The radii of C1 and C2 are derived by isosceles triangles: the distance between circles C1 and C2 is:

$$RC1C2 = \frac{R}{\cos\left(\theta_{\text{dengyao}}\right)} \tag{22}$$

By geometric relation, knowing $RC1 = 2 \times C2$, therefore:

$$RC0 = \frac{RC1_{C0}}{3}, \quad RC1 = 2 \times RC0 \tag{23}$$

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The center Angle corresponding to the arc length is calculated by the following formula:

$$\phi = 2 \times \theta_{\text{dengyao}} \tag{24}$$

Using the arc length formula, the arc lengths $S = r \times \Delta\theta$ of circular arcs C1 and C2 are respectively as follows.

$$S_{C1} = r_{C1} \times (\pi - \phi) \tag{25}$$

$$S_{C2} = r_{C2} \times (\pi - \phi) \tag{26}$$

Where r_{C1} and r_{C2} are the radii of circles C1 and C2. $\pi - \phi$ is the size of the center Angle.

4. Results and discussion

4.1 Multi-objective particle swarm optimization result

Due to the multi-objective particle swarm optimization in this paper to adjust the position of the front and back handles of each stool, there is a certain optimization error, resulting in the handle is not completely on the equidistant spiral. In this paper, the final positions of different knobs at different times were tested, and the vertical distance between the obtained position coordinates and the nearest helix was calculated. The error of 67124 points was calculated in total, as shown in the Table 1.

Table 1 Location display

	0s	60s	120s	180s	240s	300s
Tap x(m)	8.800000	5.822837	-4.024662	-3.070812	2.754696	4.298521907
Tap y(m))	0.000000	-5.747757	-6.344090	6.043200	-5.279101	2.547757173
Article I. Dragon Body x (m)	8.309445	7.422391	-1.351494	-5.269545	4.875255	0.979256817
Article I. Dragon Body y (m)	2.817615	-3.376887	-7.360837	4.214219	-3.360030	0.979256817
Section 101 The Dragon Body x (m)	-9.161864	-7.797633	-9.298665	-10.111659	0.781751	6.066360896
Section 101. Dragon Body y (m)	-7.420299	-8.229485	-5.621188	2.316446	9.827078	-7.070854873
Section 201 The Dragon Body x (m)	-9.560873	-2.604260	-1.580209	-3.193941	-11.203346	-4.204763923
Section 201 The Body of the Dragon y (m)	-10.466525	-13.553948	-13.301046	-12.624984	-5.801501	11.45072797

Since the aperture of the front and rear handles is 5.5cm, the point where the vertical distance between the front and rear handles and the helix is less than 5.5cm can still be seen as the handle pressure line pressed on the helix. By analyzing the table, it can be obtained that there are 64088 points with a distance less than 5.5cm and 3036 points with a distance greater than 5.5cm. The points with errors greater than the aperture range account for 4.52%, and the reliable points account for 95.48%. The accuracy of the model is good.

4.2 Pitch result

After the model calculation of the second question and the optimization of simulated annealing algorithm, the final minimum pitch obtained in this paper is 0.4706m.

4.3 Arc Length

Through MATLAB calculation, the radius of arc C1 is 3.0054 meters, the radius of arc C2 is 1.5027 meters, the arc length of arc C1 is 9.0808 meters, and the arc length of arc C2 is 4.5404 meters. The total arc length $\overset{\frown}{OSP}+\overset{\frown}{KRO}$ of the two sections of arc is: 13.6212 meters. Since the speed of the faucet is 1m/s, it takes 13.6212s for the faucet to walk from point to point one. Currently, the time of the point time is set as 0s, so the time for the faucet to reach the point time is 13.6212s. In addition, through the simultaneous equations, the end point of the tap and disc entry is (2.7119, -3.5911), and the starting point of the disc exit is (-2.7119, 3.5911). As shown in the Table 2.

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Table 2 Speed display

	-100 s	-99 s	-98 s	-97 s	-96 s	-95 s
Tap (m/s)	0.999978	0.99991	0.999909	0.999908	0.999908	0.999907
Section 1 Dragon Body (m/s)	0.999882	0.999814	0.999812	0.99981	0.999808	0.999805
Section 2. Dragon Body (m/s)	0.999865	0.999797	0.999795	0.999793	0.99979	0.999788
Section 3 Dragon Body (m/s)	0.999848	0.999781	0.999778	0.999776	0.999773	0.99977
Section 4 Dragon Body (m/s)	0.999831	0.999765	0.999762	0.999759	0.999756	0.999753
Section 5 Dragon Body (m/s)	0.999815	0.999749	0.999746	0.999743	0.99974	0.999737
Section 6 Dragon Body (m/s)	0.999799	0.999734	0.999731	0.999727	0.999724	0.999721
Section 7 Dragon Body (m/s)	0.999784	0.999719	0.999716	0.999712	0.999709	0.999705
Section 8 Dragon Body (m/s)	0.999769	0.999704	0.999701	0.999697	0.999693	0.99969
Section 9 Dragon Body (m/s)	0.999754	0.99969	0.999686	0.999682	0.999678	0.999674
Section 10 Dragon Body (m/s)	0.999739	0.999676	0.999672	0.999668	0.999664	0.99966

5. Conclusion

This study focused on the bench dragon, a significant element of Chinese rural culture. By formulating three research questions related to timing, pitch optimization, and trajectory, we employed a series of algorithms and geometric models. These included multi objective particle swarm optimization, genetic algorithms, and the establishment of turning trajectory models. The results indicated that the proposed methods effectively resolved the collision problems, optimized the pitch, and accurately calculated the turn trajectory, arc lengths, the dragon's position and velocity. The high accuracy rate of the model in determining handle positions near the spiral, along with the optimized pitch and calculated arc lengths, demonstrated the validity of our approach. In conclusion, this research deepens our understanding of the bench dragon's motion, offering valuable support for the design and real time simulation of dragon dance performances. It also promotes the integration of mathematics, mechanics, and cultural research. Future work could explore more complex motion scenarios, real - time adjustments, and expanded application areas.

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