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Research on Instructional Design for Middle School Plane Geometry Based on Mathematical Core Competencies: A Case Study of the Triangle Midline Theorem

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Abstract: *With the rapid advancement of science and technology, society's demand for talent places greater emphasis on cultivating character and key abilities. Plane geometry is a crucial component of the middle school mathematics curriculum, serving as a vehicle for developing students' abilities in intuitive visualization and logical reasoning. It plays a unique role in fostering students' problem awareness, improving their observation skills, logical reasoning abilities, and problem-solving capabilities. Based on mathematical core competencies, this article analyzes the entire process of plane geometry instructional design—"preparation, implementation, and reflection"—keeping the goal of "competency enhancement" central throughout, thereby concretely implementing mathematical core competencies in teaching.*

Keywords: *Mathematical Core Competencies; Plane Geometry; Instructional Design*

1. Introduction

Mathematics is a highly creative discipline. It is not only an instrumental subject that enriches thinking and helps people understand the world more profoundly through a mathematical lens but also a linguistic subject that expands cultural understanding and assists in expressing the world mathematically. Furthermore, it is a developmental subject that enhances various abilities and empowers people to transform the world using mathematical thinking. The Compulsory Education Mathematics Curriculum Standards (2022 Edition) highlights that mathematics has a crucial impact on cultivating rational thinking, scientific spirit, and the development of individual intelligence[1].

Learning plane geometry is both a process of establishing relationships between mathematics and life and a process where learners apply geometric intuition and deductive reasoning to formulate and verify conjectures. It plays a key role in cultivating core competencies such as intuitive visualization, logical thinking, and abstraction. Mathematics education aims to foster mathematical literacy through the individual. It should emphasize process and enhance thinking skills, not be confined to the teaching of knowledge and methods alone. Instead, it should elevate students' cognitive abilities and cultivate their development of dialectical and systematic mathematical qualities[2]. Junior high school is a critical and accelerated period for cognitive development. Designing teaching activities for plane geometry lessons has always been both a priority and a challenge. Only through rigorous teaching design for geometry lessons can students be enabled to actively explore, exercise their thinking, enhance their competencies, and thereby achieve the expectations and requirements set by the new curriculum standards for students.

The study of geometry began with Euclid. His work Elements established European geometry, systematizing and structuring the field. Subsequently, numerous scholars and experts dedicated themselves to geometry research, profoundly influencing later generations. Hutchins believed that the most effective and direct way to cultivate students' logical thinking abilities is through geometry teaching. In secondary mathematics education, the process of learning and exploring "Shapes and Geometry" is key to developing students' mathematical core competencies, such as reasoning ability, geometric intuition, and abstraction. However, in current geometry classroom teaching, teachers often directly present geometric concepts and theorems, followed by simple proofs, concluding with "rote practice drills" to finish the topic. This teaching approach prevents students from gaining a deep understanding of the knowledge, denies them the successful experience of geometric proof, hinders their appreciation of the joy of mathematics, and ultimately fails to cultivate mathematical thinking qualities and core competencies. This is incompatible with the educational goals proposed in the Compulsory Education Mathematics Curriculum Standards (2022 Edition). Building on the above research, this paper proposes a core competency-centered approach to teaching design for junior high school plane geometry.

2. The Path of Plane Geometry Instructional Design Guided by Competencies

Learning plane geometry requires students to understand the characteristics of basic plane geometric figures, perform standardized constructions based on descriptions, possess certain graphic analysis skills, and develop divergent thinking for investigating plane geometric properties, achieving "language in diagrams, diagrams in language, and logical speech[3]." Teachers need to understand students' prior knowledge, recognizing that students do not enter the classroom with "empty minds." They must be aware of students' current status, create appropriate contexts, provide specific problem situations, exploration scenarios, and application settings[4]. Teaching design should not only reflect students' central role but also start from their existing knowledge, experience, and mathematical background, providing mathematical activity experiences. Teachers should continuously optimize teaching design under the new curriculum concepts, making the promotion of students' application and innovation abilities the overarching path. This enables students to deeply appreciate the connection between mathematics and the real world through the lesson.

Therefore, in junior high school plane geometry classrooms, teachers should focus on creating life-like situations, providing appropriate guidance, and organizing activities involving observation, analysis, reasoning, generalization, and induction. The ultimate goal is for students to learn to express what they see and think using mathematical language and to experience the way mathematics communicates with real-world thinking[5]. The specific teaching process is shown in Figure 1.

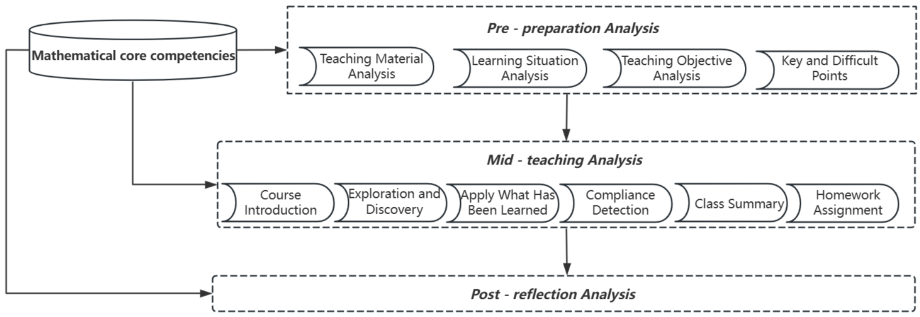

Figure 1: Teaching Process Flowchart.

3. Instructional Design Case for the Triangle Midline Theorem

3.1 Preparation Analysis

Textbook Analysis: This lesson is selected from Section 18.2 "Determining Parallelograms" in Chapter 18 of the People's Education Press Grade 8 (Lower Semester) textbook. It belongs to the content on properties of figures within the "Shapes and Geometry" domain. The Triangle Midline Theorem reflects the quantitative relationship between a triangle's midline and its three sides. Its application deepens understanding of the properties and determination of congruent triangles, as well as knowledge of central symmetry. It also serves as a fundamental basis for learning the trapezoid midline theorem and opens a new pathway for proving positional and quantitative relationships (including multiple and fractional relationships) between line segments. Therefore, this lesson's content plays a connecting role in junior high school geometry teaching.

Simultaneously, as a geometry theorem inquiry lesson, it integrates observation, activity, and application. It effectively highlights the new curriculum standard's advocated concept of cultivating core competencies through "problem discovery—problem analysis—problem solving." Furthermore, this lesson permeates mathematical thought methods such as special-to-general, transformation, and equation, creating conditions for core competencies like geometric intuition, abstraction, and reasoning

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ability, positively impacting the cultivation of students' thinking.

Student Analysis: Students possess experience in investigating the invariance of positional and quantitative relationships of segments and angles within dynamic figures in the context of parallelograms. They have the ability to add simple auxiliary lines and possess preliminary analogical transformation thinking and some reasoning ability. However, they still need experience in using parallelogram properties to handle triangle-related problems. They require strengthening in the proactive application of analogical transformation methods, active transfer of thinking, and higher-level reasoning abilities.

Teaching Objectives:

(1) Students are able to understand the concept of a triangle's midline and master the midline theorem of a triangle along with its applications.

(2) Students will experience the investigative process involving observation, conjecture, and verification for proving the triangle midline theorem. In this process, their abilities in independent and collaborative exploration will be developed. They will also gain experience in research methods such as plausible reasoning and mathematical thinking methods like transformation, and their core competencies, including geometric intuition, abstraction, and reasoning, will be fostered.

(3) The integration of mathematical history enables students to deeply appreciate the profoundness of Chinese mathematical culture, thereby enhancing their national confidence.

Teaching Focus and Difficulties:

(1) Teaching Focus: The exploration and proof of the triangle midline theorem, as well as the experience of the general approaches and methods for studying geometric figures.

(2) Teaching Difficulty: The addition of auxiliary lines to convert a triangle into a parallelogram, which is required for proving the triangle midline theorem.

3.2 Implementation Analysis

Situational Introduction

Pose a problem: As shown in Figure 2, if a wooden strip starts from position CA and rotates clockwise around point O to position DB, during this process, let the strip intersect with AB and CD at points E and F, respectively (where point E does not coincide with points A or B).

(1) How does the shape of quadrilateral $BCOP$ change? (2) At which position of point F does quadrilateral $BCOP$ become a trapezoid?

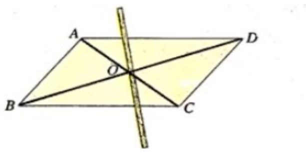

Figure 2: Diagram of the Rotating Wooden Strip.

Student Activity: Students use rulers to perform operations on worksheets. After reaching conclusions, students demonstrate and explain the process on the podium.

Whiteboard Activity: As shown in Figure 3, the transformation sequence is: trapezoid → parallelogram → trapezoid. The quadrilateral becomes a parallelogram when the point is at the midpoint.

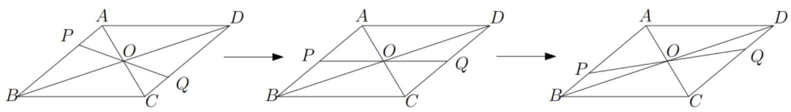

Figure 3: Quadrilateral Transformation Process.

Teacher Facilitation: Dynamic geometry software is utilized to guide students in analyzing triangles within the framework of a parallelogram. Through interactive exploration, addressing the key question" Does segment OP maintain any specific geometric relationship with components of $\triangle ABC$? "

[Design Intent] The problem design starts from parallelograms, follows general geometric solution

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approaches, stays close to students' zone of proximal development, and naturally transitions to the "generation" of the midline. It then guides them through observation, conjecture, verification, and reasoning to finally derive the theorem. This approach emphasizes the depth and process of knowledge acquisition, aligning with students' cognitive patterns.

Exploring New Knowledge

Inquiry 1: Concept Differentiation - Clarifying Definitions

Teacher Facilitation: The median and the midsegment of a triangle differ by only one character. Are they the same segment? Can you distinguish them clearly? The teacher assigns a task: draw both segments within a time limit. Guide students to verbally define the midsegment. The teacher writes the standard definition on the board simultaneously.

Student Activity: Students independently draw the median and midsegment of a triangle.

[Design Intent] Students intuitively differentiate the two segments through hands-on drawing, deepening their understanding and distinction between old and new knowledge, enabling them to grasp the concept of the triangle midsegment clearly.

Inquiry 2: Conjecture and Proof - Forming the Theorem

Teacher Facilitation: What special knowledge does this particular segment (midsegment) hold?

Student Activity: Students observe the figure, intuitively perceive, and are encouraged to boldly conjecture the relationship between segment DE and side BC.

Teacher Activity: The teacher prompts that relationships between segments should be considered in terms of both quantitative and positional aspects.

Student Conjecture: $DE//BC, DE=\frac{1}{2}BC$.

[Design Intent] Conjecture is a vital part of mathematics learning. Students attempt independently, continuously discovering problems, stimulating interest, and activating thinking.

Student Activity: Using measurement and paper-cutting/piecing methods, groups of four conduct inquiry. Students utilize prior experience to verify the conjecture.

Teacher Activity: The teacher circulates, observes, participates and organizes. After students complete the task, the teacher selectively invites groups using different methods to share their results.

[Design Intent] Students engage in measurement, cutting, and piecing activities to verify their conjecture, deepening their understanding of the concept more vividly and intuitively. Verifying and showcasing different methods emphasizes developing divergent thinking and awareness of diverse problem-solving approaches.

Teacher Facilitation: I notice some groups did not reach this conclusion, indicating measurement and paper-cutting methods have inherent limitations. What should we do next?

Verbal Responses: We can use deductive reasoning for rigorous proof.

[Design Intent] Through measurement, paper cutting/rearrangement, and other mathematical learning activities, students verify their conjectures, visually reinforcing their understanding of the concept. The verification and showcasing of multiple methods focus on cultivating students' divergent thinking and appreciation for diverse approaches to problem-solving.

Inquiry 3: Mathematical Reasoning - Verifying the Conclusion

Whiteboard Activity: In $\triangle ABC$, point D is the midpoint of side AB, and point E is the midpoint of side AC. Prove: $DE//BC$ and DE=$DE=\frac{1}{2}BC$.The teacher poses three consecutive probing questions:

(1) Recalling the piecing process, what shape did we transform the triangle into?

(2) How can we abstract the geometric model from the intuitive pieced figure?

(3) How should we add auxiliary lines?

[Design Intent] These three sequential questions prompt continuous thinking, guiding students to add appropriate auxiliary lines to construct a parallelogram for proving the theorem. This integrates

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intuitive manipulation with logical reasoning, making the proof process a natural extension of observation, conjecture, and exploration.

Teacher-Student Interaction: Students collaborate to complete the proof. Teacher circulates and guides. When students encounter obstacles, the teacher provides timely hints. Finally, uses screen sharing to randomly display students' proof processes for discussion.

Student Activity: Independently complete writing one proof method.

[Design Intent]: Students' mathematical language is standardized, enabling them to use it for communication and expression.

Teacher Activity: After presenting students' proof methods, the area-cutting and patching technique proposed by Liu Hui, a Chinese mathematician, in his Commentary on The Nine Chapters on the Mathematical Art is introduced (see Figures 4), which involves transforming the area of a triangle into that of a rectangle. This method transforms the area of a triangle into a rectangle. Dynamic IT demonstrations are utilized to help students revisit the concept of the triangle midsegment.

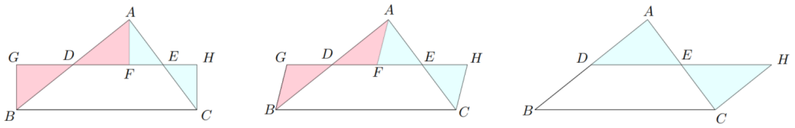

Figure 4: Cutting and Complementing Method - Image placeholder description.

[Design Intent] Students are enabled to appreciate the beauty of ancient Chinese mathematics, which stimulates their patriotism and enhances their mathematical cultural literacy and confidence.The dynamic demonstration emphasizes the role of midpoints, further deepening understanding of the midline.

Application of Knowledge

Teacher introduce a problem measuring the width of the school's inkstone pool, abstracting a real-life problem into a mathematical one, developing students' abstraction competency. This allows students to apply the triangle midsegment theorem in real-life contexts while learning foundational knowledge. Moreover, it tests students' grasp of applying the positional and quantitative relationships of the midsegment theorem and helps teachers assess the achievement of this lesson's teaching objectives.

Achievement Assessment

Assessment items should follow the basic approach to geometric research (from conjecture verification to appropriate auxiliary lines and deductive proof), guiding students on how to apply the theorem. It also helps students build a complete knowledge network and promotes the development of mathematical thinking and methods.

Lesson Summary

What content did we learn in this lesson? What process did we go through to discover the midsegment theorem? How did we complete the proof of the midsegment theorem? What insights did you gain?

[Design Intent] Problem 1 guides the review of knowledge, reinforcing students' understanding. Problem 2 further clarifies the research path for geometric figures, inspiring students to follow this path when investigating other geometric problems in the future. Problem 3 targets the difficulty point of this lesson, highlighting its difference from previous content in this unit—specifically, viewing the triangle within the parallelogram. This clarifies the method used to address this difficulty, enhancing students' metacognitive awareness overall.

Homework Assignment

Basic: Textbook P49 Review & Consolidate 1-3; P50 Comprehensive Application 8, 9.

Optional: Textbook P50 Review & Consolidate 6; P50 Comprehensive Application 10.

Exploration: Attempt to measure an inaccessible distance between two points on campus using learned methods.

[Design Intent] Differentiated homework is assigned. Basic problems assess learning; optional

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problems develop thinking, allowing students to choose based on ability; exploration problems are practical, enabling students to perceive mathematics in campus and daily life, truly applying their knowledge.

3.3 Post-Reflection Analysis

This lesson follows the main thread of "Context Creation - ConceptInquiry - Theorem Inquiry - Application - Summary & Enhancement," successfully achieving the teaching objectives. The classroom activities were diverse and highly effective, fully aligned with the new curriculum standard's requirement of unifying teacher guidance and student agency. It developed students' core competencies in geometric intuition, spatial reasoning, and logical ability. Its main highlights are explained below:

(1) Emphasis on Concept Generation: The lesson builds on students' prior knowledge of parallelograms as the cognitive foundation for creating the context. By viewing the triangle within the parallelogram, it encourages students to approach problems holistically, focusing on connections and development, thereby cultivating their scientific thinking.

(2) Emphasis on Student Agency: The lesson prioritizes students' hands-on operation and the application of accumulated activity experience. Throughout the teaching process, it consistently adhered to the principle: let students devise the approach, discuss the difficulties, attempt the reasoning, and draw the conclusions.

(3) Emphasis on Cultivating the Application Awareness of Core Competencies: Introducing the problem of measuring the campus inkstone pool extracts the midline concept from a real-world shape. This develops students' abstract thinking, enables them to better view the surrounding world through a mathematical lens, and empowers them to think about the real world mathematically.

However, in actual teaching practice, teachers should pay greater attention to students' points of difficulty, cultivate their own educational sensitivity, and flexibly address the problems students encounter. This is essential to resolve students' confusions and difficulties, thereby enhancing their learning outcomes. Teachers should encourage students to raise questions and guide them in summarizing and synthesizing these problems, fostering their problem awareness.

4. Conclusion

The instructional design for junior high school plane geometry should be grounded in the curricular values outlined in the Compulsory Education Mathematics Curriculum Standards (2022 Edition). Teaching activities should be arranged rationally, and the introductory classroom scenarios should be carefully selected to foster student thinking based on existing cognitive experiences, enabling autonomous construction. This approach helps students connect with familiar cognitive structures, develop planning and transformation thinking, and cultivate reasoning abilities.

Most importantly, teachers must effectively coordinate their guidance with students' agency: With activities as the main thread and learning by doing, it is ensured that all students are actively engaged and participating; With students as the main agents and reflection through presentation, students are given the stage to showcase their thinking; With core competencies as the goal and the awakening of insights during class, the spark of intellectual exchange is ignited.

References

[1] Ministry of Education, People's Republic of China. (2022). Mathematics Curriculum Standards for Compulsory Education (2022 Edition). Beijing Normal University Press.
[2] Han, L. Y. (2022). On the lack and countermeasures of cultivating mathematical thinking in middle school mathematics classroom teaching. Education Theory and Practice, 42(32), 50-52.
[3] Yang, M. (2022). Cultivating students' ability to analyze plane geometric figures in mathematics teaching. New Curriculum Research, (31), 60-6.
[4] Fei, W. Q. (2023). Mathematics classroom teaching design in junior high school under constructivism theory. Middle School Mathematics, (20), 34-35.
[5] Tang, M. Q. (2019). Exploring teaching strategies for mathematical activity experience aimed at students' core literacy: Taking "Surface Area of a Sphere" as an example. Middle School Teaching Research (Mathematics), (11), 6-10.