

Noisy image segmentation model combining PDE and improved variational level set method

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Abstract: During image shooting, transmission and acquisition, noise is easily introduced into the image. For pixels in an image, the introduction of noise often changes the intensity values of some pixels, resulting in a large difference in intensity between those pixels and their neighbors. Aiming at the image segmentation problem of weak edge and different types of noise, this paper proposes a new variational horizontal set noise image segmentation model, which is mainly used for the segmentation of target images of different noises. Experimental results show that the traditional PDE segmentation model method does not have obvious segmentation effect on noisy images, while the segmentation method proposed in this paper that combines PDE and improved variational level set has a better segmentation effect on different types of noisy images.

Keywords: PDE (Partial Differential Equation); Variational horizontal set; symbolic distance function; image segmentation

1. Introduction

In recent years, the image segmentation method^[1-4] Based on partial differential technology^[5-7] is a typical model driven method. Because of its good mathematical plasticity, it has been regarded as a research hotspot by researchers at home and abroad. This method constantly looks for prior information^[8] to match image features in the process of target segmentation. It is a segmentation model integrating prior information. The minimization of energy functional is the basic problem of image segmentation method based on partial differential technology^[2]. Firstly, a closed curve and its energy function^[1] are defined, and the energy functional is minimized by variational method^[6]. In the process of minimization, the driving force moving towards the target area is formed. The driving force is composed of the internal force of contour curve evolution^[8] and the pulling external force contained in image data to drive the contour to move towards the target boundary, Until the energy reaches the minimum, the contour curve of the image reaches the target boundary, and the segmentation ends. Compared with traditional image segmentation methods, the image segmentation method based on variational theory^[9-11] can extract continuous and closed target contour, and combines a lot of prior knowledge to improve the reliability and accuracy of contour extraction. It has been widely used and made great progress in edge detection, noise image segmentation^[12] and motion tracking. At the same time, this method has good human-computer interaction ability, which makes scholars in various fields intervene in the segmentation process and results in specific work fields, obtain more real segmentation results and improve the analysis result data. In recent ten years, researchers around the world have successfully applied it to various industry segments, and its related research work has achieved many valuable research results, which has become a hot field of computer vision and pattern recognition^[13].

With the development of computer technology and the deepening of human understanding of the essence of image, image processing based on partial differential equation^[14] has also achieved gradual and orderly development. The research work in this field can be traced back to keonderink^[15]'s exploration of image structure and Nagao^[16], Rudin^[17] and other research on image smoothing and image enhancement. The use of partial differential equation technology for image processing really began with Witkin, which introduced the theory of scale space. Scale space expressed a group of images on multiple scales at the same time. In their research work, the multi-scale representation of images was completed by Gauss filtering (equivalent to the evolution of images by classical heat conduction equation), This theory has become the basis of partial differential equation^[14] in the application field of image processing.

Aiming at the problem of image segmentation with weak edges and different types of noise, this paper

proposes a segmentation algorithm combining PDE and improved variational level set for noisy images. A constraint energy based on symbolic distance is introduced into the original variational level set model to solve the problem of slow evolution of variational level set function. The new method not only has the segmentation characteristics based on edge gradient information of GAC model [4], but also has the advantages of image segmentation based on global region of C-V model [8]. At the same time, it also combines the method without reinitialization proposed in literature. In the traditional PDE model, the symbolic distance function in the variational level set equation is directly based on the image gradient information. In this paper, a new variational level set function is applied to the energy functional to redefine the symbolic distance function. Experiments show that the new variational level set energy function proposed in this paper can better segment the edge information of different noisy images.

2. Basic knowledge

2.1 Variational Theory

Simply put, when the argument x is a set of numbers, $f(x)$ is called a function; When the argument is a function $u(x)$, $J(u(x))$ is called a functional. That is, the function is the mapping relationship between numbers, while the functional is the mapping relationship between functions and numbers. In order to solve the extreme value problem of this kind of functional, the variational method [26-29] is generally used, which is also called variational principle or variational theory.

Assuming that variables x , $u(x)$ and $u'(x)$ are second-order continuous differentiable in interval $[x_0, x_1]$, the simplest functional can be expressed as follows:

$$J(u(x)) = \int_{x_0}^{x_1} F(x, u(x), u'(x)) dx, \quad (1)$$

Where, J is the universal function and F is the integrand function. By selecting the appropriate integrand function F , the universal function J can obtain the maximum or minimum value. The integrand function F is called Lagrange function. The Lagrange function F here is a function of the derivatives of $u(x)$, $u'(x)$ and even $u(x)$. Since the value of J depends on the form of $u(x)$ in the above integral, J is called the functional of $u(x)$.

Suppose that in the case of one dimension, the universal function is expressed in the following form

$$E(u) = \int_{x_0}^{x_1} F(x, u, u_x) dx. \quad (2)$$

Generally, when the first derivative F' of the function is 0, the corresponding point is the extreme point. Similarly, when the first-order variational $\frac{\partial E}{\partial u}$ of the functional is 0, the corresponding function is the extreme value of the functional. In order to ensure the accuracy of the final result, $u(x) + v(x)$ can be obtained by perturbing $u(x)$.

$$F(x, u + v, u' + v') = F(x, u, u') + \frac{\partial F}{\partial u} v + \frac{\partial F}{\partial u'} v'. \quad (3)$$

Substituting formula (3) into formula (2), we can get the following formula

$$E(u + v) = E(u) + \int_{x_0}^{x_1} \left(\frac{\partial F}{\partial u} v + \frac{\partial F}{\partial u'} v' \right) dx. \quad (4)$$

According to the step-by-step integration method and the value of the perturbation term at the endpoint is 0, it can be obtained

$$\int_{x_0}^{x_1} v \cdot \frac{\partial F}{\partial u} dx = - \int_{x_0}^{x_1} v \frac{d}{dx} \left(\frac{\partial F}{\partial u'} \right) dx. \quad (5)$$

Substituting equation (5) into equation (4), the following equation can be obtained

$$E(u+v) = E(u) + \int_{x_0}^{x_1} \left(\frac{\partial F}{\partial u} v - v \frac{d}{dx} \left(\frac{\partial F}{\partial u'} \right) \right) dx. \quad (6)$$

It can be seen that when $u(x)$ perturbation term $v(x)$ is small enough, its value range will not affect the value of $E(u)$. So there

$$\frac{\partial F}{\partial u} - \frac{d}{dx} \left(\frac{\partial F}{\partial u'} \right) = 0. \quad (7)$$

This equation (7) is called Euler equation of variational problem [30].

Similarly, it can be analogized to two dimensions. After the same derivation process, the Euler equation in two dimensions can be obtained as follows

$$\frac{\partial F}{\partial u} - \frac{d}{dx} \left(\frac{\partial F}{\partial u_x} \right) - \frac{d}{dy} \left(\frac{\partial F}{\partial u_y} \right) = 0. \quad (8)$$

Through the above analysis, it can be concluded that the extreme value solution problem of energy functional $E(u)$ can be transformed into the problem of solving the corresponding partial differential equation ^[1] (PDE), that is, the problem of solving Euler Lagrange equation. However, generally, the calculation process of Euler equation is complex and the space-time complexity is large due to its nonlinear characteristics. Therefore, in order to solve the problem of solving Euler Lagrange equation, a variable "t" about time is introduced to make the initial functional iterate continuously along the opposite direction of the gradient, so as to find the minimum value of the functional. This method is the gradient downward flow equation ^[2].

In the image processing of PDE, in order to obtain the numerical solution of Euler Lagrange equation more quickly and conveniently, the gradient descending flow equation ^[2] can be used to solve it. The time-varying level set function is defined as $u(\cdot, t)$, and the purpose of solution is to reduce $E(u(\cdot, t))$ continuously. Let the perturbation term $v(\cdot)$ of $u(\cdot)$ be the change amount of the function $u(\cdot, t)$ in the process of changing with time t (from time t to $t + \Delta t$), then

$$v = \frac{\partial u}{\partial t} \Delta t. \quad (9)$$

At this point, equation (6) can be rewritten as

$$E(u(\cdot, t + \Delta t)) = E(u(\cdot, t)) + \Delta t \int_{x_0}^{x_1} \frac{\partial u}{\partial t} \left(\frac{\partial F}{\partial u} - \frac{d}{dx} \left(\frac{\partial F}{\partial u'} \right) \right) dx. \quad (10)$$

Well, just make

$$\frac{\partial u}{\partial t} = - \left[\frac{\partial F}{\partial u} - \frac{d}{dx} \left(\frac{\partial F}{\partial u'} \right) \right], \quad (11)$$

It can be guaranteed that $E(u(\cdot, t))$ is decreasing. In equation (11), we call it the gradient descending flow equation of functional (2) ^[31]. At this time, select an appropriate initial function u_0 and start to iterate until it obtains a stable solution. When $\frac{\partial u}{\partial t} = 0$ is found at this time, it is the same as the previous Euler Lagrange equation (7), so the steady-state solution of gradient descending flow equation (11) is the solution of Euler equation.

2.2 Variational level set method

The basic idea of the level set method proposed by Osher and Sethian is to express the low-dimensional evolution curve or surface through a high-dimensional function surface, so as to ensure that the topological structure of the curve or surface can be processed naturally in the evolution process, which has been widely used in image segmentation. The key of the variational level set method is to first establish an energy model, and the internal and external energy of the energy model are expressed by the level set function, and then minimize the energy function by the variational method to obtain the partial differential equation of the evolution of the level set. Compared with the traditional level set image segmentation method driven by pure PDE, the image segmentation method based on variational level set can naturally integrate additional constraint information into the energy function, such as information based on image region, edge or object shape prior knowledge [8], which can produce more robust results. Geodesic active contour (GAC) model [4] is v The model based on curve evolution theory [6] and level set method proposed by caselles et al in 1997 is considered to be a major breakthrough in the application of PDE method [14] in image segmentation.

A curve C can be defined as a two-dimensional function

$$C = \{(x, y) | \phi(x, y) = c_0\}, \quad (12)$$

Among them, function $\phi(x, y)$ is the embedded function of curve C , also known as level set function. When the constant $c_0 = 0$, i.e. $C = \{(x, y) | \phi(x, y) = 0\}$, is called the zero level set of the level set function $\phi(x, y)$.

Since the evolution of curve C leads to the change of position, the closed curve $C(t)$ changing with time can be expressed as a two-dimensional level set function $\phi(x, y)$ changing with time, i.e

$$C(t) = \{(x, y) | \phi(x, y, t) = 0\}. \quad (13)$$

The following level set evolution equation can be obtained by deriving the function $\phi(x, y, t)$ in equation (13) with respect to time t

$$\frac{d\phi}{dt} = \phi_t + \frac{\partial(x, y)}{\partial t} \cdot \nabla \phi = 0, \quad (14)$$

Where, the unit normal vector of $\frac{\partial(x, y)}{\partial t} = \frac{\partial C(t)}{\partial t} = V$ and curve $C(t)$ is $\vec{N} = \frac{\nabla \phi}{|\nabla \phi|}$.

Equation (14) can be written into the following standard level set evolution equation

$$\phi_t = -V_N |\nabla \phi|, \quad (15)$$

Where, the velocity in the normal direction of curve $C(t)$ is $V_N = V \cdot \frac{\nabla \phi}{|\nabla \phi|}$. Equation (15) is the evolution of level set function $\phi(x, y)$ under the condition of given initial value $\phi_0(x, y)$. In other words, at any time, t can determine the current curve $C(t)$ by taking out the level set of $\phi(x, y, t) = 0$.

In the above derivation, the normal adopts the form of positive sign, which means that it has been assumed

$$\begin{cases} \phi(x, y, t) = d(x, y, C(t)), & (x, y) \in \Omega_1(t) \\ \phi(x, y, t) = 0, & (x, y) \in C(t) \\ \phi(x, y, t) = -d(x, y, C(t)), & (x, y) \in \Omega_2(t) \end{cases} \quad (16)$$

Where $\Omega_1(t)$ represents the inside of the closed curve, $\Omega_2(t)$ represents the outside of the closed

curve, and $d(x, y, C(t))$ represents the Euclidean distance from point (x, y) to curve $C(t)$.

The curve evolution problem derived from the minimization of the energy functional of curve $C(t)$ usually includes the following energy function about the length of the curve

$$\min_C \{E(C) = \oint_C ds\}, \quad (17)$$

Where E is the energy functional and ds is the arc length element. Zhao et al. proposed the variational level set method to solve the above energy functional. First, define the Heaviside function $H(z)$ and the Dirac function $\delta(z)$.

$$H(z) = \begin{cases} 1, & z \geq 0 \\ 0, & z < 0 \end{cases}, \delta(z) = \frac{d}{dz} H(z), \quad (18)$$

Dirac function in the formula is the derivative of Heaviside function in the sense of distribution. According to the residual area formula of Heaviside function, the integral on the line on the right of equation (17) can be rewritten as the integral on the following

$$\oint_C ds = \int_{\Omega} |\nabla H(\phi)| dx = \int_{\Omega} |\nabla \phi| \delta(\phi) dx, \quad (19)$$

Where Ω is the entire rectangular image area. Thus, equation (17) can be converted to the energy minimization problem of ϕ

$$\min_{\phi} \{E(\phi)\} = \{\oint_C ds\} = \{\int_{\Omega} |\nabla H(\phi)| dx\} = \{\int_{\Omega} |\nabla \phi| \delta(\phi) dx\}, \quad (20)$$

The gradient descending flow of the level set function ϕ can be obtained by taking the variation of the above formula

$$\frac{\partial \phi}{\partial t} = \text{div} \left(\frac{\nabla \phi}{|\nabla \phi|} \right) \delta(\phi), \quad (21)$$

Where $\text{div}(\cdot)$ is the divergence operator, $\text{div}(\nabla \phi / |\nabla \phi|)$ is the level set expression of curvature $\kappa = -\text{div}(\vec{N})$, κ is the curvature of curve C , and \vec{N} is the unit normal vector of the curve. In actual calculation, we need to approximate $H(\phi)$ and $\delta(\phi)$ with regular $H_{\varepsilon}(\phi)$ and $\delta_{\varepsilon}(\phi)$. Generally, the following two approximate forms can be adopted

$$H_{1\varepsilon}(z) = \begin{cases} 1, & z > \varepsilon \\ \frac{1}{2} \left(1 + \frac{z}{\varepsilon} + \frac{1}{\pi} \sin\left(\frac{\pi z}{\varepsilon}\right) \right), & |z| \leq \varepsilon \\ 0, & z < -\varepsilon \end{cases} \quad (22)$$

$$\delta_{1\varepsilon}(z) = \frac{dH_{1\varepsilon}(z)}{dz} = \begin{cases} 0, & |z| > \varepsilon \\ \frac{1}{2\varepsilon} \left(1 + \cos\left(\frac{\pi z}{\varepsilon}\right) \right), & |z| \leq \varepsilon \end{cases}, \quad (23)$$

Or

$$H_{2\varepsilon}(z) = \frac{1}{2} \left(1 + \frac{2}{\pi} \arctan\left(\frac{z}{\varepsilon}\right) \right), \delta_{2\varepsilon}(z) = \frac{dH_{2\varepsilon}(z)}{dz} = \frac{1}{\pi} \frac{\varepsilon}{\varepsilon^2 + z^2}, \quad (24)$$

Where ε is a small enough normalization parameter. In actual calculation, equation (24) is generally used.

3. Thesis model

In the iterative process of numerical calculation, with the increase of the number of iterations, the level set function will deviate from the symbolic distance function ^[1]. In the traditional image segmentation based on level set method, in order to ensure the stability of GAC model ^[4] and prevent the level set function from being too flat or too steep when close to the edge, the initial level set function usually constrains the curve as a signed distance function, and must be periodically reinitialized in the iterative process of curve evolution, so that the level set function becomes a signed distance function again. If the level set function degenerates, the iterative process of numerical calculation will tend to be unstable, resulting in wrong segmentation results. Reinitializing the level set function into a symbolic distance function requires a large amount of calculation and slow segmentation speed, which will inevitably increase the computational complexity of the model. In order to avoid this situation, reference proposed to add an energy penalty term in the evolution process of level set, so that the level set function maintains an approximate symbolic distance function in the evolution process. The penalty term is expressed as

$$P(\phi) = \int_{\Omega} \frac{1}{2} (|\nabla \phi| - 1)^2 dx dy. \quad (25)$$

The meaning of equation (25) is to control the level set gradient $|\nabla \phi|$ to be stable around 1. When the gradient is large, the energy becomes larger, which will make the level set tend to be smooth and reduce the island of the level set function, so as to maintain the approximate symbolic distance function.

Reference proposed the energy penalty term of double well potential. The general energy penalty term may cause the oscillation of level set function in the evolution process, while the energy penalty term of double well potential will not have too many oscillations. The expression is

$$P_i(\phi) = \begin{cases} \int_{\Omega} \frac{1}{(2\pi)^2} (1 - \cos(2\pi |\nabla \phi|)) dx dy, & |\nabla \phi| \leq 1 \\ \int_{\Omega} \frac{1}{2} (|\nabla \phi| - 1)^2 dx dy, & |\nabla \phi| > 1 \end{cases}. \quad (26)$$

Through the variational method, the gradient descending flow can be obtained from equation (26) as

$$R(\phi) = \begin{cases} \operatorname{div} \left[\frac{\sin(2\pi |\nabla \phi|)}{2\pi |\nabla \phi|} |\nabla \phi| \right], & |\nabla \phi| \leq 1 \\ \operatorname{div} \left[\left(1 - \frac{1}{|\nabla \phi|} \right) \nabla \phi \right], & |\nabla \phi| > 1 \end{cases}. \quad (27)$$

The variational level set does not need to be reinitialized in the evolution process, which makes the level set function close to the target distance function and ensures its stability. Image segmentation is the division of the optimal region and the most significant part in the image. Mastering the overall contour information can ensure the accurate position of the image boundary and improve its accuracy, so as to achieve variational level set segmentation ^[1].

To sum up, a new variational level set segmentation model can be obtained by combining the improved CV model ^[6] with the edge stop function g_L and introducing the double well potential energy penalty term, and its energy functional is

$$\begin{aligned} E(k_1, k_2, \phi) = & \mu \int_{\Omega} g_L H(\phi) |\nabla \phi| dx dy + \nu \int_{\Omega} \delta(\phi) dx dy \\ & + \lambda_1 \int_{\Omega_1} (1 - k_1)^2 H(\phi) dx dy \\ & + \lambda_2 \int_{\Omega_2} (1 - k_2)^2 (1 - H(\phi)) dx dy + P_1(\phi). \end{aligned} \quad (28)$$

The evolution equation of level set obtained by variational method and gradient descending flow method is

$$\begin{cases} \frac{\partial \phi}{\partial t} \delta(\phi) \left[\mu \operatorname{div} \left(g_L \frac{\nabla \phi}{|\nabla \phi|} \right) \right] - \lambda_1 (1 - k_1)^2 + \lambda_2 (1 - k_2)^2 - v + R(\phi) \\ \phi(0, x, y) = \phi_0(x, y) \end{cases} \quad (29)$$

4. Experimental results and analysis

4.1 experimental environment and parameter setting

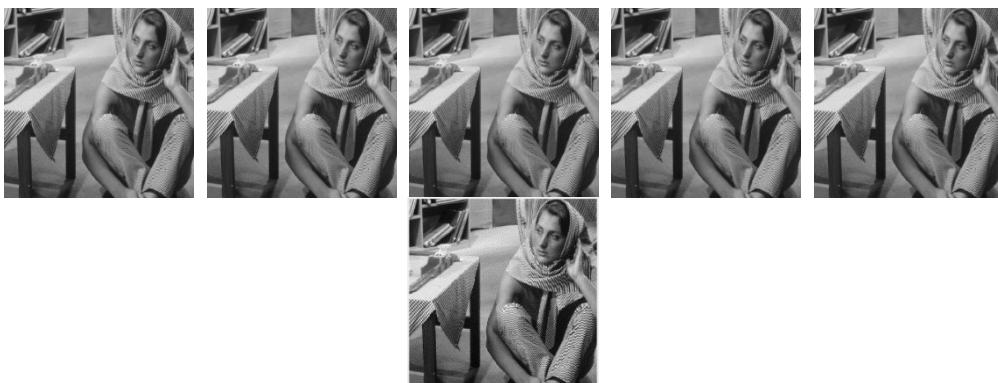
In order to verify the robustness of the image segmentation algorithm combined with PDE and the improved variational level set evolution to noise and the effectiveness of segmenting noisy images, verification experiments will be carried out on real images, synthetic images and noisy images with different noise types, and compared with the experiment of traditional CV model. All experiments are carried out on a computer with Intel Core i5-4200h CPU and windows10 operating system. The experimental simulation software adopts MATLAB r2020a, which ensures the consistency of the comparative experiment in the software and hardware environment.

Next, we will analyze and compare the classical mean filter, Gaussian filter, median filter and the model algorithm created in this paper, and compare the noise resistance, peak signal-to-noise ratio (PSNR) and accuracy, as shown in Figure 1.



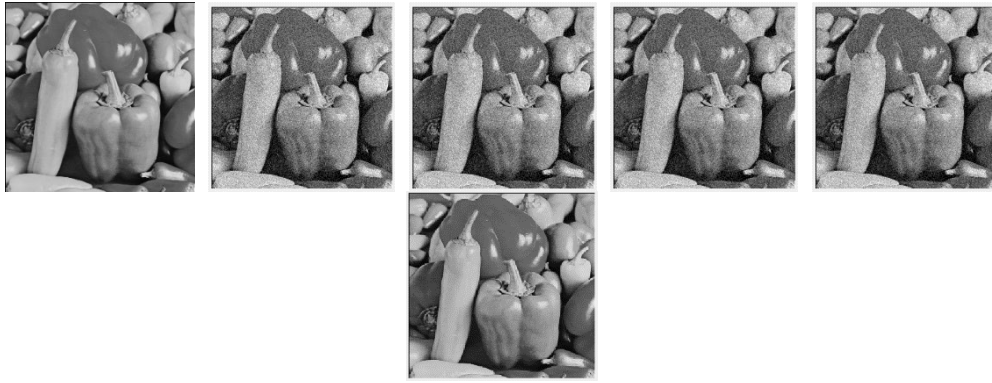
(a)Original image (b) Noisy image (c) Mean filtering (d) Gaussian filtering (e) Median filtering (f) Paper method

Figure 1.1 Comparison of Lena filtering processing



(a) Original image (b) Noisy image (c) Mean filtering (d) Gaussian filtering (e) Median filtering (f) Paper method

Figure 1.2: Comparison of filtering processing of Barbara



(a)Original image (b) Noisy image (c) Mean filtering (d) Gaussian filtering (e) Median filtering (f) Paper method

Figure 1.3: Comparison of filtering processing of peppers256

Figure 1: Comparison of filtering processing of three kinds of images

Calculate the PSNR (objective evaluation standard of image denoising) corresponding to various processing methods, and the corresponding PSNR results are shown in Table 1.

Table 1: Comparison of four noise reduction results of image denoising

| Denoising method | Mean filtering | Gaussian filtering | Median filtering | Paper method |
|-----------------------------|----------------|--------------------|------------------|--------------|
| PSNR/DB value of Lena | 25.0735 | 19.9436 | 27.0935 | 28.1324 |
| PSNR/DB value of Barbara | 25.0756 | 19.9412 | 27.0924 | 28.0125 |
| PSNR/DB value of peppers256 | 27.0746 | 19.9435 | 27.0912 | 28.1056 |

From the above figure of filtering processing results and the comparison table of four denoising results of image denoising, it can be concluded that under the constraints of experimental conditions and the selected denoising method in this paper, the denoising effect of median filter and the model algorithm created in this paper is relatively good. It can be seen that the image processed by this algorithm is clearer and better, unlike the image blurring caused by mean and Gaussian filter, The edge and detail information of things in the picture are lost, so the algorithm in this paper can better resist interference in the removal of noise. The values in the comparison table of the four denoising results of image denoising also confirm this point. Compared with the other three methods, the PSNR value of median filtering is larger. For this objective evaluation standard, the larger the PSNR value, the better the image processing effect of this method.

4.2 Performance comparison experiment of different algorithms

The segmentation results of the improved model algorithm are evaluated based on the GT (ground truth) image. The evaluation indicators include the accuracy (segmentation accuracy, SA), over segmentation rate (OR), under segmentation rate^[12] (UR) and running time(T) of the segmented image.

4.2.1 Segmentation accuracy SA

The segmentation accuracy SA is the percentage of the accurately segmented area in the real area of the GT image, and its expression is

$$SA = \left[1 - \left(\frac{|R_s - T_s|}{R_s} \right) \right] \times 100\%, \quad (30)$$

Wherein, R_s represents the reference region of the segmented image artificially calibrated; T_s represents the actual area of the image segmented by the algorithm; $|R_s - T_s|$ represents the number of pixels incorrectly segmented.

4.2.2 Over segmentation rate *OR*

The over segmentation rate *OR* is the ratio of pixels divided outside the reference area of the GT image, and its expression is

$$OR = \frac{O_s}{R_s + O_s}, \quad (31)$$

Among them, O_s refers to the number of pixels that should not be included in the segmentation result, but are actually in the segmentation result .

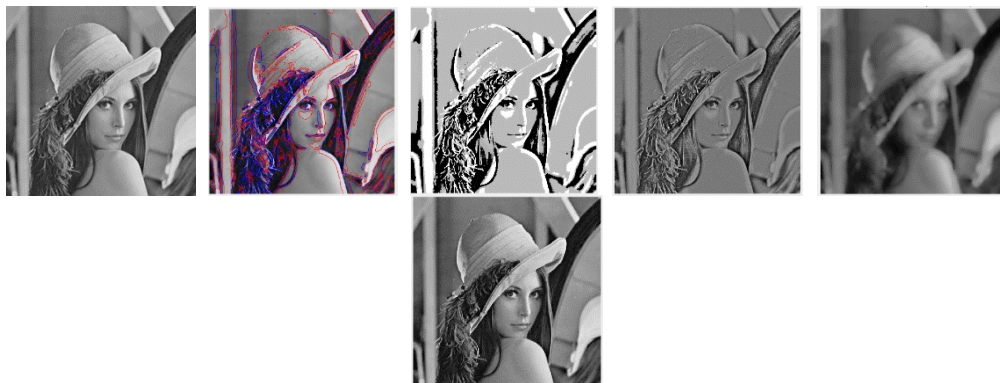
3.3.3 under split rate *UR*

Under segmentation rate *UR* is the ratio of missing pixels in the reference area of GT image, and its expression is

$$UR = \frac{U_s}{R_s + O_s}, \quad (32)$$

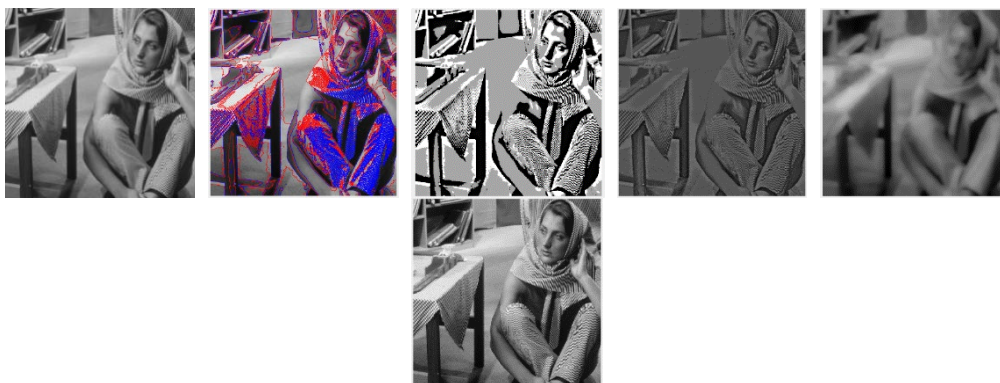
Where U_s represents the number of pixels that should be included in the segmentation result but are not included in the segmentation result .

In Fig. 2, Lena, Barbara and pepper 256 images are selected for comparative experiments of different methods, and the segmentation results are evaluated.



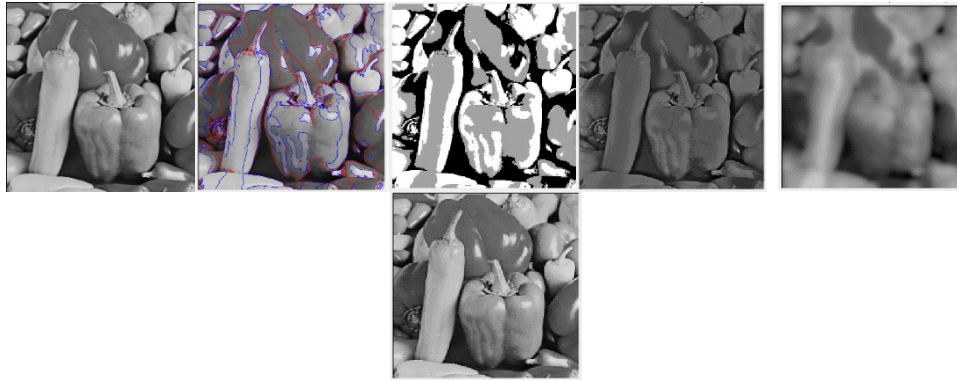
(a) Original image (b) GAC method (c) improved PDE method (d) Level set method (e) Variational level set method (f) Paper method

Figure 2.1: Comparative experiments of different Lena algorithms



(a) Original image (b) GAC method (c) improved PDE method (d) Level set method (e) Variational level set method (f) Paper method

Figure 2.2: Comparative experiments of different Barbara algorithms



a) Original image (b) GAC method(c) improved PDE method (d)Level set method (e) Variational level set method (f) Paper method

Figure 2.3: Comparative experiments of different Peppers256 algorithms

Figure 2: Comparison Experiment of different algorithms of three images

As can be seen from Figure. 2, the GAC method has poor processing effect on the edge part of the image. Although the improved PDE method and level set method can segment the target image well, the over segmentation rate and under segmentation rate are also very high. When the image contains a lot of noise and the background is complex, the algorithm is difficult to segment the background image and the target image correctly. The method proposed in this paper can segment the target image more effectively, process the details of the image edge effectively, and has a good segmentation effect for the image with uneven regional gray level or small difference between the image boundary and the background image. Table 2 lists the experimental data of image SA , OR , UR and T .

Table 2: Comparison of segmentation effects of three image segmentation methods

Table 2.1: Comparison of segmentation effects of different methods of Lena image

| method | SA | OR | UR | T |
|------------------------------|--------|--------|--------|--------|
| GAC method | 0.5431 | 0.3718 | 0.0949 | 18.153 |
| Improved PDE method | 0.8102 | 0.3217 | 0.3317 | 14.256 |
| Level set method | 0.8123 | 0.2245 | 0.2245 | 10.420 |
| Variational level set method | 0.5021 | 0.3518 | 0.2517 | 7.593 |
| Paper method | 0.9312 | 0.1324 | 0.1324 | 4.836 |

Table 2.2: Comparison of segmentation effects of different methods of Barbara image

| method | SA | OR | UR | T |
|------------------------------|--------|--------|--------|--------|
| GAC method | 0.5541 | 0.3808 | 0.0967 | 18.073 |
| Improved PDE method | 0.8262 | 0.3127 | 0.3317 | 14.546 |
| Level set method | 0.8273 | 0.2249 | 0.2249 | 10.529 |
| Variational level set method | 0.5161 | 0.3518 | 0.2607 | 7.794 |
| Paper method | 0.9422 | 0.1368 | 0.1368 | 4.753 |

Table 2.3: Comparison of segmentation effects of different methods of Peppers256 image

| method | SA | OR | UR | T |
|------------------------------|--------|--------|--------|--------|
| GAC method | 0.5431 | 0.3718 | 0.0935 | 18.170 |
| Improved PDE method | 0.8392 | 0.3217 | 0.3527 | 14.946 |
| Level set method | 0.8653 | 0.2495 | 0.2495 | 10.780 |
| Variational level set method | 0.5901 | 0.3908 | 0.2647 | 7.593 |
| Paper method | 0.9472 | 0.1395 | 0.1395 | 4.952 |

As can be seen from Figure 2 and table 2, for the image segmentation effect, the algorithm proposed in this paper shows better segmentation results, the segmentation accuracy is significantly higher than other methods, and the over segmentation rate and under segmentation rate are also smaller. GAC model

evolves based on regional information, but the effect of this model on noise suppression is relatively poor. However, the algorithm proposed in this paper adopts image-based regional evolution, which has little impact on noise points. By comparing the running time of different algorithms, it can also be seen that the efficiency of this algorithm is higher. This is because the symbolic distance penalty term and double well potential energy penalty term are added to the algorithm in this paper, which avoids the reinitialization of the variational level set function, and improves the image segmentation accuracy and efficiency.

5. Conclusion

Aiming at the variational level set model, a new noisy image segmentation algorithm combining PDE and improved variational level set is proposed in this paper. The algorithm combines the variable potential set and the variable potential set to improve the accuracy of image segmentation, and improves the robustness of the image segmentation algorithm by avoiding the initial penalty level set of the variable potential set and the variable potential set. The experimental results show that compared with the traditional level set model, the algorithm proposed in this paper can get better segmentation results in a shorter time, reduce boundary leakage and shorten the experimental time to a certain extent. However, the parameter selection and adjustment in the segmentation algorithm in this paper are still based on previous experience. Therefore, the next research work will be how to use the image information and machine learning technology to obtain the parameters adaptively.

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