

Study and Optimization of Core Loss Characteristics of Magnetic Components Based on Multi-model Algorithm

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Abstract: In this paper, decision tree, ANOVA, least squares and other modeling algorithms are used to study the core loss of magnetic components. For the excitation waveform classification, the decision tree model is used to realize high-precision classification based on the extracted waveform features; in the correction of the Steinmetz equation, the temperature factor is added to construct three kinds of correction equations, and the optimal correction equations are selected by the least squares method to improve the loss prediction accuracy; the one-factor and two-factor ANOVA models are used to investigate the effects of temperature, excitation waveform, and core material on the magnetic core loss, and the independent and synergistic effects of each factor are clarified. The effects of temperature, excitation waveform and core material on core loss are investigated using one-factor and two-factor ANOVA models, and the independent and synergistic effects of each factor are clarified. The research results provide a basis for the in-depth understanding of the core loss mechanism, and also lay a foundation for the design and optimization of magnetic components, which will help to improve the performance of power converters.

Keywords: Decision Trees, Least Squares, ANOVA

1. Introduction

In power conversion systems, the core loss of magnetic components has a significant impact on system performance [1]. However, the traditional core loss models are difficult to meet the practical needs because they cannot accurately consider the effects of multiple complex factors. In this paper, we aim to fill this gap and conduct an in-depth study using various advanced modeling algorithms. The decision tree algorithm is used to classify the excitation waveforms and mine the waveform features to achieve accurate identification; the least squares method is widely used in solving the equation parameters to provide strong support for model optimization; and the ANOVA model is used to analyze the influence mechanism of various factors on the core loss. Through these modeling algorithms, the core loss characteristics are explored comprehensively and deeply to provide a scientific basis for the optimal design of magnetic components and to promote the development of power electronics technology in the direction of high efficiency and energy saving, which is of great theoretical and practical significance in the research of power electronics field.

2. Excitation waveform classification based on decision tree modeling

First, an image of the flux density of each waveform over time is plotted based on the flux density values in one cycle. The features of each image are observed, and the feature variables of different waveforms are extracted from them. Next, the extracted feature variables are used to build a classification model on the training set, and the data in the validation set are substituted into the classification model, and the evaluation indexes such as accuracy, precision as well as confusion matrix are used to judge the effectiveness of the model. The data in the test set are then substituted into the classification model to predict the type of waveform to which each set of data belongs. Finally, the flux density over time images of some data in the test set are drawn, and the reasonableness of the model is checked against the images and compared with the prediction results.

Step 1: Draw an image of the flux density of each waveform over time, as shown in Figures 1, 2 and 3 below:

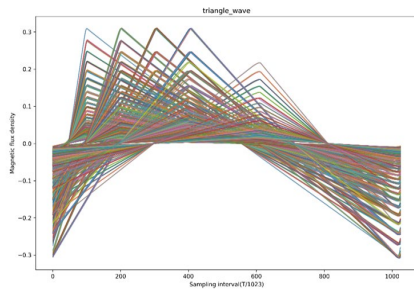


Figure 1. Sinusoidal wave flux density over time image

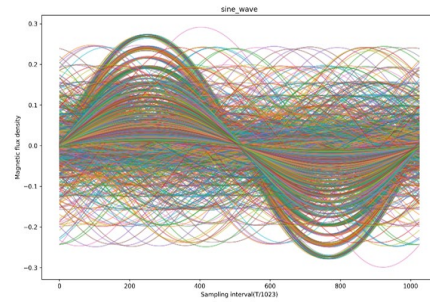


Figure 2. Triangular wave flux density image over time

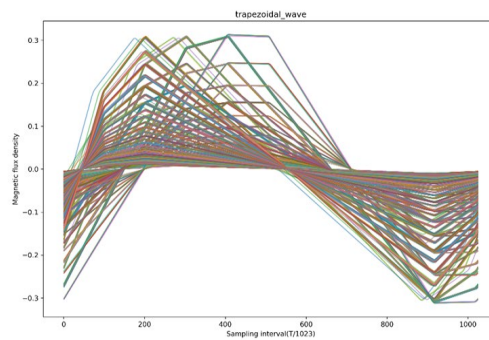


Figure 3. Trapezoidal wave flux density image over time

In the above image, the horizontal coordinate indicates the n th sampling point, and the vertical coordinate indicates the magnetic flux density corresponding to each sampling point. Based on the above image, the shape characteristics of various waveforms are analyzed. Figure 1 demonstrates the superposition of multiple sinusoidal waves with the following characteristics: the lines in the image show the periodic characteristics of the sinusoidal waveforms, the regularity of the shapes of the peaks and valleys is revealed, and in general the waveforms have symmetry in the horizontal direction, with symmetrical distributions of the upper and lower peaks and valleys. It indicates that the magnetic flux density changes periodically with time in a sinusoidal waveform. Figure 2 demonstrates a series of triangular waveforms with the following characteristics: the lines in the image have sharp waveforms, the peaks and valleys show clear linear changes, and the magnetic flux density rises and falls in a linear manner with time, with a gradual transition at the peak, showing a typical triangular waveform shape with continuous, equal-amplitude rising and falling phases. Figure 3 illustrates the superposition of multiple trapezoidal waves with the following characteristics: the lines in the image are relatively smooth, the top and bottom of the waveform are flat and exhibit some lag time, and the rising and falling portions are slightly steeper but not as sharp as a triangular wave. The magnetic flux density is also distributed in a linear manner of rise or fall with time, but there will be a section at the top and bottom that changes more slowly.

Step 2: Extract the variables that characterize the shape of the waveform and use the variables to build a model.

Five characteristic variables, mean, variance, extreme value, kurtosis and skewness, are extracted from the graph in the first step, and these five characteristic variables are utilized to build a classification model with the help of a decision tree.

The decision tree consists of nodes and directed edges, and the types of nodes include the root node at the beginning, the internal nodes in the middle, and the leaf nodes as the final output [2,3]. The root node is the starting point of the tree and contains the entire data set. The algorithm proceeds step by step through recursion in constructing the decision tree. At each recursion, the algorithm divides the dataset

based on the current optimal features or attributes to ensure that the split subset has a smaller error [4]. This process is repeated until the model reaches an optimal state.

With the help of python language, the specific data values of five feature variables, namely mean, variance, extreme value, kurtosis and skewness, were calculated for each group of magnetic flux density data, and then the classification model was built using decision tree, while the feature importance diagram was drawn as shown in Figure 4.

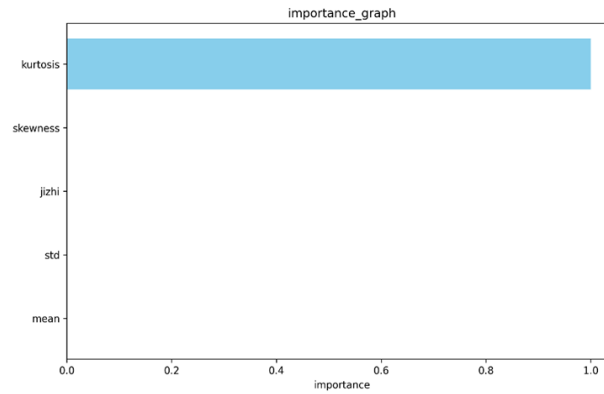


Figure 4. Feature importance diagram

As can be seen in Figure 4, the importance of kurtosis is 1, and the importance of the rest of the variables is 0. Therefore, only one feature variable, kurtosis, is retained in the subsequent classification models

Step 3: Predicting excitation waveforms using classification models.

The data in the test set were lassoed into the model to predict the excitation waveforms for each set of data, and the respective quantities of the three waves in the statistical test set are shown in Table. 1.

Table. 1. Number of waveforms

Waveform	Quantity
Sine	20
Triangle	44
Trapezoidal	16

The categorization results for the sample numbers: 1, 5, 15, 25, 35, 45, 55, 65, 75, and 80 are shown in Table. 2.

Table. 2. Waveform prediction

No.	Waveform
1	Triangle wave
5	Triangle wave
15	Sine wave
25	Triangle wave
35	Trapezoidal wave
45	Trapezoidal wave
55	Triangle wave
65	Triangle wave
75	Triangle wave
80	Sine wave

Step 4: Analyze the validity and reasonableness of the classification model.

The data in the validation set are applied to the model, and the excitation waveforms of each set of data are predicted, and the comparison with the actual values is made to obtain the confusion matrix as shown in Table.3.

Table. 3. Confusion matrix

		Actual value		
		Sine wave	Triangle wave	Trapezoidal wave
Predicted value	Sine wave	797	0	0
	Triangle wave	0	977	0
	Trapezoidal wave	0	0	706

The resulting accuracy is: $Accuracy = \frac{797 + 977 + 706}{797 + 0 + 0 + 0 + 977 + 0 + 0 + 0 + 706} = 100\%$. It shows that it is extremely easy to separate the waveforms of various excitations from the characteristic variable of kurtosis, and also shows the validity of the model.

In addition to this, in order to check the reasonableness of the model, the images of magnetic flux density with respect to time for the data sets in the test set with serial numbers 1, 5, 15, 25, 35, 45, 55, 65, 75, and 80 are plotted as shown in Figure 5.

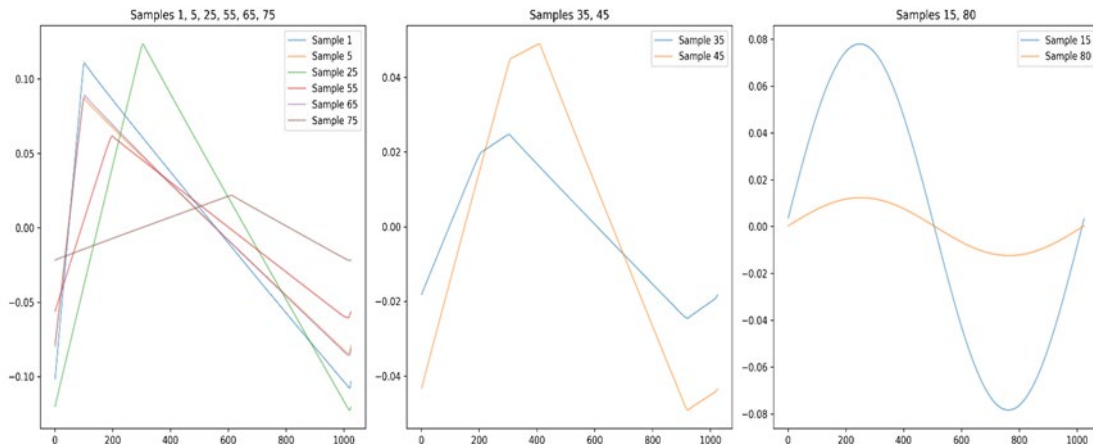


Figure 5. Magnetic flux variation with time

It can be found that in the above images, the images with serial numbers 15, 80 basically satisfy the characteristics of sine wave images, the images with serial numbers 1, 5, 25, 55, 65, 75 basically satisfy the characteristics of triangle wave images, and the images with serial numbers 35, 45 basically satisfy the characteristics of trapezoidal wave images. Meanwhile comparing the prediction results of the classification model, it is consistent with the graphical comparison results, therefore, it shows that the classification model is very reasonable.

3. Least squares based correction and analysis of the Steinmetz equation

Firstly, based on the original Steinmetz equation, the factor of temperature was added, and the original equation was modified according to three ways; secondly, a model was built according to the three modified equations, and parameter estimation was carried out by using the least squares estimation [5], and at the same time, the advantages and disadvantages of the three models were compared, and the optimal model was selected as the final modified equation; finally, the effects of the final modified equation were compared with those predicted by the original Steinmetz equation.

3.1 Model building

First, the equation is corrected according to the following three methods:

Method 1: $P = k_1 f^{\alpha_1} B_m^{\beta_1} T^{\gamma_1}$.

Method 2: $P = k_1 f^{\alpha_1} B_m^{\beta_1} e^{\gamma_1 T}$.

Method 3: After reviewing, it is understood that the temperature change has a more significant effect on the coefficient k_1 in the Steinmetz equation, and the effect on the parameters α_1 and β_1 is not so significant, therefore, the equation is corrected as follows:

$$P = k_0 \left(1 + C_1 T + C_2 T^2 \right) f^{\alpha_1} B_m^{\beta_1}. \quad (1)$$

For the modified equations in Methods 1 and 2, the values of the parameters k_1 , α_1 , and β_1 are estimated using least-squares estimation, and thus the relationship between the core loss P and the frequency f , the maximum flux density B_m , and the temperature T is obtained. For method 3, it is assumed that the parameters α_1 and β_1 are not affected by the temperature, therefore, firstly, the parameters α_1 and β_1 in the original Steinmetz equation $P = k_1 f^{\alpha_1} B_m^{\beta_1}$ are estimated using the least-squares estimation, and then the obtained values are substituted into the modified equation of method 3, which is obtained as $\frac{P}{f^{\alpha_1} B_m^{\beta_1}} = k_0 \left(1 + C_1 T + C_2 T^2 \right)$, and then the parameters C_1 and C_2 are estimated using the least squares estimation, and the final model can be obtained.

3.2 Model solving and analysis

Least squares estimation of a multiple linear regression model:

Assuming that there are k factors affecting the dependent variable y , we obtain n sets of observations $(y_i, x_{i1}, \dots, x_{ik})$, $i = 1, 2, \dots, n$, for the regression model:

$$y = X\beta + \varepsilon \quad (2)$$

Find the least squares estimate $\hat{\beta}$ of the parameter vector β . This method is to find an estimate of β that minimizes the square of the length $y - X\beta$ of the deviation vector $\varepsilon = y - X\beta$.

$$Q(\beta) = y - X\beta^2 = (y - X\beta)'(y - X\beta) \quad (3)$$

Expand this equation as:

$$Q(\beta) = y'y - 2y'X\beta + \beta'X'X\beta \quad (4)$$

Taking the partial derivative of β and ordering it to be zero yields the regular equation:

$$X'X\hat{\beta} = X'y \quad (5)$$

A sufficient condition for this system of linear equations to have a unique solution is that $X'X$ has rank $k + 1$, and we always assume that this condition holds. Thus, we get the unique solution:

$$\hat{\beta} = (X'X)^{-1} X'y \quad (6)$$

According to the extreme value theory of calculus, $\hat{\beta}$ is just a stationary point of the function $Q(\beta)$. We also need to show that $\hat{\beta}$ does minimize $Q(\beta)$. In fact, for any β , there is,

$$y - X\beta^2 = y - X\hat{\beta} + X\hat{\beta} - X\beta^2 = y - X\hat{\beta}^2 + (\hat{\beta} - \beta)'X'X(\hat{\beta} - \beta) + 2\hat{\beta} - \beta)'X'(y - X\hat{\beta}), \quad (7)$$

Since $\hat{\beta}$ satisfies the regular equation, it follows that $X'(y - X\hat{\beta}) = 0$, and thus the third term of the above equation is equal to 0. This proves that for any β , there is:

$$y - X\beta^2 = y - X\hat{\beta}^2 + (\hat{\beta} - \beta)' X'X(\hat{\beta} - \beta), \quad (8)$$

And that the third term of the above equation equals 0.

Again, since $X'X$ is a positive definite array, the second term of the above equation is always non-negative, and so:

$$Q(\beta) = y - X\beta^2 \geq y - X\hat{\beta}^2 = Q(\hat{\beta}). \quad (9)$$

The equal sign holds if and only if:

$$(\hat{\beta} - \beta)' X'X(\hat{\beta} - \beta) = 0, \quad (10)$$

Therefore, the least squares estimate of the parameter β is:

$$\hat{\beta} = (X'X)^{-1} X'y. \quad (11)$$

Moreover, the least squares estimate has many excellent properties, such as linear property, unbiasedness, and validity, where validity means that among all linear unbiased estimates, the least squares estimate is the only one that has the smallest variance. Therefore, in this paper, least squares estimation is used to fit the solution model.

Before solving the model, it is necessary to linearize the three modified equations. Taking the first modified equation as an example, it is necessary to take logarithms of the left and right sides of the equal sign at the same time to obtain $\ln(P) = \ln(k_1) + \alpha_1 \ln(f) + \beta_1 \ln(B_m) + \gamma_1 \ln(T)$, and then take logarithms of the variables P, f, B_m , and T in the original data, and utilize least squares estimation to find the values of the parameters, and then obtain the final model.

Before fitting the least squares estimation, a test for multicollinearity was carried out on the three variables and it was found that the VIF values were less than 10, indicating that there is no multicollinearity in these three variables.

In this section, the least squares estimation was carried out with the help of python to obtain the final model of Steinmetz equation and the three modified equations and respectively:

$$P = 0.16419 f^{1.59183} B_m^{2.50598} \quad P = 1.27009 f^{1.65209} B_m^{2.52534} T^{-0.59870} \\ P = 0.23667 f^{1.65232} B_m^{2.52794} e^{-0.01175T} \quad P = (0.54052 - 0.0084T + 0.00001T^2) f^{1.59183} B_m^{2.50598}.$$

And find the R^2 of the four models on the training set as 0.93701; 0.96719; 0.97416; 0.95694.

The R^2 of the four models on the validation set are: 0.93900; 0.98145; 0.98926; 0.96857.

The analysis of variance (ANOVA) using python yielded residual plots and Q-Q plots for the Steinmetz equation and the three modified equations as shown in Figure 6, Figure 7, Figure 8 and Figure 9, respectively.

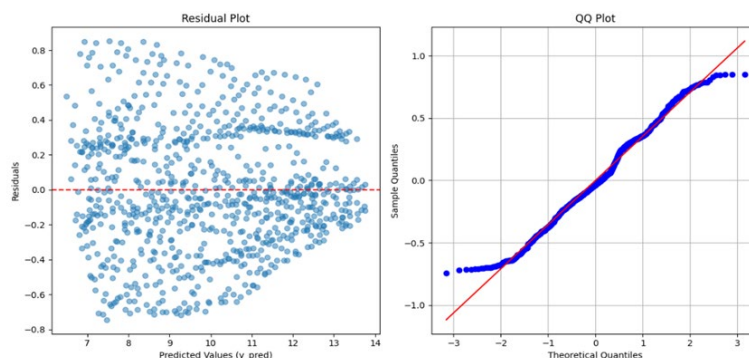


Figure 6. Residual and Q-Q plots for the Steinmetz equation

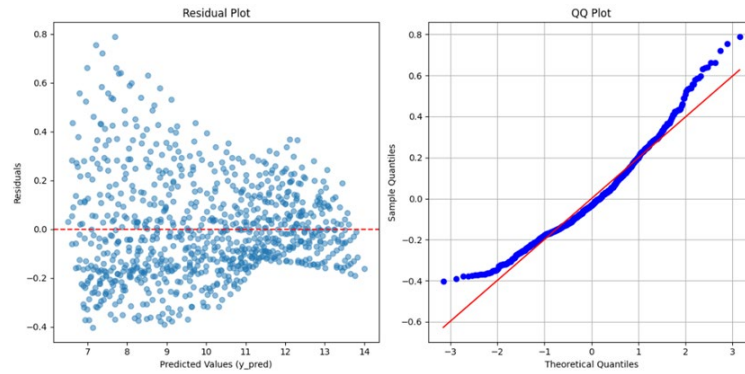


Figure 7. Residual and Q-Q plots for method 1 modified equation

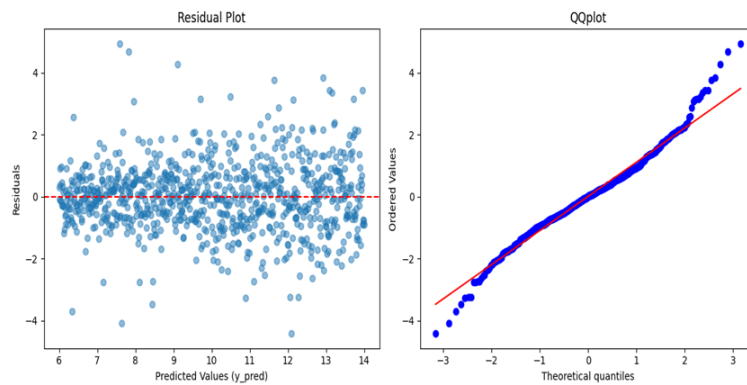


Figure 8. Residuals and Q-Q plots for method 2 modified equation

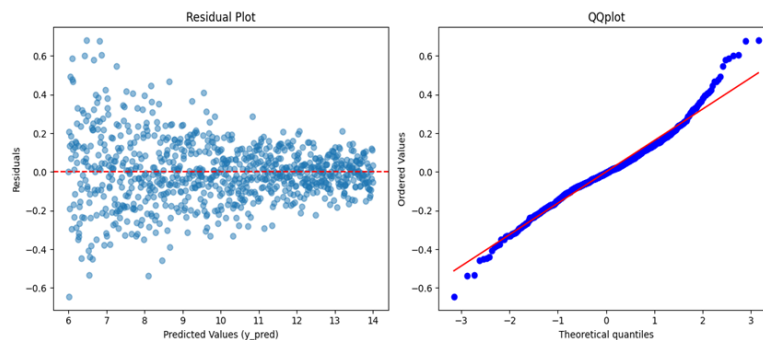


Figure 9. Residuals and Q-Q plots for method 3 modified equation

Combining the R^2 and ANOVA results of correcting the three models, it was found that Method 2 had the largest R^2 and the best residual plots and Q-Q plots on both the training and validation sets, and thus the corrected equation of Method 2 was finally chosen as the final temperature correction equation.

Comparing the corrected equation with the Steinmetz equation, it is found that the R^2 of the former is larger than that of the latter, and the residual and Q-Q plots of the former are also better than that of the latter, so the constructed corrected equation predicts the core loss better than the Steinmetz equation.

4. Exploration of core loss factors based on ANOVA modeling

A one-way ANOVA model is used to analyze how the three factors of temperature, excitation waveform and excitation material independently affect the core loss; a two-way ANOVA with an interaction effect scenario is used to explore how the temperature, excitation waveform and excitation material synergistically affect the core loss; and the effect of each factor is solved using these two models. Finally, the two-by-two interaction diagrams of the three factors are drawn to find out the conditions under which the core loss may be minimized.

4.1 Modeling

One-way ANOVA is a statistical method used to compare the differences between the means of multiple groups [6]. The specific modeling process is as follows:

Hypothesis:

Original hypothesis (H_0): the means of all groups are equal, i.e., $\mu_1 = \mu_2 = \dots = \mu_k$.

Alternative hypothesis (H_a): at least one group has different means.

Model construction and expression: the one-way ANOVA model can be expressed as:

$$Y_{ij} = \mu + \alpha_i + \epsilon_{ij}. \quad (12)$$

Where Y_{ij} denotes the j th observation of the i th group, μ denotes the overall mean, α_i denotes the effect of the i th group (i.e., it denotes the degree of influence on the response variable), and ϵ_{ij} is the error term, which is assumed to be independent and to follow the normal distribution, with a mean of 0 and a variance of σ^2 .

Two-way ANOVA is used to test the effect of two independent variables on a response variable and the interaction between them. The specific modeling process is as follows:

Hypothesis:

Original hypothesis (H_0):

The means of independent variable A are equal, i.e., $\mu_{A1} = \mu_{A2} = \dots = \mu_{Ak}$. Means of independent variable B are equal, i.e. $\mu_{B1} = \mu_{B2} = \dots = \mu_{Bk}$. There is no interaction between independent variables A and B .

Alternative hypothesis (H_a):

At least one set of independent variables A has different means. At least one set of independent variables B has different means. There is an interaction between independent variables A and B .

Model construction and expression: the two-factor ANOVA model can be expressed as:

$$Y_{ijk} = \mu + \alpha_i + \beta_j + (\alpha\beta)_{ij} + \epsilon_{ijk}. \quad (13)$$

Where Y_{ijk} denotes the k th observation at the j th level of the i th group, μ denotes the overall mean, α_i denotes the effect of independent variable A (i.e., the extent to which independent variable A influences the response variable), β_j denotes the effect of independent variable B (i.e., the extent to which independent variable B influences the response variable), and $\alpha\beta_{ij}$ denotes the effect of the interaction between independent variables A and B (i.e., the extent to which independent variables A and B synergistically influence the response variable). B synergistically on the response variable), ϵ_{ijk} is the error term, which is assumed to be independent and normally distributed with mean 0 and variance σ^2 .

4.2 Results and analysis

In this paper, the aov function in R language is utilized to perform the ANOVA on the data in Material 1 and the eta_squared function is utilized to quantify the degree of influence of the variables and the following results are obtained:

The one-way ANOVA of temperature on core loss yields the ANOVA in Table. 4.

Table. 4. One-way ANOVA of temperature on core loss

Source of variance	Sum of squares	Degree of freedom	Mean square	F-value	p-value
Different temperatures	7.513×10^{12}	3	2.504×10^{12}	17.75	1.7×10^{-11}
Error	1.748×10^{15}	12396	1.411×10^{11}		
Total	1.756×10^{15}	12399			

It can be seen that the required p-value is less than the significance level of 0.05 and the original hypothesis is rejected, thus indicating that different temperatures produce different levels of effect on the magnitude of core losses.

The one-way ANOVA analysis of excitation waveform on core loss yields the ANOVA in Table .5.

Table. 5. One-way ANOVA of excitation waveform on core loss

Source of variance	Sum of squares	Degree of freedom	Mean square	F-value	p-value
Different wave	7.52×10^{13}	2	3.76×10^{13}	277.3	$< 2 \times 10^{-16}$
Error	1.681×10^{15}	12397	1.356×10^{11}		
Total	1.756×10^{15}	12399			

It can be seen that the required p-value is less than the significance level of 0.05 and the original hypothesis is rejected, thus indicating that the different excitation waveforms have different degrees of influence on the magnitude of core losses.

The one-way ANOVA analysis of core material on core loss yields the ANOVA in Table. 6.

Table. 6. One-way ANOVA of core material and core loss

Source of variance	Sum of squares	Degree of freedom	Mean square	F-value	p-value
Different core materials	4.114×10^{13}	3	1.371×10^{13}	99.12	$< 2 \times 10^{-16}$
Error	1.715×10^{15}	12396	1.383×10^{11}		
Total	1.756×10^{15}	12399			

It can be seen that the required p-value is less than the significance level of 0.05 and the original hypothesis is rejected, thus indicating that different core materials have different degrees of influence on the magnitude of core losses.

A two-factor ANOVA was performed on the temperature and excitation waveform to obtain the ANOVA table and the quantitative results of the degree of influence, and the two-by-two interaction plots of the three factors were drawn as shown in Figure 10, Figure 11 and Figure 12.

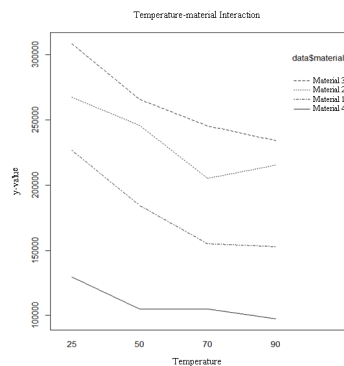


Figure 10. Interaction of temperature with material

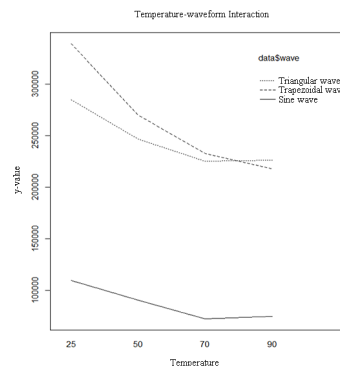


Figure 11. Interaction of temperature with waveforms

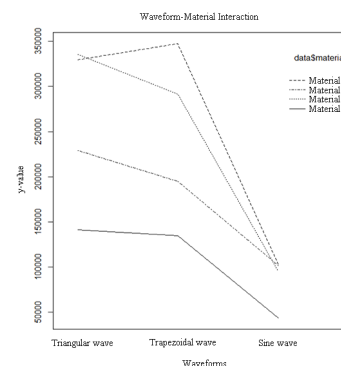


Figure 12. Interaction of waveform with material

From the three graphs, it can be seen that the core loss may be minimized when the temperature is taken to be 90 degrees Celsius, the core material is taken to be material IV, and the excitation waveform is taken to be sinusoidal.

5. Conclusions

In this study, the core loss of magnetic components is systematically analyzed by means of various modeling algorithms. The decision tree model performs well in the excitation waveform classification task, and with the extracted features such as mean and kurtosis, it can effectively differentiate between sinusoidal, triangular, and trapezoidal waveforms, and the model is highly accurate and reasonably reliable. Regarding the correction of the Steinmetz equation, the three correction equations constructed by adding the temperature factor are solved and compared by the least-squares method to determine the optimal equation, which significantly improves the prediction accuracy of the core loss at different temperatures. Using the ANOVA model, it is found that temperature, excitation waveform and core material all have a significant effect on the core loss, and there is an interaction between some of the factors. Taken together, these modeling algorithms provide an effective means to study core losses. In the future, the study can be further expanded to incorporate more influencing factors and improve the model, so as to provide more accurate support for the optimal design of magnetic components and the efficient operation of power electronic systems.

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