Analysis of China's population growth forecast based on Leslie model under the three-child policy

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Abstract: *The number and structure of the population are important factors affecting socio-economic development. China has experienced the implementation of "family planning", "comprehensive two-child" and "liberalized three-child" policies, which are all adjustments to the trend of China's population development. In this paper, we establish a mathematical model by combining relevant data. This paper establishes a mathematical model to predict the demographic situation of China in the next 10 years after the opening of the three-child policy, considering the age structure of China's population. Based on this, we analyze how to implement the new policies in medical care to further alleviate the aging process of China's population, considering the actual situation at present.*

Keywords: *Leslie model; Population projection; Aging; Python.*

1. Introduction

The number and structure of the population are important factors affecting socio-economic development. China has experienced "family planning" to "comprehensive two-child", and then "liberalization of three children" policy, which is the adjustment of China's population development trend.

The first step is to analyze the statistical characteristics of China's population in recent years through python programming, and to calculate the total mortality rate $\alpha(t)$ and total fertility rate $\beta(t)$ based on historical demographic data, and the impact of the "three-child" policy. By defining the regulation effect γ to change the total fertility rate $\beta(t)$ and constructing the Leslie model, we can predict the age structure and total population of China in the next 10 years.

Based on this premise, by analyzing the mortality rate of each age group by gender and the medical expenditure of each age group in recent years, we can reduce the mortality rate of this age group by strengthening the medical coverage for children and young and middle-aged people, so as to increase the proportion of non-elderly population and alleviate the increasingly serious population aging in China. The problem of population aging in China is becoming increasingly serious.

2. Assumptions and notations

We use the following assumptions.

● Assume that our population is a closed system and does not consider mutual migration with foreign countries.

● Assume that changes in the sex ratio over time in the next 10 years are not considered.

● Assume that the total fertility rate of our country does not exceed 3 in the next 10 years.

● Assume that the mortality pattern is independent of the year.

● Assume that women have only one child per birth.

The primary notations used in this paper are listed as Table 1.

Table 1: Notations

Symbols	Meaning
$x_i(t)$	Total number of people aged i in year t
$b_i(t)$	Fertility rate of women aged i in year t
k_i	Proportion of females aged i in the total population
h_i	Number of children born to women aged i as a percentage of women of childbearing age
$d_i(t)$	Mortality rate of the population aged i in year t
g_i	Mortality rate of the population aged i as a proportion of the mortality rate of all age groups
$\alpha(t)$	Total mortality in year t
$\beta(t)$	Total fertility rate in year t
$\beta'(t)$	Total fertility rate under the "double reduction" policy
β_0	Total fertility rate in the initial year
C	Constants Matrix
D	Death Matrix
B	Birth Matrix
γ	The regulatory effect of the "three-child" policy
θ	The regulatory effect of the "double reduction" policy
$N(t)$	Total population

3. Model construction and solving

3.1. Population projection models

3.1.1. Model building

Population projection models are relatively typical mathematical models, and the basic mathematical models describing population growth are exponential growth (Malthus) and stagnant growth (Logistic) models [1], although both models can describe the population growth pattern to some extent, they cannot take into account the influence of factors such as population age structure, sex ratio and fertility policies, for this reason, we chose the Leslie matrix population model that takes into account the above factors.

We made statistics on the age structure, sex ratio and other indicators in China according to the relevant data published by the National Bureau of Statistics [2]. Analysis of the data shows that since the 1990s, China's population has been growing slowly, while the population aged 65 and above has been growing faster and faster.

Secondly, by accessing the total fertility rate of our population [3], we obtain the trend of total fertility rate changes in China during 1960-2020. Analysis of the data shows that since 2000, the total fertility rate of our population has been increasing slowly and has been stable between 1.6 and 1.7.

In addition, we make a statistical analysis of the sex ratio of China's population in the last 20 years [4], which shows that the sex ratio (male: female) is relatively stable in the last 20 years, fluctuating between 1.04 and 1.07. Therefore, in the next modeling, we can disregard the change of sex ratio over time in the next 10 years.

At the same time, we do statistical analysis on the birth rate and death rate of our population in the last 20 years, and analyzing the data we can know that the birth rate of our population in the last 10 years has a decreasing trend with a large change, while the death rate of our population has a slow decreasing trend.

With the above preliminary analysis of the data, we next build the Leslie matrix population model [5], where we define xi (t) to denote the total number of people aged i in year t, t = 0, 1, 2, 3..., n-1, n, here we group all people older than n-1 into class n. Define bi (t) to denote the fertility rate of women aged i in year t. That is, the average number of children per woman, when i is not in the range of women's reproductive age bi = 0. From the above analysis, we can assume that the sex ratio does not change over time, and define ki to denote the proportion of women aged i in the total population. Therefore, the number of children born in year t+1 can be obtained as。

$$x_0(t+1) = \sum_{i=0}^{n} b_i(t)k_i x_i(t) \tag{1}$$

Next, we define the fertility model hi to denote the number of children born to women at age i as a

proportion of women of childbearing age, i.e.

$$h_i = \frac{b_i(t)}{\beta(t)} \tag{2}$$

$$\sum_{i=0}^{n} h_i = 1 \tag{3}$$

The above 3 equations can be combined to obtain.

$$x_0(t+1) = \beta(t) \sum_{i=0}^{n} r_i x_i(t) \tag{4}$$

$$\beta(t) = \sum_{i=0}^{n} b_i(t) \tag{5}$$

On the other hand, death also affects the number of the population as well, and the mortality rate of our population is decreasing year by year as the level of medical care improves year by year. Define di (t) to denote the mortality rate of the population at age i in year t. Then the total number of deaths in year t is.

$$y(t+1) = \sum_{i=0}^{n} d_i(t) x_i(t) \tag{6}$$

Further, we define the mortality pattern gi to denote the proportion of mortality in the population at age i among all age-specific mortality rates.

$$g_i = \frac{d_i(t)}{\alpha(t)} \tag{7}$$

$$\alpha(t) = \sum_{i=0}^{n} d_i(t) \tag{8}$$

Here we assume that the mortality pattern is age-dependent only, and α(t) denotes the sum of mortality rates at all ages in year t, and call it the sum mortality rate. Letting the survival rate si(t) = 1 - α(t)gi, the number of people aged i + 1 < n years in year t + 1 is.

$$x_{i+1}(t+1) = s_i(t) x_i(t) \tag{9}$$

Then.

$$x_n(t+1) = s_{n-1}(t) x_{n-1}(t) + s_n(t) x_n(t) \tag{10}$$

From the above analysis, we obtain the recurrence of the population size for each age group, which is written as a column vector according to the age group.

$$x(t) = [x_0(t), x_1(t), x_2(t), \cdots, x_n(t)]^{\mathrm{T}}, t = 0,1,2 \dots \tag{11}$$

Introduce the constant matrix C, the death matrix D and the birth matrix B.

$$\mathbf{C} = \begin{bmatrix} 0 & 0 & \cdots & 0 & 0 \\ 1 & 0 & \cdots & 0 & 0 \\ 0 & 1 & \cdots & 0 & 0 \\ \vdots & \vdots & & \vdots & 0 \\ 0 & 0 & \cdots & 1 & 1 \end{bmatrix} \quad \mathbf{D} = \begin{bmatrix} 0 & 0 & \cdots & 0 & 0 \\ g_0 & 0 & \cdots & 0 & 0 \\ 0 & g_1 & \cdots & 0 & 0 \\ \vdots & \vdots & & \vdots & 0 \\ 0 & 0 & \cdots & g_{n-1} & g_n \end{bmatrix} \quad \mathbf{B} = \begin{bmatrix} r_0 & r_1 & \cdots & r_{n-1} & r_n \\ 0 & 0 & \cdots & 0 & 0 \\ 0 & 0 & \cdots & 0 & 0 \\ \vdots & \vdots & & \vdots & 0 \\ 0 & 0 & \cdots & 0 & 0 \end{bmatrix}$$

Then the recursive formula for the vector of population grouped by age is.

$$x(t+1) = [\mathbf{C} - \alpha(t)\mathbf{D}]x(t) + \beta(t)\mathbf{B}x(t) \tag{12}$$

where [C-α(t)D]x(t) denotes the number of people surviving at each age in year t, and β(t)Bx(t) is the number of new children born in year t+1.

Now that China has fully implemented the "three-child" policy, the future changes in China's total fertility rate are regulated by this policy, and we define the regulation effect as γ and assume that China's total fertility rate does not exceed 3 in the next 10 years and satisfies the following growth law.

$$\beta(t) = \frac{3}{1+\left(\frac{3}{\beta_0}-1\right)e^{\frac{\gamma}{10}(t-t_0)}} \tag{13}$$

where β0 represents the total fertility rate in the initial year (i.e., 2020, the year before the implementation of the "three-child" policy).

The age-grouped population growth model gives a complete description of the population development at x(t), and finally we get the total population N(t).

$$N(t) = \sum_{i=0}^{n} x_i(t) \tag{14}$$

3.1.2. Model Results

Using the total fertility rate in 2020 as the initial fertility rate, according to.

$$\beta(t) = \frac{3}{1+\left(\frac{3}{\beta_0}-1\right)e^{\frac{\gamma}{10}(t-t_0)}} \tag{15}$$

To forecast the total fertility rate of China in the next ten years, here we choose $\gamma=0.1$, $\gamma=0.3$, $\gamma=0.5$, and the forecast results are as Table 2.

Table 2: Projected integrated fertility rate under different γ values of the "three-child" policy

Year	γ=0.1 Predicted value	γ=0.3 Predicted value	γ=0.5 Predicted value
2020	1.7	1.7	1.7
2021	1.716	1.716	1.716
2022	1.726	1.737	1.751
2023	1.737	1.748	1.789
2024	1.748	1.769	1.813
2025	1.76	1.781	1.856
2026	1.771	1.813	1.904
2027	1.783	1.842	1.963
2028	1.794	1.883	2.031
2029	1.803	1.921	2.106
2030	1.815	1.972	2.174
2031	1.826	2.034	2.253

After predicting β(t), according to.

$$x(t) = [x_0(t), x_1(t), x_2(t), \cdots, x_n(t)]^T, t = 0,1,2 \ldots \tag{16}$$

$$N(t) = \sum_{i=0}^{n} x_i(t) \tag{17}$$

Projection of our total population in the next 10 years as shown in Table 3.

Table 3: Projected total population under different γ values of the "3-child" policy

Year	γ=0.1 Predicted value	γ=0.3 Predicted value	γ=0.5 Predicted value
2020	14.1258	14.1398	14.1689
2021	14.1436	14.1429	14.2086
2022	14.1689	14.2076	14.3192
2023	14.1936	14.2596	14.3982
2024	14.2169	14.3436	14.5129
2025	14.2489	14.4163	14.6476
2026	14.3156	14.5329	14.7359
2027	14.3698	14.6186	14.8962
2028	14.4278	14.7223	14.9638
2029	14.4986	14.8125	15.1029
2030	14.5428	14.8624	15.2493
2031	14.5689	14.9523	15.3567

From the above results, we can see that the model fits well, and the total fertility rate will increase under the "three-child" policy, the rate of increase depends on the regulation effect of the "three-child" policy γ, and then the population of China will also increase in varying degrees in the next 10 years.

3.2. Projections of the future size of the aging population

We can find that since the 1990s, China's aging population has been growing faster and faster, and the proportion of China's aging population to the total population has increased from 4% at the beginning of the country to 12% today, and, compared with the proportion of the world's aging population, China's aging rate is significantly higher than the world average.

Based on the population sub-age projection model established in 3.1, that.

$$x(t + 1) = [C - \alpha(t)D]x(t) + \beta(t)Bx(t) \tag{18}$$

We make a projection of China's old (64+) population in the next few decades, here we take $\gamma = 0.3$,

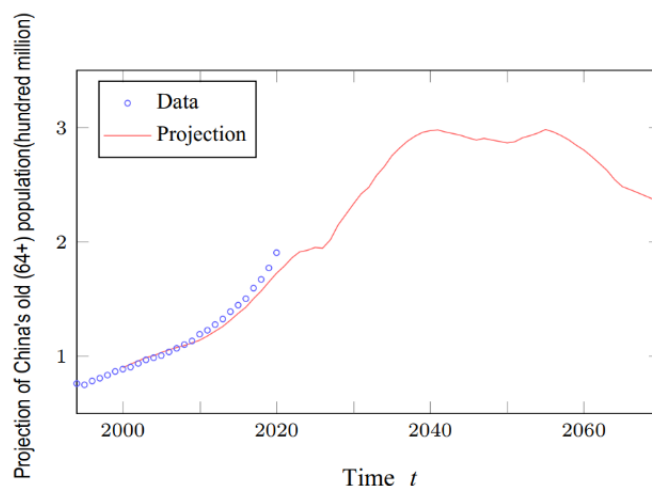and get the projection results in Figure 1.

*Figure 1: China's Future Aging Population Projections*

From Figure 1, we can see that the aging population in China will keep increasing significantly until 2040, which shows that China is facing an increasingly serious problem of population aging. Next, we analyze the statistical analysis of China's age groups in 2018 according to the available data as shown in Table 4.

Table 4: Sex-specific mortality rates by age group

| Age group	Male mortality rate	Female mortality rate (‰)
0-4	0.0004857	0.0004153
5-9	0.0002922	0.0001814
10-14	0.0002978	0.0001583
15-19	0.0004314	0.0002122
20-24	0.000641	0.0002923
25-29	0.0007465	0.0003409
30-34	0.0009098	0.0004251
35-39	0.0012161	0.0005765
40-44	0.0017233	0.0008398
45-49	0.0024594	0.0012264
50-54	0.0036237	0.0020265
55-59	0.0056898	0.003561
60-64	0.010372	0.0063714
65-69	0.0174941	0.0112489
70-74	0.0295969	0.0200063
75-79	0.0494179	0.0352291
80-84	0.0816993	0.0611249
85-89	0.134493	0.1049073
90-94	0.2137577	0.1767941
95 above	0.4172711	0.3688998

Mortality rates for both men and women begin to rise gradually after age 50, and always at a higher rate for men than for women. Through further studies, we found that the likelihood of people moving from a healthy to an unhealthy state increases with age. Based on the China Health and Nutrition Survey (hereinafter referred to as CHNS) panel data, we also found a similar pattern of health transition, as shown in Figure 2.

As seen, the probability of remaining healthy decreases with age, meaning that older adults are less likely to maintain optimal health. At the same time, we also find that the probability of the least healthy states also increases with age.
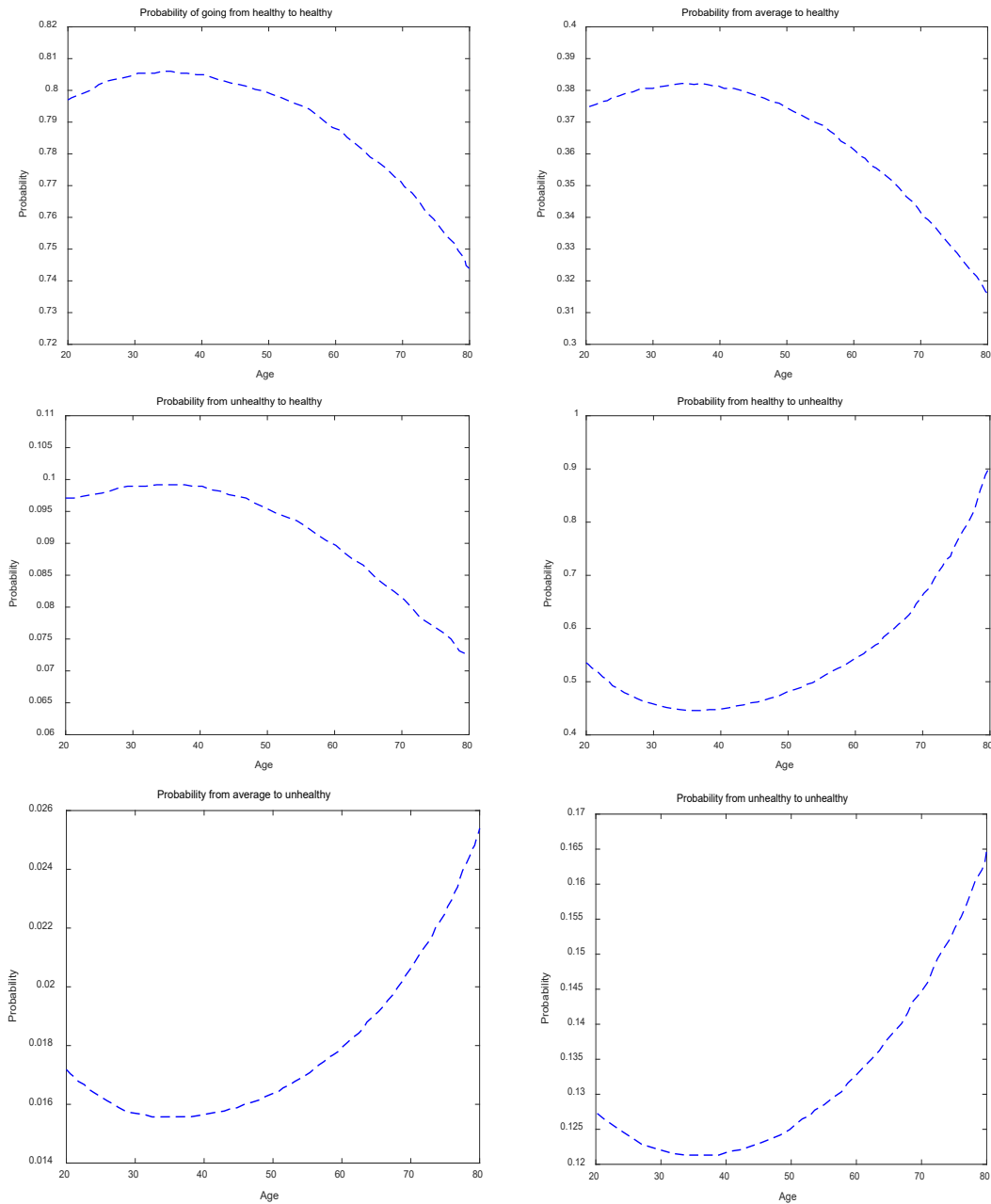
Figure 2: Probability of health problem transformation at different ages

Numerous studies have shown that health care costs largely increase exponentially with age, and based on the 2015 CHNS, we statistically analyzed the relationship between health care costs and age for people with different levels of education in China, as shown in Figure 3.
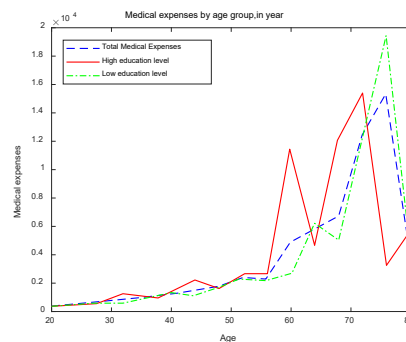
Figure 3: Medical costs by age group in 2015

From Figure 3, we can easily find that we find that the medical costs are relatively low before retirement (55-60 years old) for both the high and low education groups, while after retirement, there is a significant climb.

After the above analysis, combined with the relevant literature, we propose the following recommendations in terms of medical policies to ease the aging process in China,

(1) Promote the construction of medical and health service system for the elderly, and strengthen the health management and health promotion of common and chronic diseases.

(2) Promote the development of medical care integration, encourage the active participation of social forces, and promote the organic integration of medical services such as prevention, treatment, rehabilitation and hospice care for the elderly with the elderly services provided at home, in the community and in institutions.

(3) Promote the development of long-term care services, and establish a multi-level long-term care guarantee system for the elderly of different economic and health levels.

(4) Improve the medical insurance system for children and young and middle-aged people to reduce the mortality rate of this group, so as to increase the proportion of the population in this group and ease the aging process in China.

4. Conclusion

In this paper, we consider the influence of population age structure, sex ratio and fertility policy when building the Leslie model, which makes the model fit better. In addition, three values of the total fertility rate are reasonably selected to predict the regulation effect of the "three-child" policy, which makes the final total population model more comprehensive. This paper uses multiple mathematical software to calculate and visualize the data to complement each other and make the results more accurate and distinct. However, the construction of multiple models for population prediction is not used, resulting in a single model that cannot compare the prediction results between different models. Therefore, exponential growth (Malthus) and stagnant growth (Logistic) models can be additionally constructed, and finally, the prediction effects of the three models on the total population of China in the next 10 years can be compared and analyzed.

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