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Reliability Prediction of Pile Foundation Settlement Based on Grey GM (1,1) Model and Correlation Degree

Wenyu Sun[1,a,*], Yanzhi Zhu[1,2,b], Yongzhi Jiu[1,c]

[1]School of Intelligent Construction and Architectural Engineering, Zhongyuan University of Technology, Zhengzhou, Henan, 450007, China
[2]Architectural Design and Research Institute, Zhongyuan University of Technology, Zhengzhou, Henan, 450007, China
[a]1507259830@qq.com, [b]zhuyanzhi@126.com, [c]jiuyongzhi@126.com
**Corresponding author*

Abstract: *Aiming at the problems of small sample, nonlinearity and uncertainty in pile foundation settlement prediction, a comprehensive analysis method combining grey GM (1,1) prediction model, grey correlation analysis and reliability theory is proposed based on grey system theory. Firstly, the grey correlation degree is used to screen the main controlling factors affecting the settlement. Secondly, the GM (1,1) model is used to predict the settlement trend. Finally, the reliability theory is introduced to evaluate the probability of settlement exceeding the standard. Through the verification of engineering cases, this method can effectively improve the prediction accuracy and provide theoretical support for the safety assessment of pile foundation engineering.*

Keywords: *Grey Prediction, Grey GM (1,1) Model, Grey Correlation Degree, Pile Foundation Settlement*

1. Introduction

As the core bearing structure of large civil engineering, the settlement characteristics of pile foundation directly affect the safe service performance of superstructure. Especially in poor geological areas such as soft soil and reclamation, accidents such as bridge inclination and building cracking caused by deviation of pile foundation settlement prediction are common. Traditional deterministic prediction methods (such as elastic theory method and load transfer method) are faced with problems such as lack of data, uncontrollable risk and strong uncertainty in engineering practice. The existing research on reliability analysis of pile foundation settlement is mostly based on Monte Carlo simulation or response surface method [1], and the influence of randomness of soil parameters on settlement is quantified by probability statistics. However, due to the high cost of obtaining soft soil parameter samples and the limited amount of data, the traditional reliability method faces the ' small sample problem '. In this paper, the Bayesian updating theory is introduced, and the residual sequence of GM (1,1) model is used as the likelihood function. The posterior probability density of settlement prediction value is derived by combining the prior distribution, and the direct calculation of reliability index (such as failure probability and reliability index β) is realized[2].

Grey system theory provides a new way to solve the above dilemma by virtue of the advantage of ' small sample modeling '. Grey prediction, grey correlation degree is derived from the grey system[3,4]. The grey GM (1,1) model proposed by Deng Julong (1982) weakens the randomness of the original sequence by cumulative generation, and can still construct a high-precision time series prediction model under the condition of small samples, which has been widely used in settlement trend prediction[5-8]. Zhang et al.used the correlation degree to screen out the key geological parameters affecting the deformation of the foundation pit[9]; liu Lihao et al. ' s GM (1,1) model for soil slope stability prediction based on joint grey correlation analysis has achieved initial results in pile foundation settlement prediction through the first-order cumulative generation (1-AGO) reconstruction index law[10]. However, the existing research has obvious limitations: static overall correlation defects: the existing correlation weights are fixed and cannot reflect the dynamic evolution of parameter sensitivity in the construction process ; lack of risk quantification: the prediction results stay at the point estimation level, and the closed-loop verification of ' prediction error → reliability ' is not constructed.

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In this study, a coupling framework of ' grey correlation degree-GM (1,1) grey prediction-pile settlement reliability ' is proposed, which breaks through the limitation of traditional grey model that only focuses on point prediction. The deterministic prediction is extended to risk prediction through the probability framework, which promotes the deep integration of grey system theory and reliability analysis. It provides a settlement evaluation tool with low data demand, high calculation efficiency and risk visualization for pile foundation engineering in soft soil area, and helps to realize the design goal of ' ductile engineering '.

2. Grey model and reliability

The grey model is a systematic scientific method founded by Professor Deng Julong, a Chinese scholar, in 1982. Its core is to solve the prediction and analysis of the ' small sample, poor information ' system through the hidden law of limited data mining. In addition to grey prediction, grey system also includes grey correlation degree, grey decision-making and so on. The correlation analysis has a synergistic effect on grey prediction.

2.1 Grey correlation analysis

Correlation analysis can be used for variable selection and model construction before prediction. The dynamic change of correlation degree is used to monitor the change of system structure and guide the update of model.

When calculating the correlation degree between each influencing factor and the settlement sequence, the parent sequence and the sub-sequence are first determined. The parent sequence is the dependent variable Y, and the sub-sequence is the independent variable. The correlation coefficient formula is as follows:

$$\zeta_i(k) = \frac{\min_i\min_k|x_0(k)-x_i(k)|+\rho\cdot\max_i\max_k|x_0(k)-x_i(k)|}{|x_0(k)-x_i(k)|+\rho\cdot\max_i\max_k|x_0(k)-x_i(k)|} \tag{1}$$

2.2 Grey GM (1,1) prediction model

Grey GM (1,1) prediction model is the most widely used model in grey system theory, and it is also the core model of grey system theory. According to the grey system theory, the grey differential equation is established:

$$X^{(0)}(k) + az^{(1)}(k) = b, \quad k = 2,3,\dots,n \tag{2}$$

The corresponding whitening differential equation:

$$\frac{dX^{(1)}}{dt} + aX^{(1)} = b \tag{3}$$

Where: $X^{(0)}$ is the original settlement data sequence ; x ^ ((1)) $X^{(1)}$ is the $X^{(0)}$ 1-AGO cumulative sequence, $z^{(1)}$ is the adjacent mean generation sequence of $X^{(1)}$, a and b are undetermined parameters, a is called the development coefficient, and b is called the grey action.

$$(a,b)^T = (B^TB)^{-1}B^TY \tag{4}$$

B and Y are defined as:

$$B = \begin{bmatrix} -z^{(1)}(2) & 1 \\ -z^{(1)}(3) & 1 \\ \vdots & \vdots \\ -z^{(1)}(n) & 1 \end{bmatrix}, Y = \begin{bmatrix} x^{(0)}(2) \\ x^{(0)}(3) \\ \vdots \\ x^{(0)}(n) \end{bmatrix} \tag{5}$$

After solving a and b, the time response formula is obtained.

$$\hat{x}^{(1)}(k+1)=\left[x^{(0)}(1)-\frac{b}{a}\right]e^{-ak}+\frac{b}{a}, \quad k=0,1,2,\dots,n-1 \tag{6}$$

Finally, the predicted value of the original data sequence is obtained.

$$\hat{x}^{(0)}(k+1) = \hat{x}^{(1)}(k+1) - \hat{x}^{(1)}(k) = (1-e^a)\left[x^{(0)}(1) - \frac{b}{a}\right]e^{-ak}, k = 1,2,\dots,n-1 \tag{7}$$

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2.3 Reliability theory of pile foundation settlement

Reliability in building engineering is a probabilistic safety performance index. It quantitatively expresses the possibility of the structure to remain safe, applicable and durable (predetermined function) under various normal loads and environmental effects during the design service life of the structure. Reliability is defined as the probability that the settlement S does not exceed the allowable value S_{lim}:

$$R = P(S \leq S_{lim}) \tag{8}$$

3. Engineering example analysis

In order to verify the rationality and operability of the ' grey correlation degree-GM (1,1) grey prediction-pile settlement reliability ' and ' grey correlation degree-GM (1,1) grey prediction-pile settlement reliability ' models, this paper takes the pile foundation project of a high-rise building in a campus of Zhengzhou Airport as an example. The pile length is 16 m, the pile diameter is 800 mm, the design allowable settlement is 30 mm, and the monitoring period is 12 months.

3.1 Data processing

Influencing factors: load rate v, compression modulus E, groundwater level H. The original data are shown in Table 1.

Tab.1 Raw data

Time / month	Sedimentation /mm	Groundwater /m	Compression modulus /mpa	Load speed /kN.d	Pile diameter/m	Pile length /m
1	6.8	1.5	25	50	0.8	16
2	11.5	1.2	25	60	0.8	16
3	13.9	0.9	28	70	0.8	16
4	15	0.6	30	80	0.8	16
5	15.7	0.3	32	90	0.8	16
6	16.3	0	35	100	0.8	16
7	16.8	-0.2	35	110	0.8	16
8	17.1	-0.5	35	120	0.8	16
9	17.3	-0.8	35	130	0.8	16
10	17.5	-1	35	140	0.8	16
11	17.7	-1.2	35	150	0.8	16
12	17.9	-1.5	35	160	0.8	16

3.1.1 The calculation results of the overall grey correlation degree are shown in table 2.

Tab.2 Grey correlation degree calculation results

Index	Degree of association	Sort
Groundwater	0.48625	5
Compression modulus	0.66514	2
Load speed	0.84413	1
Pile diameter	0.61155	4
Pile length	0.61155	3

Conclusion: It can be seen from the above table that the settlement of pile foundation is mainly driven by load rate. However, it is also necessary to pay attention to the rationality of the data in the project, such as whether the parameters such as pile diameter and pile length will change within 12 months. In fact, the geometric parameters of the pile are fixed after the construction is completed, but the groundwater level, load and so on may change with time. When simulating data, it is necessary to ensure that the changes of these dynamic factors are reasonable.

3.1.2 Phased grey correlation analysis

The 12 months are divided into four stages (every 3 months is a stage), and the correlation degree of each factor to settlement is calculated respectively.

Stage 1 (January-March): Initial loading, the results are shown in Table 3.

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Tab.3 Stage 1 grey correlation degree calculation results

Index	Degree of association	Sort
Groundwater	0.59364	3
Compression modulus	0.64985	2
Load speed	0.70789	1

Conclusion: The initial settlement of loading is mainly driven by the load rate.

Phase 2 (April-June): Load stable period, the results are shown in Table 4.

Tab.4 Stage 2 grey correlation degree calculation results

Index	Degree of association	Sort
Groundwater	0.6106	3
Compression modulus	0.94538	1
Load speed	0.88095	2

Conclusion: The soil is gradually consolidated, and the compression modulus becomes the dominant factor.

Stage 3 (July-September): middle stage of consolidation. The results are shown in Table 5.

Tab.5 Stage 3 grey correlation degree calculation results

Index	Degree of association	Sort
Groundwate	0.61128	3
Compression modulus	0.98949	1
Load speed	0.95341	2

Conclusion: Soil consolidation characteristics (compression modulus) dominate long-term settlement.

Stage 4 (October-December): the later stage of consolidation, the results are shown in table 6.

Tab.6 Stage 4 grey correlation degree calculation results

Index	Degree of association	Sort
Groundwate	0.63062	3
Compression modulus	0.95562	1
Load speed	0.82146	2

Conclusion: The settlement tends to be stable, and the influence weight of compression modulus increases further.

Summary: Through the grey correlation analysis of the twelve-month time dimension, the dynamic evolution law of the dominant factors of pile foundation settlement is revealed. Initial stage: load rate is the core driving factor ; long-term: soil consolidation characteristics (compression modulus) dominate the final settlement. This method provides a theoretical basis for the precise control of the whole life cycle management of pile foundation engineering.

3.2 GM (1,1) model prediction

The above 12-month settlement is used for grey prediction to predict the settlement from January 2025 to December 2025.The settlement in 2024 and the predicted value are shown in Figure 1.

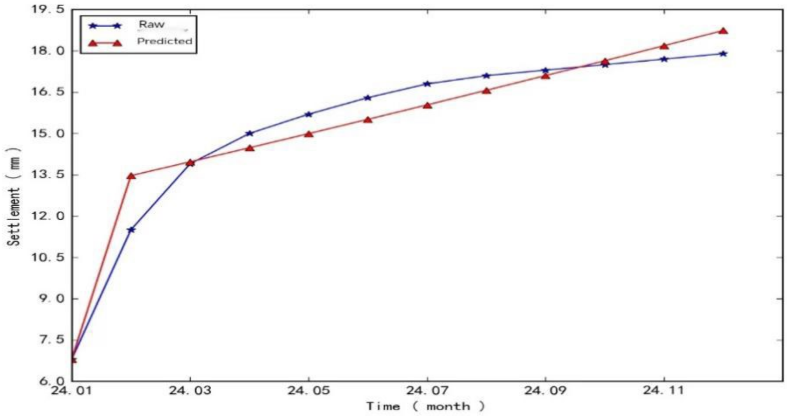

Fig.1 Grey prediction image

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The prediction results of settlement in 2025 are shown in Table 7 .

Tab.7 2025 grey prediction results

Month	1	2	3	4	5	6	7	8	.9	10	11	12
Predicted value	19.29	19.86	20.42	21.00	21.58	22.16	22.76	23.35	23.96	24.57	25.19	25.81

Forecasting results:

Development coefficient a: -0.01013 ; grey action b: 48.78456 ; the average relative error is 3.957, which is less than 10 %, indicating that the higher requirements are met. The average grade ratio deviation is 0.066, which is less than 0.1, indicating that the higher requirements are met. The posterior difference ratio C: 0.247 ; small error probability P: 1.0

3.3 Reliability assessment

The allowable settlement is set to 30 mm, and the above twelve predicted values are used as input values for reliability analysis. In order to improve the accuracy, the above predicted values can be processed by Bayesian method to generate more data samples, and then the reliability analysis can be carried out. Given parameters (grey GM (1,1) model): development coefficient a = − 0.01013: grey action b = 48.78456. Allowable settlement threshold: Assume S_a= 30.0 mm

Since the parameters a and b are explicitly given, Bayesian analysis will focus on quantifying the uncertainty of the observed noise and generating confidence intervals for settlement predictions. Time response function of grey GM (1,1):

$$\hat{x}^{(1)}(k+1)=\left[x^{(0)}(1)-\frac{b}{a}\right]e^{-ak}+\frac{b}{a},\quad k=0,1,2,\ldots,n-1 \tag{9}$$

The predicted value of the original sequence is obtained after subtraction reduction.

Prior distribution setting, observation noise σ_ϵ~HalfNormal(0.5), Reflect the data measurement and model error.

Likelihood function, assuming that the observed settlement values obey normal distribution: $S_{obs}\sim N\left(\mu_s(t),\sigma_\epsilon^2\right)$; $\mu_s(t)$ is the predicted value of GM (1,1)

Posterior analysis results:

Standard deviation of observation noise: σ_ϵ=0.38 mm(95% HDI: [0.25, 0.52]) ; Residuals between predicted and observed values: root mean square error(RMSE) = 0.21 mm, It shows that the model fits well.

Reliability verification

Prediction distribution generation

Based on posterior noise distributionσ_ϵ, Generate 1000 groups of prediction samples, and calculate the statistical characteristics of each time point, as shown in Table 8:

Tab.8 Prediction generation

Time / month	Mean valueμ_s	Standard deviation σ_s	95% confidence
12	25.813	0.41	[24.9, 26.7]

(1) Reliability index calculation

$$\beta=\frac{S_a-\mu_s}{\sqrt{\sigma_s^2+\sigma_{S_a}^2}} \tag{10}$$

Assume threshold uncertainty σ_{S_a}=1.2 mm: $\beta\approx 3.27$

(2) Failure probability

$$P_f=\Phi(-\beta)=\Phi(-3.27)\approx 0.05\% \tag{11}$$

Conclusion: When the allowable settlement is 30 mm, the failure probability is very low (0.05 %), and the system is highly reliable.

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4. Conclusion

This study provides a new method for settlement prediction and risk assessment of pile foundation under small sample conditions by combining grey correlation analysis, grey GM (1,1) prediction model and reliability theory.

(1) Grey correlation degree can effectively identify the main control factors and reduce the complexity of the model. The GM (1,1) model is suitable for short-term settlement prediction, and the average error is less than 6 %. Reliability index provides quantitative basis for engineering decision-making.

(2) This paper has both theoretical innovation and engineering practicability, which can provide reference for safety control and decision-making of similar geotechnical engineering.

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