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Research on Independent Innovation Investment of Firms under Mixed Multi-Oligopoly

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Abstract: *Considering the spillover between independent innovation investment of firms, research and development subsidy policies of government, we construct an independent innovation investment model of mixed multi-oligopoly including one public firm and many private firms. We obtain optimal independent innovation investment of public firm and private firms, and analyze the relationship among the technology spillover, research and development subsidy and the optimal independent innovation investment. Furthermore, this paper discusses the relationship between independent innovation investment and non-independent innovation investment when social welfare reached optimal. Finally, take into account the difference of all firms scale and the difference of innovation investment, we derive the Nash equilibrium solution of independent innovation investment of firms, and analyze the effects of the change of the firms scale on independent innovation investment.*

Keywords: *Independent innovation investment; Public and private firms; Scale difference; Spillover; Innovation game*

1. Introduction

With the rapid development of society, the market competition becomes more fiercely, enterprises must have their own independent intellectual property rights to have a place in the world market. Otherwise, they cannot build core competitive advantages and take part in the fierce competition in the world. In order to establish core competitive advantage, the firm must enhance independent innovation research and development(R&D) ability through research in-depth, and actively develop new products, new technology and new process. Independent innovation capability is a basis of measuring the core competitiveness of a country and region[1]. Therefore, research on independent innovation investment of firms has an important significance. In recent years, some scholars had done some research in view of innovative R&D investment and technology spillovers, Nasierowski and Arcelus(2003) researched that weak companies will put more resources into imitation, plagiarism due to technical monopoly, and thus reduce the independent innovation investment[2]. Fjell and Heywood(2004) discussed that when the role of private firms changed in the mixed multi-oligarch model, the relationship among R&D subsidies, R&D output and social welfare, as well as the influence of R&D subsidies to R&D decision of the firms[3]. Tomaru(2007) investigated the effects of foreign firm and privatization of the domestic public firm on the production efficiency and social welfare, while the domestic firm undertaked R&D investment to reduce costs in Cournot competition, showed that foreign firm reduced production efficiency of the domestic firm but raised the domestic social welfare[4]. Wei and Lu(2011) analyzed the independent innovation investment problem of firms with technology spillover, but he did not consider the difference of firms scale and incentive policy of the government[5]. From the firm nature, some researches considered the privatization of public firms, such as Wang, L.F.S and Chen(2011) established a mixed oligopoly competition model with foreign penetration, derived the equilibrium output and social welfare under Cournot competition market, and examined the impact of foreign penetration on privatization in a mixed oligopolistic market[6]. Han(2010) introduced the technology spillover of foreign investment in mixed oligopoly model, and analyzed the impacts of technology spillover of foreign investment on the privatization of public firms[7]. Some researches considered the innovative R&D subsidy, Sheng(2008) compared the effect of innovation investment subsidies and production subsidies under homogenous duopoly Cournot competition[8]. José Gil-Moltó and Poyago-Theotoky(2011) examined the use of subsidies to R&D in a mixed and a private duopoly market, showed that the socially optimal R&D subsidy was increasing in the degree of spillovers but it

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was lower in the private duopoly, and also found that privatization of the public firm reduced R&D activity and welfare in the duopoly market[9]. Dai and Cheng(2015) researched the effect of public subsidies on corporate R&D investment, showed that public subsidies for the total R&D investment of firms and private R&D investment respectively follow an S-shaped and inverted U-shaped relationship[10]. Bogoroditskaya(2021) researched the tax evasion and R&D subsidy in a mixed market[11]. Mukherjee(2023) showed that merger may increase investments in product innovation[12].

In this paper, the difference from previous literatures is that we take into account the privatization of public firm, R&D subsidy of government and the difference of all firms scale, and analyze the independent innovation investment problem of firms. First, we construct a independent innovation investment model of mixed multi-oligopoly including one public firm and many private firms in the case of technology spillover and R&D subsidy, and obtain optimal independent innovation investment of public firm and private firms, and analyze the relationship among the technology spillover, research and development subsidy and the optimal independent innovation investment. Furthermore, this paper discusses the relationship between independent innovation investment and non-independent innovation investment when social welfare reached optimal. Finally, take into account the difference of all firms scale and the difference of innovation investment, we derive the Nash equilibrium solution of independent innovation investment of firms, and analyze the effects of the change of the firms scale on independent innovation investment. In Section 2, we give a independent innovation investment model of mixed multi-oligopoly including one public firm and many private firms. In Section 3, We investigate the relationship between independent innovation investment and non-independent innovation investment when social welfare reached optimal, and analyze the effects of the change of the firms scale on independent innovation investment. Section 4 concludes.

2. Model Description

Considering an industry which consists of one public firm and n private firms on the market, the public firm is partial privatization denoted by firm 0. Firms carry on the independent R&D investment for their maximize goal, x_i is of firm i, $x_i \geq 0$; x_T denotes the independent innovation investment of the whole society, $x_T = \sum_{i=0}^{n} x_i$. α is a coefficient of independent innovation investment contribution for output, $0 < \alpha < 1$, β is a coefficient of non-independent innovation investment contribution for output, $0 < \beta < 1$; it exists spillover among firms in the process of firms carrying out R&D and innovation. The technology spillover coefficient is w, $0 < w < 1$, and the production function of firm i can be expressed as

$$q_i = [(1-w)x_i + wx_T]^{\alpha} y_i^{\beta}$$

The unit cost while firms making independent innovation investment is $\overline{c}_x$, and the unit cost while firms making non-independent innovation investment is $\overline{c}_y$.At the same time, the government provides the R&D subsidy s per unit for firms to encourage them to conduct research and development innovation, give R&D subsidy, and the cost function of firm i is given by

$$c_i = (\overline{c}_x - s)x_i + \overline{c}_y y_i, \quad i = 0,1,2,\cdots,n$$

Then, the total outputs and total costs of all R&D investment firms is

$$q_T = \sum_{i=0}^{n} q_i, \quad c_T = \sum_{i=0}^{n} c_i = (\overline{c}_x - s)x_T + \overline{c}_y \sum_{i=0}^{n} y_i$$

3. Model Analysis

3.1 The analysis of output maximization and social welfare maximization

First of all, the firm conducts R&D investment in order to maximum its output considering from the

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firm itself, therefore, to maximize its own output, x_i , y_i must satisfy the follow equations

$$\begin{cases} \max q_i = [(1-w)x_i + wx_T]^{\alpha} y_i^{\beta} \\ \text{s.t.} \quad c_i = (\overline{c}_x - s)x_i + \overline{c}_y y_i \end{cases}$$

Construct the Lagrangian function $L_i = q_i + \lambda(c_i - \overline{c}_x x_i - \overline{c}_y y_i + sx_i)$, By Lagrange multiplier method, we can obtain the following three equations

$$\frac{\partial L_i}{\partial x_i} = \alpha(1-w)[(1-w)x_i + wx_T]^{\alpha-1} y_i^{\beta} - \lambda\overline{c}_x + \lambda s = 0 \tag{1}$$

$$\frac{\partial L_i}{\partial y_i} = \beta[(1-w)x_i + wx_T]^{\alpha} y_i^{\beta-1} - \lambda\overline{c}_y = 0 \tag{2}$$

$$\frac{\partial L_i}{\partial \lambda} = c_i - \overline{c}_x x_i - \overline{c}_y y_i + sx_i = 0 \tag{3}$$

Solving equations (1)(2)(3), we get:

$$x_i = \frac{\alpha(1-w)c_i}{(\alpha+\beta-\alpha w)(\overline{c}_x - s)} - \frac{\beta w}{\alpha+\beta-\alpha w}\sum_{j\neq i}^{n} x_j \tag{4}$$

Hence, we can get the following conclusion:

Conclusion 1: When firms undertaking the independent R&D investment to maximize profits with technology spillover, the independent R&D investment of firm and other firms change in the opposite direction. That is to say, If the independent R&D investment of other firms increase, the independent R&D investment of the firm will reduce. The more R&D subsidy firms obtain, the higher enthusiasm for firms carry out independent R&D investment.

And from $\frac{\partial x_i}{\partial w} = -\frac{\beta c_i}{(\alpha+\beta-\alpha w)^2(\overline{c}_x - s)} - \frac{\beta(\alpha+\beta)}{(\alpha+\beta-\alpha w)^2}\sum_{j\neq i}^{n} x_j < 0$, we can get the following conclusion 2:

Conclusion 2: When firms undertaking the independent R&D investment to maximize profits with technology spillover, the independent R&D investment of firm and technology spillover coefficient changes inversely. If other firms spill over more in technology innovation, the firm will reduce the independent R&D investment.

Then, in the view of the perspective of social welfare maximization, we analyze the conditions the R&D investment should meet when social welfare reached optimal. As the firm 0 is a public firm with partial privatization, we assume that the degree of privatization of public firm is θ , whereas the firm 0 maximizes the weighted of profits and social welfare, $\max q_p = \theta q_0 + (1-\theta)q_T$. From the perspective of the total social welfare, it should meet the following conditions while social welfare is optimal.

$$\begin{cases} \max q_p = \theta q_0 + (1-\theta)q_T \\ \text{s.t} \quad c_T = (\overline{c}_x - s)x_T + \overline{c}_y \sum_{i=0}^{n} y_i \end{cases}$$

Constructing the Lagrangian function $L_T = q_p + \lambda[c_T - (\overline{c}_x - s)x_T - \overline{c}_y \sum_{i=0}^{n} y_i]$

The optimal conditions is $\begin{cases} \frac{\partial L_T}{\partial x_T} = 0 \\ \frac{\partial L_T}{\partial y_i} = 0 \end{cases}$, and we get $\frac{\sum_{i=1}^{n} \frac{\partial q_i}{\partial x_T}}{\frac{\partial q_i}{\partial y_i}} = \frac{\overline{c}_x - s}{\overline{c}_y}$.

We derive the following equation after conversion

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$$\frac{\frac{\partial q_i}{\partial x_T}}{\frac{\partial q_i}{\partial y_i}} = \frac{\overline{c}_x - s}{\overline{c}_y} - \frac{\sum_{j\neq i}^{n} \frac{\partial q_j}{\partial x_T}}{\frac{\partial q_i}{\partial y_i}} \tag{5}$$

Therefore, we have the conclusion 3:

Conclusion 3: When the welfare of whole social achieve to the optimal, marginal yield ratio of the independent R&D investment and the non-independent R&D investment satisfy equation(5), and it has nothing to do with the privatization degree of public firm, the R&D subsidy reduces the requirement to achieve Pareto optimal when firms undertaking the independent R&D investment.

3.2 The analysis of independent innovation investment while the scale of firms is different

In this part, we take into account the difference of scale and independent innovation investment of firms, normalize the cost of all private and public firms. Supposing the cost of firm i is $c_i = \frac{ic}{n(n+1)}$, and the independent innovation investment of firm i is x_i, $i = 1,2,\cdots,n.$, the cost and the independent innovation investment of public firm 0 respectively is $c_0 = \frac{\lambda c}{n(n+1)}$ and x_0. By the equation (4), we can obtain the reaction function of public firm 0 and private firms.

$$\begin{cases} x_0 = \frac{\alpha(1-w)\lambda c}{n(n+1)(\alpha+\beta-\alpha w)(\overline{c}_x - s)} - \frac{\beta w}{\alpha+\beta-\alpha w}\sum_{i=1}^{n} x_i \\ x_i = \frac{\alpha(1-w)ic}{n(n+1)(\alpha+\beta-\alpha w)(\overline{c}_x - s)} - \frac{\beta w}{\alpha+\beta-\alpha w}\sum_{\substack{i=0 \\ j\neq i}}^{n} x_j \end{cases} \tag{6}$$

By the equation (6), we get

$$\begin{cases} x_0 = \frac{\alpha c[2\lambda(\alpha+\beta-\alpha w+(n-1)\beta w) - n(n+1)\beta w]}{2n(n+1)(\alpha+\beta)(\alpha+\beta-\alpha w+n\beta w)(\overline{c}_x - s)} \\ x_i = \frac{\alpha c[2i(\alpha+\beta-\alpha w+n\beta w) - 2\lambda\beta w - n(n+1)\beta w]}{2n(n+1)(\alpha+\beta)(\alpha+\beta-\alpha w+n\beta w)(\overline{c}_x - s)} \end{cases}$$

From the above equations, we know that if $\lambda > \frac{n(n+1)\beta w}{2[\alpha+\beta-\alpha w+(n-1)\beta w]}$, we have $x_0 > 0$, and if

$$\lambda < \frac{2i(\alpha+\beta-\alpha w+n\beta w) - n(n+1)\beta w}{2\beta w},$$

we can derive $x_i > 0$.

Hence, when $\frac{n(n+1)\beta w}{2[\alpha+\beta-\alpha w+(n-1)\beta w]} < \lambda < \frac{2i(\alpha+\beta-\alpha w+n\beta w) - n(n+1)\beta w}{2\beta w}$, the Nash equilibrium solution of independent innovation investment of firms is

$$\begin{cases} x_0 = \frac{\alpha c[2\lambda(\alpha+\beta-\alpha w+(n-1)\beta w) - n(n+1)\beta w]}{2n(n+1)(\alpha+\beta)(\alpha+\beta-\alpha w+n\beta w)(\overline{c}_x - s)} \\ x_i = \frac{\alpha c[2i(\alpha+\beta-\alpha w+n\beta w) - 2\lambda\beta w - n(n+1)\beta w]}{2n(n+1)(\alpha+\beta)(\alpha+\beta-\alpha w+n\beta w)(\overline{c}_x - s)} \end{cases} \tag{7}$$

Whereas, if $\lambda \geq \frac{2i(\alpha+\beta-\alpha w+n\beta w) - n(n+1)\beta w}{2\beta w}$, we know $x_i = 0$,and

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$$x_0 = \frac{\alpha c(\lambda - i)}{n(n+1)(\alpha+\beta)(\overline{c}_x - s)}$$

Therefore, when $\lambda \geq \frac{2i(\alpha+\beta-\alpha w+n\beta w)-n(n+1)\beta w}{2\beta w}$, the Nash equilibrium solution of independent innovation investment of firms is

$$x_0 = \frac{\alpha c(\lambda - i)}{n(n+1)(\alpha+\beta)(\overline{c}_x - s)},\ x_i = 0,\ i = 1,2,\cdots,n. \tag{8}$$

By equation(8), we obtain that if $\lambda > i$, thus $x_0 > x_i$, $\frac{\partial x_0}{\partial \lambda} > 0$, $\frac{\partial x_i}{\partial \lambda} < 0$.

Conclusion 4: In Nash equilibrium of the independent R&D investment, while the scale of public firm is greater than the scale of private firms, the independent innovation R&D investment of public firm is higher than the private firms. Along with the enlarge gap among the scale, the independent innovation R&D investment of public firm will increase, the independent innovation R&D investment of private firms will reduce inversely. When the scale of public firm is large enough, the independent innovation R&D investment of private firms will be reduced to zero, and utterly depend on the independent innovation R&D investment of public firm. Meanwhile, Increase their R&D subsidies will stimulate firms to undertake independent innovation R&D.

The first order partial derivatives of the equation(7) to i is $\frac{\partial x_i}{\partial i} = \frac{\alpha c}{n(n+1)(\alpha+\beta)(\overline{c}_x - s)} > 0$. This shows that, when independent innovation investment of firms in Nash equilibrium, the scale of private firm will effect the independent innovation investment of firms for private firm, the greater the scale difference among the private firms, the more private firms invest on R&D.

And when $\frac{n(n+1)\beta w}{2[\alpha+\beta-\alpha w+(n-1)\beta w]} < \lambda < \frac{2i(\alpha+\beta-\alpha w+n\beta w)-n(n+1)\beta w}{2\beta w}$, from equation(7), we can derive the total R&D investment of society.

$$x_0 + \sum_{i=1}^{n} x_i = \frac{\alpha c(1-w)(2\lambda + n^2 + n)}{2n(n+1)(\alpha+\beta-\alpha w+n\beta w)(\overline{c}_x - s)} \tag{9}$$

When $\lambda \geq \frac{2i(\alpha+\beta-\alpha w+n\beta w)-n(n+1)\beta w}{2\beta w}$, we know $x_i = 0$,so the total R&D investment of society is

$$x_0 = \frac{\alpha c(\lambda - i)}{n(n+1)(\alpha+\beta)(\overline{c}_x - s)} \tag{10}$$

From equation(9) and (10), we find that the total independent innovation investment has noting to do with the differences among the private firms, but is related to the scale differences between public and private firms.

In other words, when independent innovation investment of firms in Nash equilibrium, the total independent innovation investment has noting to do with the scale differences among the private firms, but is related to the scale differences between public and private firms. On the contrary, the increased number of private firms in the social system can reduce the overall investment in independent research and development in society.

4. Conclusions

In the context of implementing innovation-driven development strategy, research on independent innovation investment of firms has the vital significance. This paper takes into account the spillover between independent innovation investment of firms, research and development subsidy policies of government, and constructs an independent innovation investment model of mixed multi-oligopoly

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including one public firm and many private firms. We obtain optimal independent innovation investment of public firm and private firms, and analyze the relationship among the technology spillover, research and development subsidy and the optimal independent innovation investment. Furthermore, this paper discusses the relationship between independent innovation investment and non-independent innovation investment when social welfare reached optimal. Finally, considering the difference of all firms scale and the difference of innovation investment, we derive the Nash equilibrium solution of independent innovation investment of firms, and analyze the effects of the change of the firms scale on independent innovation investment. The research shows that, when firms undertaking the independent R&D investment to maximize profits with technology spillover, the independent R&D investment of firm and other firms change in the opposite direction, the independent R&D investment of firm and technology spillover coefficient changes also inversely. In Nash equilibrium of the independent R&D investment, while the scale of public firm is greater than the scale of private firms, the independent innovation R&D investment of public firm is higher than the private firms. Along with the enlarge gap among the scale, the independent innovation R&D investment of public firm will increase, the independent innovation R&D investment of private firms will reduce inversely. When the scale of public firm is large enough, the independent innovation R&D investment of private firms will be reduced to zero, and utterly depend on the independent innovation R&D investment of public firm. Meanwhile, Increase their R&D subsidies will stimulate firms to undertake independent innovation R&D.

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