Multi-Objective Optimization Model and Genetic Algorithm Based on "FAST" Active Reflecting Surface Dynamic Optimization Research

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Abstract: *FAST requires the active reflector to be dynamically adjusted to an ideal paraboloid in real time when tracking the celestial motion, but the real paraboloid is not ideal due to the interconnection of the main cable nodes and the adjustment constraint of the reflector panel. First, the ideal paraboloid is determined when the object S is located directly above the reference sphere, and the reflecting panel adjustment is considered.*

Keywords: *Circular traversal; Single-objective optimization; Multi-objective optimization; Genetic algorithm*

1. Introduction

It is known that the object to be observed, S, is located directly above the reference sphere, and in combination with the consideration of the reflection panel adjustment factor, the ideal paraboloid will be determined in this paper. In addition, the most ideal paraboloid and the actual optimal paraboloid closest to the ideal paraboloid are determined when the object to be observed S is located at $\alpha = 36.795$, $\beta = 78.169$, and the data such as the main rope node number are saved. The ideal paraboloid can be solved according to the model. The optimal paraboloid in practice is solved by establishing an optimization model with the distance variation between adjacent nodes and the expansion range of the actuator as the constraint, and the minimum deviation of the main cable node position from the ideal paraboloid after adjustment as the target.

2. Assumptions and notations

Table 1: The symbol conventions

Symbols	Description
F	denotes the radius difference between the focal plane and the reference sphere
h	denotes the value of the vertex of the parabola on the z-axis
vi	denotes the coordinates of the main rope node on the ideal parabolic curve
vi0	denotes the coordinates of the main cable node on the reference sphere
Δu	Indicates the retraction of the actuator
Q	indicates Qi is the main cable node after adjusting the reflection panel in practice
f1	Indicates the acceptance ratio of the feeder compartment after adjustment
f2	Indicates the acceptance ratio of the reflecting sphere of the reference sphere

We use the following assumptions.

(1) Assume that the signal of electromagnetic wave and the reflected signal are propagated in a straight line.

(2) Assume that the force analysis of the main cable nodes and actuators are not considered, and only the distance constraint is considered.

(3) Assume that each reflective panel receives the same electromagnetic wave concept.

(4) Assume that the electromagnetic wave signal propagates parallel to the reflecting panel.

(5) Assume that there is no obstacle in the propagation of electromagnetic waves. The primary

notations used in this paper are listed as follows.

The symbol conventions are shown in Table 1.

3. Model construction and solving

3.1. Ideal parabolic model based on circular traversal

The initial position of the reflecting surface of the sky eye is on a spherical surface, called the reference sphere, but the whole reflecting surface is not fixed, but its working paraboloid shape can be constantly adjusted dynamically, in the observation process, the shape of the working reflecting surface is changed in real time, the main rope node outside the irradiation range is not actively adjusted, the feeder illumination part is adjusted to a certain accuracy of the rotating paraboloid, so that the celestial electromagnetic waves by reflections can have a better reception effect[1]. The first question of the celestial position is located directly above the center of the sphere, so here we simplify the model by considering that the vertex coordinates of the ideal paraboloid are located on the line connecting the center of the sphere. We obtain different parabolas with the feed point as the focal point by circularly traversing the vertex coordinates of the parabola. Considering the symmetry of the parabola and the constraint that the reflector cannot be adjusted substantially, the distance between the parabola and the reference sphere is minimized as the optimization objective to select the best and the worst parabolas from the traversal, and finally the equation of the ideal rotating paraboloid is derived [2].

3.1.1. Model Building

Adjust the constraint based on reflection panel to get the ideal paraboloid [3].

Step1: Loop traversal of vertex coordinates.

Select a symmetric curve on the reference sphere, find out all the main cable nodes on the curve, and find out the distance between each node, then take the feeder module as the focus, and traverse the vertices up and down in steps Δh at a time to find out the parabolic equation, because the parabola is derived from the fluctuation of the main cable nodes up and down, that is, only change the original main cable point z coordinate, through the parabolic equation and the original main cable point xy, the coordinates of the first main cable point can be found, and then the distance between each main cable point can be found, if the corresponding distance does not change by more than 0.07%, the curve meets the requirements, otherwise it does not meet the requirements.

Parabolic equation

$$x^2 = 2PZ \tag{1}$$

$$x^2 = 2PZ = 4(F - h)Z \tag{2}$$

To transform the parabolic equation in this coordinate system into the parabolic equation in the original coordinate system, the Z-axis should be translated upward by 300 units.

$$x^2 = 4(F - h)(Z + 300) \tag{3}$$

Step2: The objective function of the optimal ideal paraboloid

In the barrel principle, when the short board is determined, there is not much requirement for the length of other boards, and similarly here When the distance between any node Ni on the ideal paraboloid and the corresponding node on the reference sphere is minimized, then the other nodes the smaller the distance between the parabola and the reference sphere, the smaller the magnitude of the adjustment of the reflective panel. The smaller the distance between the parabola and the reference sphere, the smaller the magnitude of the adjustment needed for the reflection panel, and the faster the adjustment time, which can better achieve the function of real-time monitoring. So, to select the right the distance between the paraboloid and the reference sphere is the minimum to the maximum to obtain the ideal paraboloid [4].

$$\min \max \left\| v_i - v_i^0 \right\| \tag{4}$$

Step3: Constraint analysis of the optimal ideal paraboloid

The distance between neighboring nodes may change slightly after the coordinates of the main cable

nodes are adjusted, and the change is no more than 0.07%, let V be the set of main cable nodes vi in the parabola and V0 be the set of main cable points vi0 in the datum.

$$\| \ \| v_i - v_j \| - \| v_i^0 - v_j^0 \| \le 0.07\% \| v_i^0 - v_j^0 \| \tag{5}$$

Parabolic equation determination.

$$x^2 + y^2 = 4(F - h)(Z + 300) \tag{6}$$

3.1.2. Model solving

Step1: Circular traversal algorithm

With the point C as the origin of the 3D coordinate axis, and the lowest point of the sphere of the reference state as the initial point on the z-axis, traverse up and down the range of up to 10m from the origin with a traversal step of 0.1m, take the distance between the current value on the z-axis and the point P on the feeder pod as the focus, bring the coordinates of the traversed focus into the parabola, calculate the difference between the point on the current reference state and the point on the parabola, determine a maximum difference value on each parabola, and filter the maximum error value to be the optimal parabola. difference value, and filter out the maximum error value of the minimum that is the optimal parabola.

Step2: optimal paraboloid indeed

Parabolic traversal diagram is shown in Figure 1.

The equation of the ideal paraboloid is obtained by traversing the focus of the curve through a circular traversal algorithm.

$$z + 300 = 0.00178\left(x^2 + y^2\right) \tag{7}$$

The top and front views of the ideal paraboloid are shown in Figure 2 and 3.
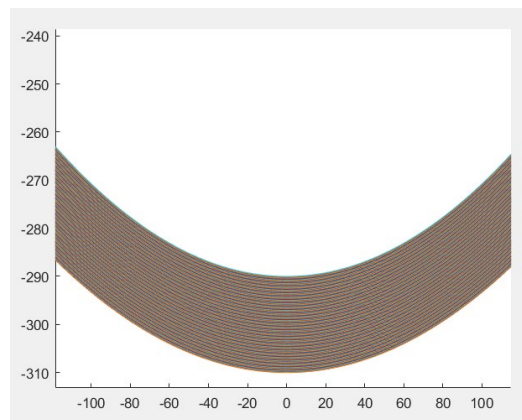

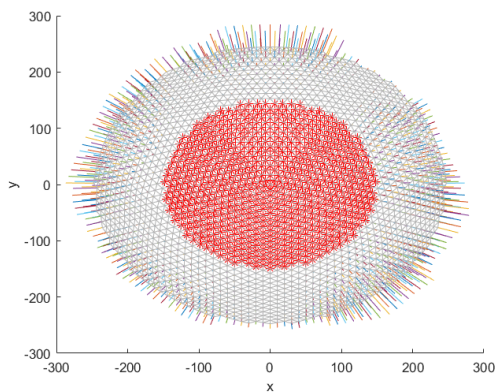*Figure 1: Parabolic traversal diagram*
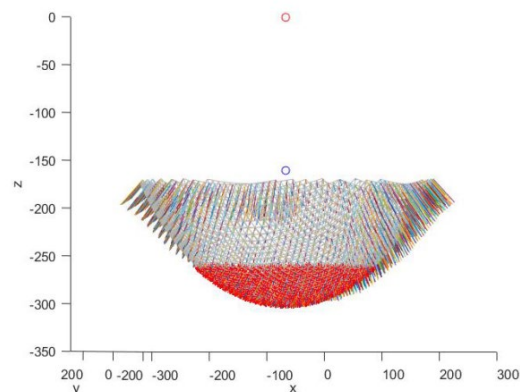

Figure 2: Ideal paraboloid top view

Figure 3: Ideal parabolic front view

3.2. Transformation of the follow-on coordinate system with the global coordinate system

Step1: Convert the global coordinate system

Coordinate conversion to the follower coordinate system If we want to derive the vertex coordinates of the ideal paraboloid [5], it is troublesome to solve the problem directly in the global coordinate system, so we establish the coordinate system that changes with the celestial body. The ideal paraboloid in problem 1 is the optimal paraboloid equation with the feeder bin as the focus, and thus can be considered as the optimal shape. Therefore, when the celestial body is constantly undergoing position change, any 300 m aperture working paraboloid within 500 m aperture should be adjusted to the same shape as the ideal paraboloid. In order to better solve the equation of the ideal paraboloid when the celestial body is in different positions, a follower coordinate system OXYZ is established, and the ideal paraboloid is transformed by coordinates to find the corresponding equation. In order to obtain the equations of the paraboloid in the global coordinate system, we need to obtain the transformation relationship between the follower coordinate system and the global coordinate system. We will translate the system around the z-axis by F units to coincide with the origin of the global coordinate system, and the relationship between the two coordinate systems is shown in Figure 4.
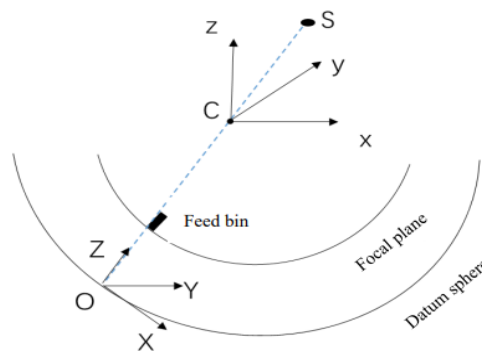

Figure 4: Relationship between two coordinate systems

The diagram of the relationship between the two coordinate systems shows that the transformation from the global coordinate system C_{xyz} to the translational followed coordinate system C_{xyz} requires 2 rotational transformations, first rotating around the z-axis with the rotation matrix T_z, and then rotating around the y-axis by $0.5\pi - \beta$, the rotation matrix T_y.

In order to specifically derive the coordinates of the principal cable node in the translational followed coordinate system, we must derive the transformation matrix T and then find the specific coordinates according to the coordinate transformation formula. The coordinate rotation matrix for the rotation around the y-axis is C_y.

$$C_y = \begin{bmatrix} \cos\theta & 0 & \sin\theta \\ 0 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{bmatrix} \tag{8}$$

The coordinate rotation matrix for rotation about the z-axis is C_z.

$$C_n = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{9}$$

Therefore, in this problem, substituting the rotation angle gives.

$$T_y = \begin{bmatrix} \cos\left(\frac{\pi}{2}-\beta\right) & 0 & \sin\left(\frac{\pi}{2}-\beta\right) \\ 0 & 1 & 0 \\ -\sin\left(\frac{\pi}{2}-\beta\right) & 0 & \cos\left(\frac{\pi}{2}-\beta\right) \end{bmatrix} \tag{10}$$

$$T_z = \begin{bmatrix} \cos\alpha & -\sin\alpha & 0 \\ \sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{11}$$

thus

$$T = T_z T_y = \begin{bmatrix} \cos\alpha\cos\left(\dfrac{\pi}{2}-\beta\right) & -\sin\alpha & \cos\alpha\sin\left(\dfrac{\pi}{2}-\beta\right) \\ \sin\alpha\cos\left(\dfrac{\pi}{2}-\beta\right) & \cos\alpha & \sin\alpha\sin\left(\dfrac{\pi}{2}-\beta\right) \\ -\sin\left(\dfrac{\pi}{2}-\beta\right) & 0 & \cos\left(\dfrac{\pi}{2}-\beta\right) \end{bmatrix} \tag{12}$$

Step2: Convert the coordinates under the follower coordinate system to the global coordinate system

In the follower coordinate system, after we determine the ideal paraboloid, we need to adjust the main cable node of the reference sphere.

The adjustment of the coordinates is made in the following coordinate system, so we also need to convert the new adjusted coordinates to the global coordinate system.

The first step is to translate the z-axis upward by F units, and then simply reverse the rotation according to step 1.

The rotation matrix is the inverse of step 1, and since the transformation matrix is an orthogonal array, the inverse matrix is equal to the transpose matrix.

3.3. Ideal paraboloid based on the model and its coordinate transformation

3.3.1. Building

Step1: Coordinate transformation of the ideal paraboloid

The ideal paraboloid is transformed by coordinates to find the corresponding equation.

$$\begin{cases} x = \cos\alpha\cos\left(\dfrac{\pi}{2}-\beta\right)x' - \sin\alpha\, y' + \cos\alpha\sin\left(\dfrac{\pi}{2}-\beta\right)z' \\ y = \sin\alpha\cos\left(\dfrac{\pi}{2}-\beta\right)x' + \cos\alpha\, y' + \sin\alpha\sin\left(\dfrac{\pi}{2}-\beta\right)z' \\ z = -\sin\left(\dfrac{\pi}{2}-\beta\right)x' + \cos\left(\dfrac{\pi}{2}-\beta\right)z' \end{cases} \tag{13}$$

Step2: Coordinates of the vertex of the ideal paraboloid

Theoretically The vertex of the paraboloid should theoretically find the inflection point of the paraboloid, but we cannot solve for the surface inflection point, therefore, we can associate the feeder module P with the celestial body S with the ideal paraboloid equation to find the intersection point is the vertex coordinate.

3.3.2. Solving

Step1: Transformation matrix of dynamic and global coordinate systems

The transformation matrix T between the dynamic and global coordinate systems is derived from the relationship between the coordinate systems and the celestial bodies.

$$T = \begin{bmatrix} 0.1856 & -0.1388 & -0.9728 \\ 0.5990 & 0.8008 & 0 \\ 0.7790 & -0.5826 & 0.2318 \end{bmatrix} \tag{14}$$

Step2: Ideal paraboloid equation and its vertex coordinates

Based on the model, the ideal paraboloid equation is found by coordinate transformation as follows.

$$0.0537x^2 + 1.0001y^2 + 0.9462z^2 - 0.1926xy - 0.4507xz = 543.99x + 202.65z + 167760 \quad (15)$$

The top view of the ideal paraboloid and the front view of the ideal paraboloid are shown in Figure 5 and Figure 6.
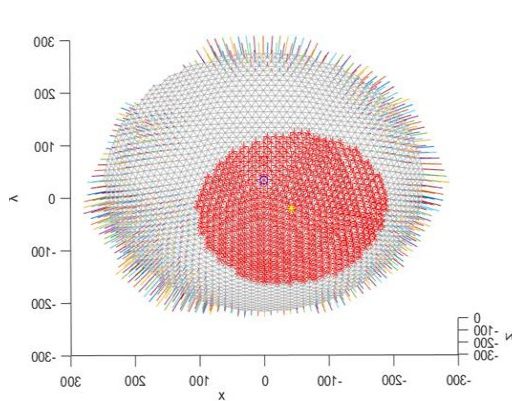

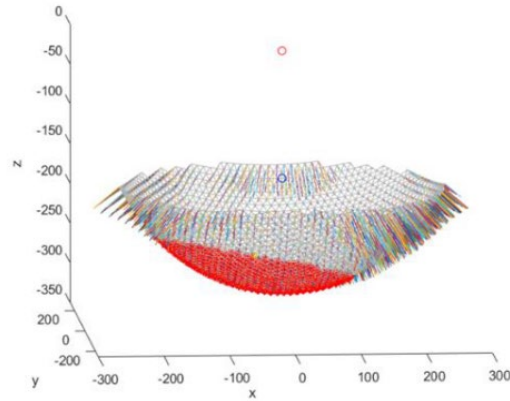
Figure 5: Top view of an ideal paraboloid   *Figure 6: Ideal parabolic front view*

3.4. Dynamic optimization-seeking model for active reflective surfaces

3.4.1. Building

Dynamic adjustment of the main cable network traversing the optimization search.

Step1: The main cable node in the working paraboloid under the determined celestial position.

In the adjustment of the main cable node, we need to determine the main cable node in the working paraboloid of 300m aperture. For simplicity, we take the linear equation of the feeder module P and the celestial body S, and find the intersection point O by associating it with the reference sphere, and the coordinates of the feeder module P=(-lcosαcosβ,-lcosβsinα,-lsinβ), where l=(1-0.466R)=0.534R connects the celestial body to the source point and determines the angle between the celestial body and the coordinate axis by the elevation angle of the celestial body on the positive x-axis and the positive z-axis, we can determine the angle between the celestial body and the horizontal plane as the angle of elevation of the celestial body B. Since the receiving aperture range of the feeder module is 300m, the geometric relationship can be found by O', we can determine that O' point is located in the midpoint of the caliber 300m, with O' as the center point, 150m as the radius to draw a spherical diagram, from the diagram can be obtained under the celestial position of the working parabolic nodes fall in the O Since the Fast eye is a paraboloidal shape, the main cable nodes in the working paraboloid at the celestial position can be obtained by calculating the main cable nodes in the 150m range around the O' point.

Step2: Dynamic adjustment of the main cable node in the dynamic coordinate system

Adjustment coordinates of the main cable node.

In the following coordinate system, the initial position of the main cable node is (X_0, Y_0, Z_0), ΔX is the displacement in X-axis direction, ΔY is the displacement in Y-axis direction and ΔZ is the displacement in Z-axis direction. The adjusted coordinates of the main cable node are $[X, Y, Z] = [X_0+\Delta X, Y_0+\Delta Y, Z_0+\Delta Z]$.

$$\Delta u = \sqrt{\Delta X^2 + \Delta Y^2 + \Delta Z^2} \quad (16)$$

Step3: Iterative optimization of the displacement of the main cable node

Proper adjustment of the length of the main cable node can dynamically change the shape of the mesh, so that the working part of it to the ideal paraboloid, so we traverse the displacement of the main cable node based on the genetic algorithm, the active reflective surface work requires the mesh of the working lighting area to be adjusted to the specified paraboloid shape, so the deviation of the adjusted main cable node position relative to the ideal paraboloid should be minimum, if the adjusted main cable If the

adjusted main cable nodes are all located on the ideal paraboloid, the error is 0,. Equivalently, it can be considered that the closer the adjusted main cable nodes should be to the ideal paraboloid, i.e., the higher the accuracy.

Objective function 1

$$\min \sum_{i=1}^{n} \left(Q_i - Q_i^0 \right)^2 \qquad (17)$$

Minimum deviation of the main cable node position relative to the ideal paraboloid after adjustment

Objective function 2 is based on objective function 1

In the case of closer to the ideal paraboloid, the smaller the adjustment relative to the reference paraboloid, the better, objective function 2 as a supplement to objective function 1, the comprehensive can determine the surface closest to the ideal paraboloid.

$$\min \max \left\| Q_i - V_i^0 \right\| \qquad (18)$$

Constraint analysis

The expansion and contraction of the actuator does not exceed its range -0.6~+0.6m.

$$\mid \Delta u \mid \le 0.06 \qquad (19)$$

The actual adjustment of the distance between the main cable nodes does not exceed 0.07%.

$$\left\| Q_i - Q_j \right\| - \left\| Q_i^0 - Q_j^0 \right\| \le 0.07\% \left\| Q_i^0 - Q_j^0 \right\| \qquad (20)$$

3.4.2. Solving

Algorithm steps.

Step1: Initialize the population to 50 and set the number of iterations to 100.

Step2: Determine the chromosomes, set the coordinates of the points on the paraboloid.

Step3: The chromosomes are randomly crossed two by two, and the mutation operation.

Step4: Calculate the fitness function for selection (the first 50 with the largest evaluation value, i.e., the best from the current (the top 50 with the highest evaluated value, i.e. the smallest distance from the current best paraboloid, are the target) for the next iteration of evolution.

Step6: When the number of iterations is reached, the algorithm stops and outputs the optimal value.

The displacement of the nodes during adjustment is very small, the largest node expansion 0.599m the smallest node expansion is 0.042m, which is easy to achieve in engineering. The displacement of the nodes outside the lighting part is small, but the coordinated movement of these nodes can ensure that the tension distribution of the adjusted cable network is relatively uniform.

4. Conclusion

In this paper, we find the corresponding relationship in the global coordinate system and establish the following coordinate system, so that the position of the celestial body which is not easily determined in the global coordinate system can be analyzed by coordinate transformation. In addition, when changing the position of the main rope points on the paraboloid, we cleverly transform the position of the main rope nodes on the paraboloid to the parabola for solving the problem according to the symmetry, which simplifies the dimensional analysis of the problem. However, when the working paraboloid is adjusted to be close to the ideal paraboloid, the coincidence with the ideal paraboloid needs to be improved.

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