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Research on Sonic Boom Localization of Rocket Debris Based on Multilateration

Zhenjia Zhang[1], Yuting Tang[2]

[1]*College of Engineering, Heilongjiang Bayi Agricultural University, Daqing, 163000, China*
[2]*College of Agriculture, Heilongjiang Bayi Agricultural University, Daqing, 163000, China*

Abstract: *As the world continues to explore space through aeronautics, rocket launch activities have become more frequent. However, the existing debris tracking methods are limited by inaccurate precision and high cost, so a more efficient, precise, and economical tracking technology has become the current research focus. This paper studies the spatial positioning of single and multiple debris separately. The mathematical model for single debris positioning is established based on the multi-side measurement model and the time difference of arrival, and the accurate value is obtained by optimizing it using the least squares method. The multi-debris positioning method takes the data matching approach, matching the series of vibration waves received by the monitoring equipment with the debris, optimizing the single debris positioning mathematical model, and obtaining precise data through the Hungarian Algorithm optimization. Based on the known data, the relative position of the debris to detector A is calculated as 37947.7463 meters east, 5801.3071 meters south, and 963.2116 meters high, with the sound wave occurring 6.5357 seconds earlier than the actual.*

Keywords: *Multilateration, Hungarian Algorithm, Least Squares method, Time Difference of Arrival*

1. Introduction

With the rapid advancement of the aerospace sector, the frequency of rocket launches has surged, encompassing areas such as manned spaceflight, meteorology, launch vehicles, and the pursuit of space exploration. The generation of rocket debris is an inevitable consequence of these launches, and accurately predicting its landing point poses significant challenges. If timely prevention and recovery measures are not implemented, it may result in unforeseen losses. Consequently, precise and prompt localization of rocket debris has emerged as a critical area of research. Conventional techniques for locating rocket debris primarily rely on satellite positioning systems, radar tracking, and visual observation. However, these methods exhibit considerable limitations regarding accuracy, real-time capabilities, and cost-effectiveness. Therefore, developing a more efficient, accurate, and economically viable approach to debris localization holds substantial theoretical research value and practical significance.

This study aims to investigate and optimize the rocket debris positioning method utilizing multi-alteration technology. For the analysis of single debris positioning, a mathematical model is established based on Time Difference of Arrival (TDOA) and multi-lateration principles. The localization of a target node using time difference of arrival (TDoA) measurements received and is still receiving considerable attention[1].TDOA sound source localization employs time differences for accurate positioning[2]. A laser multi-point measurement system is usually composed of multiple laser trackers[3]. Laser tracking measurement technology has found extensive applications across various domains including machinery, aerospace, maritime industries, gear measurement, and calibration, as well as vehicle manufacturing[4]. The model constructed from existing data effectively determines the positional coordinates of debris. The nonlinear least squares method is employed to refine these results, yielding data that closely approximates true values. This method has the advantages of fast convergence and easy implementation[5]. For multiple debris positioning studies, a mathematical model is developed using seismic wave data generated by each detected piece of debris, this involves matching these pieces with data received from detectors. Combine the model with a framework that minimizes the square of the difference between the actual and theoretical arrival times of a sonic boom. Additionally, the Hungarian Algorithm is introduced to enhance data optimization processes. The Hungarian Algorithm (HA) has long been recognized as a standard solution for assignment problems[6].

This study focuses on the localization of space rocket debris, involving a comprehensive series of

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efforts aimed at determining the impact point of the debris. Initially, the positioning of a single piece of debris was examined as a foundational aspect of this research, followed by an analysis of the more complex multi-debris localization problem. Utilizing methods such as TDOA and multilateration models, a mathematical model for spatial positioning that is closely related to it has been established, various optimization algorithms were employed to enhance the accuracy of results about actual values.

2. Multilateration Method for Determining the Sonic Boom Location of Rocket Debris

The data for this study was obtained from http://www.m2ct.org/. The investigation into the positioning of single debris was grounded in multilateration theory to develop a mathematical model. In this research, the location of an acoustic shock wave was determined by analyzing the arrival times recorded by multiple monitoring devices. A critical aspect of this study involved calculating the three-dimensional coordinates of the rocket debris at the moment of the acoustic shock wave, utilizing both the travel time of the shock wave to reach each monitoring device and the known speed of sound to ascertain these spatial coordinates. The foundational theories encompass sound wave propagation models and spatial geometric relationships. The sound wave propagation model can be articulated as a function relating travel time to relative position, with a sound speed set at 340 m/s.The geometric relationship is illustrated in Formula 1, where “(x_1,y_1,z_1)” and “(x_2,y_2,z_2)” represent two points' coordinates in space, while “d” denotes their relative position.

$$\mathbf{d} = \sqrt{(\mathbf{x_2} - \mathbf{x_1})^2 + (\mathbf{y_2} - \mathbf{y_1})^2 + (\mathbf{z_2} - \mathbf{z_1})^2} \tag{1}$$

Given that the location coordinates in the data utilized for this study are expressed in latitude and longitude, they were converted to Cartesian coordinates to facilitate subsequent calculations. Specifically, each degree of latitude was approximated as 11.263 km, while each degree of longitude was approximated as 977304 km. Following this conversion, a fundamental set of equations was established, wherein one equation represents the time difference associated with sound wave propagation, as illustrated in Formula 2. Here,t0 denotes the moment of the acoustic event;(x， y， z) indicates the coordinates of the sound wave source;(x_m,y_m,z_m) represents the position of the m-th device; and t_m signifies the time at which the sound wave reaches that device.

$$t_m = t_0 + \frac{\sqrt{(x - x_m)^2 + (y - y_m)^2 + (z - z_m)^2}}{340} \tag{2}$$

The solution methodology employs the least squares method or other numerical optimization algorithms to derive the most accurate data for (x,y,z,t_0). Spatial positioning necessitates solving for three unknown coordinates (x， y， z) and t0 at which the sonic boom reaches the monitoring equipment, resulting in a total of four unknowns. Consequently, a minimum of four sets of independent (x， y， z， t0) data is required; this implies that at least four monitoring devices must be utilized to provide these independent datasets for solving the objective function. Each equation represents a nonlinear relationship involving the unknowns. To ensure that each equation has a unique solution, it is essential that the selected devices are neither collinear nor coplanar and that their relative positions are as dispersed as possible to mitigate instability or multiple solutions. To enhance the accuracy of locating unknown points, additional monitoring devices may be employed; increasing the number of detection devices can also guarantee stable and precise system operation by supplying redundant data for any anomalous output from specific devices.

A mathematical model was developed, and the equations were solved using Python code. To enhance the accuracy of the results. Three additional monitoring devices were incorporated alongside the existing four, resulting in a total of seven devices for data collection. The time and location of the sonic boom produced by the rocket debris were ascertained utilizing data from these seven devices. The A detector was designated as the coordinate reference point, with relative position offsets calculated to derive the solution. It is observed that the sonic boom from the rocket debris occurred approximately 37,947.7463 meters east of device A,5,801.3071 meters south, and at an elevation of 963.2116 meters. The value of t0 is negative, indicating that the occurrence of the sonic boom preceded actual time by 6.5357 seconds. The three-dimensional visualization is presented in Figure 1, where red stars denote debris and black dots represent monitoring devices.

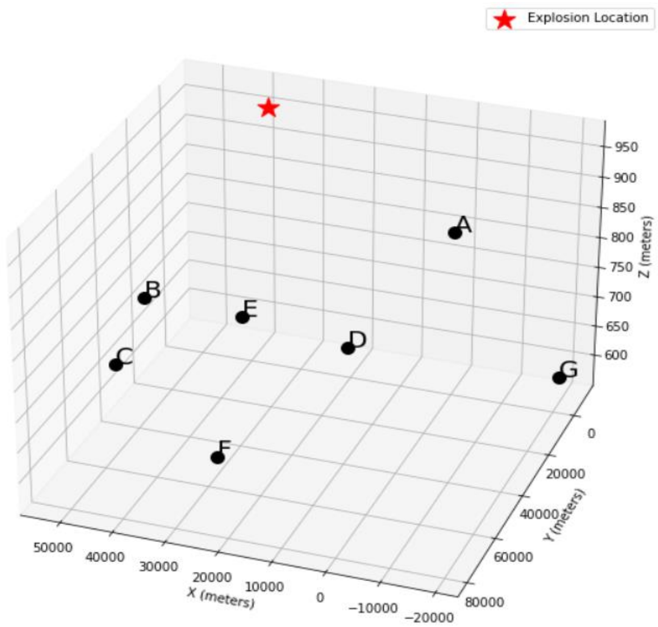

Figure 1: A 3D perspective view of the wreckage relative to the detection equipment.

3. Multi-Source Sonic Boom Localization and Data Matching Based on TDOA

The research on multi-debris localization focuses on optimizing the single debris location model while further expanding the TDOA and multi-variable measurements. This study involves multiple debris-generating sonic booms at different locations simultaneously, with each monitoring device capturing distinct shock waves from various debris sources. Consequently, the core objective of this research is to utilize the multi-group vibration wave data received by the monitoring devices to ascertain both the spatial coordinates and sonic boom timing for each piece of debris. Initially, the relative time differences between the acoustic emission source and each element of the sensor array are estimated; subsequently, distance differences between the acoustic emission source and sensors are calculated based on these time differences. Finally, search or geometric algorithms are employed to determine the location of the acoustic emission source. The mathematical relationship can be expressed in Formula 3, where t_{mn} represents the time taken by device m to receive a sonic boom from debris n, t_{0n} denotes when debris n generates its sonic boom, (x_n, y_n, z_n) indicates the position of debris n, and (x_m, y_m, z_m) signifies the position of device m

$$c \cdot (t_{mn} - t_{0n}) = \sqrt{(x_m - x_n)^2 + (y_m - y_n)^2 + (z_m - z_n)^2} \tag{3}$$

The solution process entails matching a series of sonic boom time data received from all monitoring devices with multiple debris sonic boom sources. Utilizing the optimized mathematical model, the difference between the theoretical and actual times of sonic boom occurrence can be analyzed to determine the corresponding location of each event, as expressed in Formula 4, where 'c' denotes the speed of sound.

$$\textbf{minimize} \sum_{m=1}^{j} \sum_{n=1}^{i} \left(t_{mn} - \left(t_{0n} + \frac{1}{c}\sqrt{(x_n - x_m)^2 + (y_n - y_m)^2 + (z_n - z_m)^2}\right)\right)^2 \tag{4}$$

In the investigation of multiple debris localization, the actual problem is reformulated as a system of equations to be solved using the nonlinear least squares method. And thus, the coordinates of the sonic boom and the time of its occurrence can be obtained. The results derived from the theoretical model may exhibit certain deviations; therefore, the Hungarian Algorithm is employed to optimize these computational outcomes.

To ensure the accuracy of solving this problem, additional equipment was deployed to provide sufficient measurement data, thereby enhancing the conformity of the obtained data with a normal distribution. To assess the feasibility of the model, a case study was introduced for validation. It was assumed that the coordinates of the monitoring equipment remained fixed, as illustrated in Table 1, while random sonic boom occurrence times were generated, as presented in Table 2. These random sonic boom

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occurrence times were then matched, and the results are displayed in Table 3 and Figure 2. Due to the varying spatial positions of the detectors and debris, the sequence in which seismic waves emitted by identical debris are received by different detectors also differs. This is illustrated in Table 2, where the first piece of debris emits seismic waves that are detected by Detectors A and B at the second and third time slots, respectively. By correlating the unique time sequences recorded by each detector with their corresponding debris, we can derive Table 3 and Figure 2, which present the arrival times of seismic signals from each piece of debris as detected by the sensors along with their respective matching results.

Table 1: Coordinates of the detection device

Equipment	Lon	Lat	Alt
A	110.241	27.204	824
B	110.783	27.456	727
C	110.762	27.785	742
D	110.251	28.025	850
E	110.524	27.617	786
F	110.467	28.081	678
G	110.047	27.521	575

Table 2: Randomly generated sonic boom reception time

Equipment		Time Matches	Wreckage Matches
0	A	[Time1,Time2,Time3,Time4,]	[3,1,2,4]
1	B	[Time1,Time2,Time3,Time4,]	[4,3,1,2]
2	C	[Time1,Time2,Time3,Time4,]	[4,3,2,1]
3	D	[Time1,Time2,Time3,Time4,]	[4,3,1,2]
4	E	[Time1,Time2,Time3,Time4,]	[1,3,2,4]
5	F	[Time1,Time2,Time3,Time4,]	[4,3,1,2]
6	G	[Time1,Time2,Time3,Time4,]	[1,3,2,4]

Table 3: Matching Results of Data

Equipment		Time1	Time2	Time3	Time4
0	A	150.936355	111.565374	230.432363	158.328941
1	B	259.260655	266.900281	278.740169	166.141001
2	C	246.042444	248.342920	110.128823	104.564835
3	D	194.893175	266.644883	114.906284	138.599163
4	E	99.571362	135.860739	101.472072	130.776822
5	F	185.728153	194.319047	257.607974	202.571378
6	G	236.668653	218.162256	122.248048	169.905004

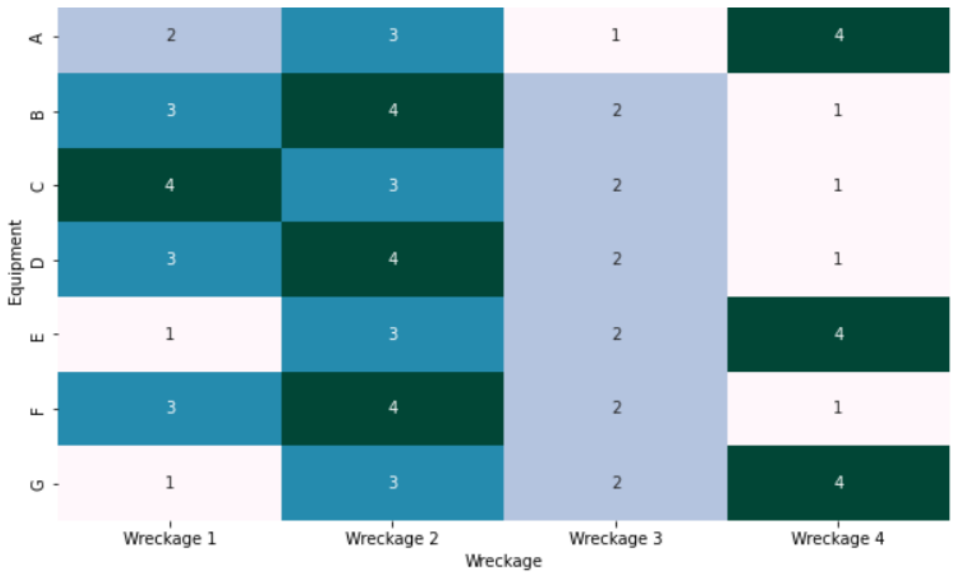

Figure 2: Matched result heatmap

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4. Conclusions

This study established a mathematical model for acoustic emission source localization based on the Time Difference of Arrival and multi-sensor measurement techniques. Through analytic geometry methods, it was determined that a minimum of four monitoring devices is required to accurately obtain the spatial coordinates and sonic boom timing of a single rocket debris. The model employed the Least Squares method to reverse-calculate the time data received by the monitoring devices, thereby estimating the position coordinates of the debris. Simulation results validated the accuracy of this model, demonstrating its capability to successfully determine the position of debris using time data collected from monitoring devices. In this study, the TDOA method was extended and the Hungarian algorithm was incorporated to optimize the results, thereby a framework for minimizing the sum of the square differences between the actual arrival time of the sonic boom and the theoretical arrival time was established. The model developed in this research is capable of distinguishing among the series of sonic boom data received by monitoring equipment and matching them with their corresponding debris.

This study offers a research framework and methodology applicable to multi-target positioning fields. Through investigations into the spatial localization of both single and multiple debris, the established model can accurately ascertain debris locations. Experimental results validate the feasibility of the positioning method employed in this research.

References

[1] Khalaf-Allah M. Particle filtering for three-dimensional TDoA-based positioning using four anchor nodes [J]. Sensors, 2020, 20(16): 4516.

[2] Xu Xuehui, Mei Zhenlong. Design and Realization of Sound Source Positioning and Tracking System Based on Generalized Cross-Correlation Time Delay Estimation (TDOA) Algorithm [J].Journal of Wuhan Polytechnic College, 2024, 23(2): 25-34.

[3] Yan Yang, Xia Wenjie, Lin Hu. Research on the Layout of Calibration Points for Laser Tracking Multi-edge Asynchronous Measurement System [J]. Chinese Journal of Measurement Science, 2023, 44(8): 1188-1195.

[4] Li Xiaoyu, Lin Hu, Xue Zi, et al. Optimization Method of Self-calibration for Laser Tracking Multi-point Measurement [J].Journal of Instrument and Control, 2021, 42(2): 10-17.

[5] Tao Yupan, Yu Jinghu, Ding Yiming. Discussion on Parameter Resolution of Nonlinear Least Squares Method in Exponential Mixture Model [J]. Statistics and Decision-Making, 2020, 36(17): 24-27.

[6] Cheng Guo. Intelligence Decision Method of Public Security Based on Assignment Problem [J]. Western Academic Journal, 2023, (22): 26-29.